Macroeconometric modelling for policy

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Abstract

The first part of this chapter sets out a coherent approach to dynamic macroeconometric model building; the second part demonstrates the approach through building and evaluating a small econometric model; the final part demonstrates various usages of the model for policy.

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"I think it should be generally agreed that a model that does not generate many properties of actual data cannot be claimed to have any 'policy implications'..."

Clive.W. J. Granger (1992), p. 4.

1 Introduction

Depending upon its properties, a macroeconometric model can highlight various aspects of economic policy: communication of policy actions, structuring of economic debate, policy simulations, testing of competing theories, forecasting, stress testing, and so on. From an academic perspective, the desired properties of a model are also legion, but the end result will depend upon the preferences for coherence along dimensions like: theory foundations (microfoundations/aggregation/general/partial), econometric methods (Bayesian/classical), and model properties (size/robustness/non-linearities/transparency/dynamics)

Satisfying the different needs and desires of policy-making could therefore entail a collection of models. Such a model collection could include more or less calibrated theory models, Structural and Bayesian Vector Autoregressions (VARs), Simultaneous Equation Models(SEMs), and Dynamic Stochastic General Equilibrium (DSGEs) models—see Pagan (2003) for an overview. A choice of model(s) for the event at hand could then be made on the basis of strengths and weaknesses of the various candidates. The inherent weaknesses of the main candidates are well known. If one were to follow Ambrose Bierce, and write a "Devil's Dictionary" of macroeconometrics, some of the entries could read: Structural VARs: How to estimate models inefficiently; SEMs: Estimates of something; Bayesian estimation: See calibration; DSGEs: Sophisticated naivety. Therefore, a choice of model(s) for the event at hand should be made on the basis of strengths and weaknesses of the various model classes.

The profession's collective understanding of the causes and possible remedies of model limitations, both in forecasting or in policy analysis, has improved markedly over the last decades. The Lucas (1976) critique and the Clements and Hendry (1999) analysis of the sources of forecast failures with macroeconometric models are milestones in that process. Interestingly, the methodological ramifications of those two critiques are different: The Lucas-critique have led to the current dominance of representative agents based macroeconomic models. Hendry (2001a), on the other hand, concludes that macroeconometric systems of equations, despite their vulnerability to regime shifts, but because of their potential adaptability to breaks, remain the best long-run hope for progress in macroeconomic forecasting. Since monetary policy can be a function of the forecasts, as with inflation forecast targeting, cf. Svensson (1997), the choice of forecasting model(s) is important.

The class of macroeconometric models we present in this chapter requires coherent use of economic theory, data, and mathematical and statistical techniques. This approach of course has a long history in econometrics, going back to Tinbergen's first macroeconometric models, and have enjoyed renewed interest in the last decades. Recent advances in econometrics and in computing means that we now are much better tools than say 20 years ago, for developing and maintaining macroeconometric models in this tradition—see Garratt et al. (2006) for one recent approach.

Regardless of underlying theory, a common aim of macroeconometric model building is identification of invariant relationships, if they exist at all, see Haavelmo (1944, Chapter II). A well-specified macroeconometric model is a good starting point of such a quest, since it provides an ideal test bed for further overidentifying restrictions of microeconomic behaviour. Such a strategy is in particular relevant with the challenges from behavioural economics, with implications like: time inconsistency (hyperbolic discounting), changing expectations (learning), asset bubbles (herd behaviour), and so on.

Macroeconomic models of the representative agent, intertemporal optimizing, type are said to have structural interpretations, with 'deep structural parameters' that are immune to the Lucas critique. However, when the model's purpose is to describe the observed macroceconomic behaviour, its structural properties are conceptually different. Heuristically, we take a model to have structural properties if it is invariant and interpretable—see Hendry (1995b). Structural properties are nevertheless relative to the history, the nature and the significance of regime shifts. There is always the possibility that the next shocks to the system may incur real damage to a model with high structural content hitherto. The approach implies that a model's structural properties must be evaluated along several dimensions, and the following seem particularly relevant:

1. Theoretical interpretation.

2. Ability to explain the data.

3. Ability to explain earlier findings, i.e., encompassing the properties of existing modes.

4. Robustness to new evidence in the form of updated/extended data series and new economic analysis suggesting e.g., new explanatory variables.

Economic analysis (#1) is an indispensable guidance in the formulation of econometric models. Clear interpretation also helps communication of ideas and results among researchers, in addition to structuring debate. However, since economic theories are necessarily simplifying abstractions, translations of theoretical to econometric models must lead to problems like biased coefficient estimates, wrong signs of coefficients, and/or residual properties that hampers valid inference. The main distinction seems to be between seeing theory as representing *the* correct specification, (leaving parameter estimation to the econometrician), and viewing theory as a guideline in the specification of a model which also accommodates institutional features, attempts to accommodate heterogeneity among agents, addresses the temporal aspects for the data set and so on—see Granger (1999).

Arguments against "largely empirical models" include sample dependency, lack of invariance, unnecessary complexity (in order to fit the data) and chance finding of "significant" variables. Yet, ability to characterize the data (#2) remains an essential quality of useful econometric models, and given the absence of theoretical truisms, the implications of economic theory have to be confronted with the data in a systematic way.

We use cointegration methods on linearized and discretized dynamic systems to estimate theory-interpretable and identified steady state relationships, imposed in the form of equilibrium-correction models. We also make use of an automated model-selection approach to sift out the best theory-interpretable and identified dynamic specifications. Hoover and Perez (1999), Hendry and Krolzig (1999) and Doornik (2008) have shown that automated model selection methods has a good chance of finding a close approximation to the data generating process, and that the danger of over-fitting is in fact (surprisingly) low. Conversely, acting *as if* the specification is given by theory alone, with only coefficient estimates left to "fill in", is bound to result in the econometric problems noted above, and to a lower degree of relevance of the model for the economy it claims to represent.

In order to develop scientific basis for policy modelling in macroeconometrics, a new model's capability of encompassing earlier findings should be regarded as an important aspect of structure (#3). There are many reasons for the coexistence of contested models for the same phenomena, some of which may be viewed as inherent (limited number of data observations, measurement problems, controversy about operational definitions, new theories). Nevertheless, the continued use a corroborative evaluation (i.e., only addressing goodness of fit or predicting the stylized fact correctly) may inadvertently hinder cumulation of evidence taking place. One suspects that there would be huge gains from a breakthrough for new standards of methodology and practice in the profession.

Ideally, empirical modelling is a cumulative process where models continuously become overtaken by new and more useful ones. By useful, we understand in particular models that are relatively invariant to changes elsewhere in the economy, i.e., they contain autonomous parameters, see Haavelmo (1944), Johansen (1977), Aldrich (1989), Hendry (1995b). Models with a high degree of autonomy represent structural properties: They remain invariant to changes in economic policies and other shocks to the economic system, as implied by #4 above.[1] However, structure is partial in two respects: First, autonomy is a relative concept, since an econometric model cannot be invariant to every imaginable shock. Second, all parameters of an econometric model are unlikely to be equally invariant, and only the parameters with the highest degree of autonomy represent structure. Since elements of structure typically will be grafted into equations that also contain parameters with a lower degree of autonomy, forecast breakdown may frequently be caused by shifts in these non-structural parameters.[2]

A strategy for model evaluation that puts a lot of emphasis on forecast behaviour, without a careful evaluation of the causes of forecast failure *ex post*, runs a risk of discarding models that actually contain important elements of structure. Hence, for example Doornik and Hendry (1997) and Clements and Hendry (1999, Ch. 3) show that the main source of forecast failure is location shifts (shifts in means of levels, changes, etc.), and not shifts in such coefficients that are of primary concern in policy analysis, i.e., the derivative coefficients of behavioural equations. Therefore, a rough spell in terms of forecasting performance does not by itself disqualify the model's relevance for policy analysis. If the cause of the forecast failure is location shifts, they can be attenuated *ex post* by intercept correction or additional differencing 'within' the model, Hendry (2004). With these add-ons, and once the break-period is in the information set, the model forecast will adapt to the new regime and improve again. Failure to adapt to the new regime, may then be a sign of a deeper source of forecast failure, in the form of non-constant derivative coefficients which also undermines the models relevance for policy analysis.[3] In general, without adaptive measures, models with high structural content will lose regularly to simple forecasting rules, see e.g., Clements and Hendry (1999), Eitrheim et al. (1999). Hence different models may be optimal for forecasting and for policy analysis, which fits well with the often heard recommendation of a suite of monetary policy models.

Structural breaks are always a main concern in econometric modelling, but like any hypothesis of theory, the only way to judge the significance of a hypothesized break is by confrontation with the evidence in the data. Moreover, given that an encompassing approach is followed, a forecast failure is not only destructive but represent a potential for improvement, if successful respecification follows in its wake, cf. Eitrheim et al. (2002). In the same vein, one important intellectual rationale for DSGE models is the Lucas critique. If the Lucas critique holds, any "reduced-form" equation in a model is liable to be unstable also over the historical sample, due to regime shifts and policy changes that have taken place in the economy. Hence according to the Lucas critique, parameter instability may be endemic in any model that fails to obey the Rational Expectations Hypothesis (REH), with the possible consequence that without integration of REH, the model is unsuited for policy analysis. However, as stated by Ericsson and Irons (1995), the Lucas critique is only a possibility theorem, not a truism, and that the implications of the Lucas critique can be

[1] see e.g., Hendry (1995a, Ch. 2,3 and 15.3) for a concise definition of structure as the invariant set of attributes of the economic mechanism.

[2] This line of thought may lead to the following practical argument against large-scale empirical models: Since modelling resources are limited, and some sectors and activities are more difficult to model than others, certain euations of any given model are bound to have less structural content than others, i.e., the model as a whole is no better than its weakest (least structural) equation.

[3] See Nymoen (2005) for an analysis of a recent failure in inflation forecasting.

tested, see also for example Hendry (1988), Engle and Hendry (1993) and Ericsson and Hendry (1999). In Bårdsen et al. (2003) we have shown, by extensive testing of a previous version, that the Lucas critique has little force for our system of equations. This finding is consistent with the international evidence presented in Ericsson and Irons (1995) and Stanley (2000). On the basis of these results, our model is more consistent with agents adopting robust forecasting rules, in line with the analysis and suggestions of Hendry and Mizon (2000). In that case the Lucas critique does not apply, although the degree of autonomy remains an issue that needs to be evaluated as fully as possible, given the information available to us.

This chapter documents the approach we use to dynamic macroeconometric model building and policy analysis. To make the analysis applied, the approach is illustrated through a model of the Norwegian economy. Our approach is of course applicable to other economies, but we know more about market characteristics, policy changes and institutional development in Norway, than in any other country or economic area. And since such factual knowledge is an indispensable and complementary aid and to formal econometrics in the building of an empirical model, we prefer to work with the economy we have most knowledge about.

The rest of the chapter consists of three main parts. The first part sets out a coherent approach to dynamic macroeconometric model building; the second part demonstrates the approach through building and evaluating a small econometric model; the final part demonstrates various tools for policy analysis using the model. The first part involves three steps in going from general to specific. The first step is theoretical and establishes a framework for linearizing and discretizing an approximation to a general theory model with constant steady state values. The second step is to estimate, and solve, the steady-state model in the form of overidentifying cointegrating relationships and common trends. The third step is to identify and estimate the dynamic structure of the model.

The second part of the chapter illustrates the approach set out in the first part by the construction and evaluation of a small open-economy model.

The final part demonstrates five tools for policy using the model: tractability; simulations of policy responses; optimal policy considerations; theory evaluation, and forecasting. The first use is illustrated by introducing a method to construct stylized versions of complex models. The second use is illustrated by evaluating responses to monetary policy shocks. The third use shows how important model specification is for the derivation of optimal monetary policy. The fourth aspect is illustrated by testing the New Keynesian Phillips Curve. The final use evaluates possible sources affecting forecast performance.

2 A modelling framework

As the values of all major economic variables are announced regularly, it is easy to believe that a local approximation to a Data Generating Process (DGP) can exist. It is an interesting philosophical question whether the true generating mechanism can (ever) be completely described, but the usefulness of the concept does not hinge on the answer to that question. The main point is that once the real economic world, in its enormous, ever-changing, complexity, is accepted as a premise for macroeconomic modelling, it follows that the main problems of macroeconometrics are model specification and model evaluation, rather than finding the best estimator under the assumption that the model is identical to the data generating process.

The local DGP is changing with the evolution of the real world economy—through technical progress, changing pattern of family composition and behaviour and political reform. Sometimes society evolves gradually and econometric models are then usually able to adapt to the underlying real-life changes, i.e. the without any noticeable loss in "usefulness" Often, however, society evolves so quickly that estimated economic relationships break down and cease to be of any aid in understanding the current macro economy and in forecasting its development even over the first couple of years. In this case we speak of a changing local approximation in the form of a regime shift in the generating process, and a structural break in the econometric model. Since the complexity of the true macroeconomic mechanism, and the regime shifts also contained in the mechanism, lead us to conclude that any model will at best be a local approximation to the data generating process, judging the quality of, and choosing between, the approximations becomes central.

In the rest of this section we present our approach to finding a local approximation useful for policy.[4]

2.1 Linearization

Consider a very simple example of an economic model in the form of the differential equation

$$\frac{dy}{dt} = f(y,x) \ , \qquad x = x(t) \ , \tag{1}$$

in which a constant input $x = \bar{x}$ induces $y(t)$ to approach asymptotically a constant state $\bar{y}$ as $t \to \infty$. Clearly $\bar{x}$ and $\bar{y}$ satisfy $f(\bar{y},\bar{x}) = 0$. For example, standard DSGE models usually take this form, with the models having deterministic steady state values. The usual procedure then is to expand the differential (or difference) equation about this steady-state solution (see, for example, Campbell (1994) or Uhlig (1999)). Employing this procedure yields

$$f(y,x) = f(\bar{y},\bar{x}) + \frac{\partial f(\bar{y},\bar{x})}{\partial y}(y-\bar{y}) + \frac{\partial f(\bar{y},\bar{x})}{\partial x}(x-\bar{x}) + R \tag{2}$$

where

$$R = \frac{1}{2!}\left(\frac{\partial^2 f(\xi,\eta)}{\partial x^2}(x-\bar{x})^2 + 2\frac{\partial^2 f(\xi,\eta)}{\partial x \partial y}(x-\bar{x})(y-\bar{y}) + \frac{\partial^2 f(\xi,\eta)}{\partial y^2}(y-\bar{y})^2\right)$$

and (ξ,η) is a point such that ξ lies between y and $\bar{y}$ while η lies between x and $\bar{x}$. Since $\bar{y}$ and $\bar{x}$ are the steady-state values for y and x respectively, then the expression for $f(y,x)$ takes the simplified form

$$f(y,x) = a\,(y-\bar{y}) + \delta\,(x-\bar{x}) + R \tag{3}$$

where $a = \partial f(\bar{y},\bar{x})/\partial y$ and $\delta = \partial f(\bar{y},\bar{x})/\partial x$ are constants.

If f is a linear function of y and x then $R = 0$ and so

$$f(x,y) = a\left(y - \bar{y} + \frac{\delta}{a}(x-\bar{x})\right) = a\,(y - bx - c) \ , \tag{4}$$

in which $b = -\delta/a$ and $c = \bar{y} + (\delta/a)\bar{x}$.

2.2 Discretization

For a macroeconometric model, a discrete representation is usually practical, and it can be worked out as follows. Let $t_1,\ t_2,\ \cdots,\ t_k,\ \cdots$ be a sequence of times spaced h apart and let $y_1,\ y_2,\ \cdots,\ y_k,\ \cdots$ be the values of a continuous real variable $y(t)$ at these times. The backward-difference operator $\triangle$ is defined by the rule

$$\triangle y_k = y_k - y_{k-1} \ , \qquad k \geq 1 \ . \tag{5}$$

[4]Section 2.1-2.4 draws on Bårdsen et al. (2004a).

By observing that $y_k = (1-\triangle)^0\, y_k$ and $y_{k-1} = (1-\triangle)^1\, y_k$, the value of y at the intermediate point $t = t_k - sh$ $(0 < s < 1)$ may be estimated by the interpolation formula

$$y(t_k - sh) = y_{k-s} = (1-\triangle)^s\, y_k\ , \qquad s \in [0,1]\ . \tag{6}$$

When s is not an integer, $(1-\triangle)^s$ should be interpreted as the power series in the backward-difference operator obtained from the binomial expansion of $(1-x)^s$. This is an infinite series of differences. Specifically

$$(1-\triangle)^s = 1 - s\triangle - \frac{s(1-s)}{2!}\triangle^2 - \frac{s(1-s)(2-s)}{3!}\triangle^3 - \cdots\ . \tag{7}$$

With this preliminary background, the differential equation

$$\frac{dy}{dt} = f(y,x)\ , \qquad x = x(t)\ , \tag{8}$$

may be integrated over the time interval $[t_k, t_{k+1}]$ to obtain

$$y(t_{k+1}) - y(t_k) = \triangle y_{k+1} = \int_{t_k}^{t_{k+1}} f(y(t), x(t))\, dt \tag{9}$$

in which the integral on the right hand side of this equation is to be estimated by using the backward-difference interpolation formula given in equation (7). The substitution $t = t_k + sh$ is now used to change the variable of this integral from $t \in [t_k, t_{k+1}]$ to $s \in [0,1]$. The details of this change of variable are

$$\int_{t_k}^{t_{k+1}} f(y(t), x(t))\, dt = \int_0^1 f(y(t_k+sh), x(t_k+sh))\,(h\, ds) = h\int_0^1 f_{k+s}\, ds$$

where $f_{k+s} = f(y(t_k+sh), x(t_k+sh))$. The value of this latter integral is now computed using the interpolation formula based on (7). Thus

$$\begin{aligned}\int_0^1 f_{k+s}\, ds &= \int_0^1 (1-\triangle)^{-s}\, f_k\, ds \\ &= \int_0^1 \left(f_k + s\triangle f_k + \frac{s(1+s)}{2!}\,\triangle^2 f_k + \frac{s(1+s)(2+s)}{3!}\,\triangle^3 f_k + \cdots \right) ds \\ &= f_k + \frac{1}{2}\,\triangle f_k + \frac{5}{12}\,\triangle^2 f_k + \frac{3}{8}\,\triangle^3 f_k + \cdots\ .\end{aligned}$$

The final form for the backward-difference approximation to the solution of this differential equation is therefore

$$\triangle y_{k+1} = hf_k + \frac{h}{2}\,\triangle f_k + \frac{5h}{12}\,\triangle^2 f_k + \frac{3h}{8}\,\triangle^3 f_k + \cdots\ . \tag{10}$$

2.3 Equilibrium correction representations and cointegration

The discretization scheme (10) applied to the linearized model (3), with $k = t-1$ and $h = 1$, gives the equilibrium correction model, EqCM, representation

$$\begin{aligned}\triangle y_t = a\,(y - bx - c)_{t-1} + R_{t-1} + \frac{1}{2}\,a(\triangle y_{t-1} - b\,\triangle x_{t-1}) + \frac{1}{2}\,\triangle R_{t-1} \\ + \frac{5}{12}\,a(\triangle^2 y_{t-1} - b\,\triangle^2 x_{t-1}) + \frac{5}{12}\,\triangle^2 R_{t-1} + \cdots\ .\end{aligned}$$

At this point two comments are in place. The first is that an econometric specification will mean a truncation of the polynomial both in terms of powers and lags. Diagnostic testing is therefore imperative to ensure a valid local approximation, and indeed to test that the statistical model is valid, see Hendry (1995a, p. 15.1) and Spanos (2008). The second is that the framework allows for flexibility regarding the form of the steady state. The standard approach in DSGE-modelling has been to filter the data, typically using the so-called Hodrick-Prescott filter, to remove trends, hopefully achieving stationary series with constant means, and then work with the filtered series. Another approach, popular at present, is to impose the theoretical balanced growth path of the model on the data, expressing all series in terms of growth corrected values. However, an alternative approach is to estimate the balanced growth paths in terms of finding the number of common trends and identifying and estimating cointegrating relationships. The present approach allows for all of these interpretations.

To illustrate the approach in terms of cointegration, consider real wages to be influenced by productivity, as in many theories.[5] Assume that the logs of the real wage rw_t and productivity z_t are each integrated of order one, but found to be cointegrated, so

$$rw_t \sim I(1), \Delta rw_t \sim I(0) \tag{11}$$

$$z_t \sim I(1), \Delta z_t \sim I(0) \tag{12}$$

$$(rw - \beta z)_t \sim I(0). \tag{13}$$

Letting $y_t \equiv (rw - \beta z)_t$ and $x_t \equiv \Delta z_t$ then gives

$$\Delta rw_t = -ac + a(rw - \beta z)_{t-1} + \frac{a}{2} \triangle (rw - \beta z)_{t-1} + \beta \Delta z_t - ab \triangle z_{t-1} - \frac{ab}{2} \Delta^2 z_{t-1} + \cdots .$$

2.4 System representations

The approach easily generalizes to a system representation. For ease of exposition, we illustrate the two-dimensional case for which $y_1 \to \bar{y}_1$ and $y_2 \to \bar{y}_2$ as $t \to \infty$. Expanding with respect to y_1 and y_2 about their steady-state values yields

$$\begin{bmatrix} f_1(y_1, y_2) \\ f_2(y_1, y_2) \end{bmatrix} = \begin{bmatrix} f_1(\bar{y}_1, \bar{y}_2) \\ f_2(\bar{y}_1, \bar{y}_2) \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1(\bar{y}_1, \bar{y}_2)}{\partial y_1} & \frac{\partial f_1(\bar{y}_1, \bar{y}_2)}{\partial y_2} \\ \frac{\partial f_2(\bar{y}_1, \bar{y}_2)}{\partial y_1} & \frac{\partial f_2(\bar{y}_1, \bar{y}_2)}{\partial y_2} \end{bmatrix} \begin{bmatrix} y_1 - \bar{y}_1 \\ y_2 - \bar{y}_2 \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \end{bmatrix},$$

where $[R_1, R_2]'$ denotes the vector

$$\frac{1}{2!} \begin{bmatrix} \frac{\partial^2 f_1(\zeta, \eta)}{\partial y_1^2} (y_1 - \bar{y}_1)^2 + 2 \frac{\partial^2 f_1(\zeta, \eta)}{\partial y_1 \partial y_2} (y_1 - \bar{y}_1)(y_2 - \bar{y}_2) + \frac{\partial^2 f_1(\zeta, \eta)}{\partial y_2^2} (y_2 - \bar{y}_2)^2 \\ \frac{\partial^2 f_2(\zeta, \eta)}{\partial y_1^2} (y_1 - \bar{y}_1)^2 + 2 \frac{\partial^2 f_2(\zeta, \eta)}{\partial y_1 \partial y_2} (y_1 - \bar{y}_1)(y_2 - \bar{y}_2) + \frac{\partial^2 f_2(\zeta, \eta)}{\partial y_2^2} (y_2 - \bar{y}_2)^2 \end{bmatrix}$$

so that

$$\begin{bmatrix} \frac{\partial y_1}{\partial t} \\ \frac{\partial y_2}{\partial t} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} y_1 - \bar{y}_1 \\ y_2 - \bar{y}_2 \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}.$$

[5] Presently, we let the unemployment rate be constant and disregard it for simplicity. We return to the role of the rate of unemployment in paragraph 2.6 below.

The backward-difference approximation to the solution of the system of differential equations gives the system in EqCM form[6], namely,

$$\begin{bmatrix} \triangle y_1 \\ \triangle y_2 \end{bmatrix}_t = \begin{bmatrix} -\alpha_{11}c_1 \\ -\alpha_{22}c_2 \end{bmatrix} + \begin{bmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{bmatrix} \begin{bmatrix} y_1 - \delta_1 y_2 \\ y_2 - \delta_2 y_1 \end{bmatrix}_{t-1} + \begin{bmatrix} R_1 \\ \\ R_2 \end{bmatrix}_{t-1}$$
$$+ \frac{1}{2} \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix}_{t-1} + \begin{bmatrix} \Delta R_1 \\ \\ \Delta R_2 \end{bmatrix}_{t-1}$$
$$+ \frac{5}{12} \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \Delta^2 y_1 \\ \Delta^2 y_2 \end{bmatrix}_{t-1} + \begin{bmatrix} \Delta^2 R_1 \\ \\ \Delta^2 R_2 \end{bmatrix}_{t-1}$$
$$+ \frac{3}{8} \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \Delta^3 y_1 \\ \Delta^3 y_2 \end{bmatrix}_{t-1} + \begin{bmatrix} \Delta^3 R_1 \\ \\ \Delta^3 R_2 \end{bmatrix}_{t-1} + \cdots .$$

with

$$\begin{array}{ll} c_1 = (\bar{y}_1 + \delta_1 \bar{y}_2)\,, & \delta_1 = \frac{\alpha_{12}}{\alpha_{11}} \\ c_2 = (\bar{y}_2 + \delta_2 \bar{y}_1)\,, & \delta_2 = \frac{\alpha_{21}}{\alpha_{22}} \end{array}$$

As before, the variables y_1 and y_2 can be considered as stationary functions of non-stationary components—cointegration is imposed upon the system. Consider the previous example, assuming linearity, so $R_i = 0$, and ignoring higher order dynamics for ease of exposition:

$$\begin{bmatrix} \triangle y_1 \\ \triangle y_2 \end{bmatrix}_t = \begin{bmatrix} -\alpha_{11}c_1 \\ -\alpha_{22}c_2 \end{bmatrix} + \begin{bmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{bmatrix} \begin{bmatrix} y_1 - \delta_1 y_2 \\ y_2 - \delta_2 y_1 \end{bmatrix}_{t-1}$$
$$\begin{bmatrix} \triangle (rw - \beta z) \\ \Delta^2 z \end{bmatrix}_t = \begin{bmatrix} -\alpha_{11}c_1 \\ -\alpha_{22}c_2 \end{bmatrix} + \begin{bmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{bmatrix} \begin{bmatrix} (rw - \beta z) - \delta_1 \Delta z \\ \Delta z - \delta_2 (rw - \beta z) \end{bmatrix}_{t-1}$$

or multiplied out:

$$\begin{aligned} \triangle rw_t &= -\alpha_{11}c_1 + \alpha_{11}(rw - \beta z)_{t-1} + \beta \Delta z_t - \alpha_{12} \Delta z_{t-1} \\ \Delta z_t &= -\alpha_{22} \left(\bar{y}_2 + \frac{\alpha_{21}}{\alpha_{22}} \bar{y}_1 \right) + (\alpha_{22} - 1)\, \Delta z_{t-1} - \alpha_{21}(rw - \beta z)_{t-1} \end{aligned}$$

So if $\alpha_{21} = 0$ and $|\alpha_{22} - 1| < 1$ the system simplifies to the familiar exposition of a bivariate cointegrated system with z being weakly exogenous for β:

$$\begin{aligned} \triangle rw_t &= -\alpha_{11}c_1 + \alpha_{11}(rw - \beta z)_{t-1} + \beta \Delta z_t - \alpha_{12} \Delta z_{t-1} \\ \Delta z_t &= -\alpha_{22}\, \bar{z} + (\alpha_{22} - 1)\, \Delta z_{t-1} \end{aligned}$$

and where the common trend is a productivity trend.

2.5 From a discretized and linearized cointegrated VAR representation to a dynamic SEM in three steps

We will keep this section brief, as comprehensive treatments can be found many places—for example in Hendry (1995a), Johansen (1995, 2006), Juselius (2007), Garratt et al. (2006), and Lütkepohl (2006)—and only make some comments on issues in each step in the modelling process we believe merit further attention.

[6] See Bårdsen et al. (2004a) for details.

2.5.1 First step: the statistical system

Our starting point for identifying and building a macroeconometric model is to find a linearized and discretized approximation as a data-coherent statistical system representation in the form of a cointegrated VAR

$$\Delta \mathbf{y}_t = \mathbf{c} + \Pi \mathbf{y}_{t-1} + \sum_{i=1}^{k} \Gamma_{t-i} \Delta \mathbf{y}_{t-i} + \mathbf{u}_t, \tag{14}$$

with independent Gaussian errors $\mathbf{u}_t$ as a basis for valid statistical inference about economic theoretical hypotheses.

The purpose of the statistical model (14) is to provide the framework for hypothesis testing, the inferential aspect of macroeconometric modelling. However, it cannot be postulated directly, since the cointegrated VAR itself rests on assumptions. Hence, validation of the statistical model is an essential step: Is a model which is linear in the parameters flexible enough to describe the fluctuations of the data? What about the assumed constancy of parameters, does it hold over the sample that we have at hand? And the Gaussian distribution of the errors, is that a tenable assumption so that (14) can supply the inferential aspect of modelling with sufficient statistics. The main intellectual rationale for the model validation aspect of macroeconometrics is exactly that the assumptions of the statistical model requires separate attention, Johansen (2006),Spanos (2006) In practice, one important step in model validation is to make the hypothesized statistical model subject to a battery of misspecification tests using the OLS residuals $\hat{\mathbf{u}}_t$ as data.[7]

As pointed out by Garratt et al. (2006), the representation (14) does not preclude forward-looking behaviour in the underlying model, as rational expectations models have backward-looking solutions. The coefficients of the solution will be defined in specific ways though, and this entails restrictions on the VAR which can utilized for testing rational expectations, see Johansen and Swensen (1999) and Johansen and Swensen (2004).

Even with a model which for many practical purpose is small scale it is usually too big to be formulated in "one go" within a cointegrated VAR framework. Hence, model (14) for example is not interpretable as one rather high dimensional VAR, with the (incredible) long lags which would be needed to capture the complicated dynamic interlinkages of a real economy. Instead, as explained in Bårdsen et al. (2003), our operational procedure is to partition the (big) simultaneous distribution function of markets and variables: prices, wages, output, interest rates, the exchange rate, foreign prices, and unemployment, etc. into a (much smaller) simultaneous model of wage and price setting—the labour market—and several sub-models of the rest of the macro economy. The econometric rationale for specification and estimation of single equations, or of markets, subject to exogeneity conditions, before joining them up in a complete model is discussed in Bårdsen et al. (2003), and also in Bårdsen et al. (2005, Ch. 2).

2.5.2 Second step: the overidentified steady state

The second step of the model building exercise will then be to identify the steady state, by testing and imposing overidentifying restrictions on the cointegration space:

$$\Delta \mathbf{y}_t = \mathbf{c} + \boldsymbol{\alpha}\boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{k} \Gamma_{t-i} \Delta \mathbf{y}_{t-i} + \mathbf{u}_t,$$

[7] The distinction between the ineferential and model validation facets of modelling is due to Spanos (2008), who conclusively dispels the charge that misspecification testing represents an illegitimate "re-use" of the data already used to estimate the parameters of the statistical model, see also Hendry (1995b, p. 313-314).

thereby identifying both the exogenous common trends, or permanent shocks, and the steady state of the model.

Even though there now exists a literature on identification of cointegration vectors, it is worthwhile to reiterate that identification of cointegrating vectors cannot be data-based. Identifying restrictions have to be imposed *a priori*. It is therefore of crucial importance to have a specification of the economic model and its derived steady state before estimation. Otherwise we will not know what model and hypotheses we are testing and, in particular, we could not be certain that it was identifiable from the available data set

2.5.3 Third step: the dynamic SEM

The final step is to identify the dynamic structure:

$$\mathbf{A}_0\Delta\mathbf{y}_t = \mathbf{A}_0\mathbf{c} + \mathbf{A}_0\boldsymbol{\alpha}\boldsymbol{\beta}^{'}\mathbf{y}_{t-1} + \sum_{i=1}^{k}\mathbf{A}_0\Gamma_{t-i}\Delta\mathbf{y}_{t-i} + \mathbf{A}_0\mathbf{u}_t,$$

by testing and imposing overidentifying restrictions on the dynamic part—including the forward-looking part—of the statistical system.

First, the estimated parameters and therefore the interpretation of the model dynamics are dependent upon the dating of the steady-state solution. However the steady-state multipliers are not—see Bårdsen and Fisher (1993, 1999)

Third, the economic interpretations of the derived paths of adjustment are not invariant to the identification of the dynamic part of the model, whereas the steady-state parts of the model are—again see Bårdsen and Fisher (1993, 1999).

2.6 Example: The supply side of a medium term macroeconomic model

One main focus of an empirical macro model is always going to be the supply side. We end the section on methodology by giving an extended example of the theoretical and econometric specification of a labour-market model that we later include in a macro model, intended for medium term analysis and forecasting. The first step is the specification of the relevant economic theory to test. We next develop the theoretical relationships into hypotheses about cointegration, that can be tested in a statistical model and identified as steady state relationships, Step 1 and 2 above. We also go through Step 3 in detail. Throughout the rest of the chapter we let lower-case letters denote natural logarithms of the corresponding upper-case variable names, so $x_t \equiv \ln(X_t)$.

2.6.1 Economic theory

A main advance in the modelling of labour markets rests on the perception that firms and their workers are engaged in a partly cooperative and partly conflicting sharing of the rents generated by the operation of the firm. In line with this assumption, *nominal wages* are modelled in game theoretic framework which fits the comparatively highly level of centralization and coordination in Norwegian wage setting (see e.g., Nymoen and Rødseth (2003) and Barkbu et al. (2003) for a discussion of the degree of coordination).

The modelling of nominal wage setting in a game theoretic framework is a theoretical advance with several implications. Linked up with an assumption of monopolistically competitive firms, it represents an incomplete competition model of the supply-side, which we refer to as ICM in the following.[8] In applications, the gap between the formal relationships of the theory and the empirical relationships that may be present in the data must be closed. The modelling assumption about $I(1)$-ness introduced above is an important

[8] See Bårdsen et al. (2005, Ch 5 and 6).

part of the bridge between theory and data. This is because $I(1)$-ness allows us to interpret the theoretical wage and price equations as hypothesized cointegration relationships. From that premise, a dynamic model of supply side in equilibrium-correction form follows logically.

There is a number of specialized models of "non-competitive" wage setting. Our aim here is to represent the common features of these approaches by extending the model in Nymoen and Rødseth (2003) with monopolistic competition among firms.

We start with the assumption of a large number of firms, each facing downward-sloping demand functions. The firms are price setters, and equate marginal revenue to marginal costs. With labour being the only variable factor of production (and constant returns to scale) we have the price setting relationship

$$Q_i = \frac{El_Q Y}{El_Q Y - 1} \frac{W_i(1 + T1_i)}{Z_i},$$

where $Z_i = Y_i/N_i$ is average labour productivity Y_i is output and N_i is labour input. W_i is the wage rate in the firm, and $T1_i$ is a payroll tax rate. $El_Q Y > 1$ denotes the absolute value of the elasticity of demand facing each firm i with respect to the firm's own price. In general, $El_Q Y$ is a function of relative prices, which provides a rationale for inclusion of e.g., the real exchange rate in aggregate price equations. However, it is a common simplification to assume that the elasticity is independent of other firms prices and is identical for all firms. With constant returns technology, aggregation is no problem, but for simplicity we assume that average labour productivity is the same for all firms and that the aggregate price equation is given by

$$Q = \frac{El_Q Y}{El_Q Y - 1} \frac{W(1 + T1)}{Z} \tag{15}$$

The expression for real profits (Π) is therefore

$$\Pi = Y - \frac{W(1 + T1)}{Q} N = (1 - \frac{W(1 + T1)}{Q} \frac{1}{Z})Y.$$

We assume that the wage W is settled in accordance with the principle of maximizing of the Nash product:

$$(V - V_0)^{\mho} \Pi^{1-\mho} \tag{16}$$

where V denotes union utility and V_0 denotes the fall-back utility or reference utility. The corresponding break-point utility for the firms has already been set to zero in (16), but for unions the utility during a conflict (e.g., strike, or work-to-rule) is non-zero because of compensation from strike funds. Finally $\mho$ represents the relative bargaining power of unions.

Union utility depends on the consumer real wage of an unemployed worker and the aggregate rate of unemployment, thus $V(\frac{W}{P}, U, A_\nu)$ where P denotes the consumer price index.[9] The partial derivative with respect to wages is positive, and negative with respect to unemployment ($V'_W > 0$ and $V'_U \leq 0$). A_ν represents other factors in union preferences. The fall-back or reference utility of the union depends on the overall real wage level and the rate of unemployment, hence $V_0 = V_0(\frac{\bar{W}}{P}, U)$ where $\bar{W}$ is the average level of nominal wages which is one of factors determining the size of strike funds. If the aggregate rate of unemployment is high, strike funds may run low in which case the partial derivative of V_0

[9] It might be noted that the income tax rate $T2$ is omitted from the analysis. This simplification is in accordance with previous studies of aggregate wage formation, see e.g., Calmfors and Nymoen (1990) and Nymoen and Rødseth (2003), where no convincing evidence of important effects from the average income tax rate $T2$ on wage growth could be found.

with respect to U is negative ($V'_{0U} < 0$). However, there are other factors working in the other direction, for example that the probability of entering a labour market programme, which gives laid-off workers higher utility than open unemployment, is positively related to U.

With these specifications of utility and break-points, the Nash product, denoted $\mathcal{N}$, can be written as

$$\mathcal{N} = \left\{ V(\frac{W}{P}, U, A_{\nu}) - V_0(\frac{\bar{W}}{P}, U) \right\}^{\mho} \left\{ (1 - \frac{W(1+T1)}{Q} \frac{1}{Z})Y \right\}^{1-\mho}$$

or

$$\mathcal{N} = \left\{ V(\frac{RW}{P_q(1+T1)}, U, A_{\nu}) - V_0(\frac{\bar{W}}{P}, U) \right\}^{\mho} \left\{ (1 - RW\frac{1}{Z})Y \right\}^{1-\mho}$$

where $RW = W(1+T1)/Q$ is the producer real wage, and $P_q(1+T1) = P(1+T1)/Q$ is the so called *wedge* between the consumer and producer real wage.

Note also that, unlike many expositions of the so called ‘bargaining approach’ to wage modelling, for example Layard et al. (1991, Chapter 7), there is no aggregate labour demand function—employment as a function of the real wage—subsumed in the Nash product. In this we follow Hahn and Solow (1997, Ch. 5.3), who point out that bargaining is usually over the nominal wage and not over employment.

The first order condition for a maximum is given by $\mathcal{N}_{RW} = 0$ or

$$\mho \frac{V'_W(\frac{RW}{P_q(1+T1)}, U, A_{\nu})}{V(\frac{RW}{P_q(1+T1)}, U, A_{\nu}) - V_0(\frac{\bar{W}}{P}, U)} = (1-\mho)\frac{\frac{1}{Z}}{(1 - RW\frac{1}{Z})}. \tag{17}$$

In a symmetric equilibrium, $W = \bar{W}$, leading to $\frac{RW}{P_q(1+T1)} = \frac{\bar{W}}{P}$ in equation (17), the aggregate bargained real wage RW^b is defined implicitly as

$$RW^b = F(P_q(1+T1), Z, \mho, U), \tag{18}$$

or, using the definition

$$RW^b \equiv W^b(1+T1)/Q$$

we obtain the solution for the bargained nominal wage:

$$W^b = \frac{Q}{(1+T1)} F(P_q(1+T1), Z, \mho, U) \tag{19}$$

Letting lower-case letters denote logs of variables, a log-linearization of (19), gives:

$$\begin{aligned} w^b &= m_w + q_t + (1-\delta_{12})(p-q) + \delta_{13}z - \delta_{15}u - \delta_{16}T1. \\ &0 \le \delta_{12} \le 1,\ 0 < \delta_{13} \le 1,\ \delta_{15} \ge 0, \quad 0 \le \delta_{16} \le 1. \end{aligned} \tag{20}$$

The elasticity of the wedge variable $(p-q)$ is $(1-\delta_{12})$ in (20). In econometric models of wage setting in manufacturing, the hypothesis of $\delta_{12} = 1$, is typically not rejected, meaning that the wedge variable drops out and the bargained nominal wage is linked one-to-one with the producer price q, see e.g., Nymoen and Rødseth (2003). However, at the aggregate level, a positive coefficient of the wedge is typically reported. This may be due to measurement problems, since GDP is an income variable the price deflator q is not a good index of ‘producer prices’. That said, the estimated importance of the wedge may also reflect that the economy wide average wage is influenced by the service sector, where wage claims are linked to cost of living considerations, implying that $(1-\delta_{12})$ is different from zero.

Irrespective of the split between q and p, productivity z is found to be a main determinant of the secular growth in wages in bargaining based systems, so we expect the elasticity

δ_{13} to be close to one. The impact of the rate of unemployment on the bargained wage is given by the elasticity $-\delta_{15} \leq 0$. Blanchflower and Oswald (1994) provide evidence for the existence the empirical law that the value of $-\delta_{15}$ is 0.1, which is the slope coefficient of their *wage-curve*. Other authors instead emphasize that the slope of the wage-curve is likely to depend on the level of aggregation and on institutional factors. For example, one influential view holds that economies with a high level of coordination and centralization is expected to be characterized with a higher responsiveness to unemployment (a higher $-\delta_{15}$) than uncoordinated systems that give little incentive to solidarity in wage bargaining, Layard et al. (2005, Ch. 8). Finally, from the definition of the wedge, one could set $\delta_{16} = \delta_{12}$ but we keep δ_{16} as a separate coefficient to allow for separate effects of the payroll tax on wages.

Equation (20) is a general proposition about the bargaining outcome and its determinants, and can serve as a starting point for describing wage formation in any sector or level of aggregation of the economy. In following we regard equation (20) as a model of the average wage in the total economy, and as explained above we therefore expect $(1 - \delta_{12}) > 0$, meaning that there is a wedge effect in the long-run wage equation.

Equation (15) already represents a price setting rule based upon so called normal cost pricing. Upon linearization we have

$$q^f = m_q + (w + T1 - z)\,. \tag{21}$$

where we use q^f as a reminder that this is a theoretical equation for firms' price-setting.

2.6.2 Cointegration and long-run identification

At this point we show how the two theoretical relationships (20) and (21) can be transformed into hypothesized relationships between observable time series. As explained in section 2.3, our maintained modelling assumption is that the real-wage and productivity are $I(1)$ series. The rate of unemployment is assumed to be $I(0)$, possibly after removal of deterministic shifts in the mean.

Using subscript t to indicate period t variables, equation (20) defines w_t^b as an $I(1)$ variable. Next define:

$$ecm_t^b = rw_t - rw_t^b \equiv w_t - w_t^b,$$

Under the null-hypothesis that the theory is correct, the 'bargained wage' w_t^b cointegrates with the actual wage, hence $ecm_t^b \sim I\,(0)$, which is a testable hypothesis. We can the write the long-run wage equation following from bargaining theory as:

$$w_t = m_w + q_t + (1 - \delta_{12})\,(p_t - q_t) + \delta_{13} z_t - \delta_{15} u_t - \delta_{16} T1_t + ecm_t^b. \tag{22}$$

With reference to equation (21), a similar argument applies to price setting. The 'firm side' real wage can be defined as

$$rw_t^f \equiv w_t + T1_t - q_t^f = -m_q + z_t,$$

and the difference between the actual real wage and the real wage implied by price setting becomes

$$ecm_t^f = rw_t - rw_t^f = w_t + T1_t - q_t - \{-m_q + z_t\}.$$

Hence, the implied long–run price setting equation becomes

$$q_t = m_q + (w_t + T1_t - z_t) - ecm_t^f \tag{23}$$

where $ecm_t^f \sim I\,(0)$ for the equation to be consistent with the modelling assumptions.

The two cointegrating relationships (22) and (23) are not identified in general. But in several cases of relevance, identification is unproblematic, see Bårdsen et al. (2005, p. 81).

Here we consider a case which is relevant for an aggregated model of the supply side in an open economy. Equation (22) and (23) can then be combined with a definition of the consumer price index p_t,

$$p_t = (1-\zeta)\,q_t + \zeta pi_t + \eta T3_t,\ 0 < \zeta < 1, \quad 0 < \eta \leq 1, \tag{24}$$

where the import price index pi_t naturally enters. The parameter ζ reflects the openness of the economy.[10] Also, the size of the parameter η will depend on how much of the retail price basket is covered by the indirect tax-rate index $T3_t$. By substitution of (24) in (22), and of (23) in (24), the system can be specified in terms of w_t and p_t:

$$\begin{aligned} w_t = m_w + \left\{1 + \zeta\frac{\delta_{12}}{(1-\zeta)}\right\} p_t \\ - \frac{\delta_{12}\zeta}{(1-\zeta)} pi_t - \frac{\delta_{12}\eta}{(1-\zeta)} T3_t + \delta_{13} z_t - \delta_{15} u_t - \delta_{16} T1_t + ecm_t^b \end{aligned} \tag{25}$$

$$p_t = (1-\zeta) m_q + (1-\zeta)\{w_t + T1_t - z_t\} + \zeta pi_t + \eta T3_t - (1-\zeta) ecm_t^f \tag{26}$$

By simply viewing (25) and (26) as a pair of simultaneous equations, it is clear that the system is unidentified in general. However, for the purpose of modelling the aggregate economy, we choose the consumer price index p_t as the representative domestic price index by setting $\delta_{12} = 0$. In this case, (26) is unaltered, while the wage equation becomes

$$w_t = m_w + p_t + \delta_{13} z_t - \delta_{15} u_t - \delta_{16} T1_t + ecm_t^b \tag{27}$$

The long-run price equation (26) and the long-run wage equation (27) are identified by the order condition.

2.6.3 VAR and identified equilibrium correction system

The third stage in the operationalization is the equilibrium-correction system, where we follow Bårdsen and Fisher (1999). In brief, we allow wage growth Δw_t to interact with current and past price inflation, changes in unemployment, changes in tax-rates, and previous deviations from the desired wage level consistent with (27)

$$\begin{aligned} \Delta w_t - \alpha_{12,0}\Delta q_t = c_1 + \alpha_{11}(L)\,\Delta w_t + \alpha_{12}(L)\,\Delta q_t + \beta_{12}(L)\,\Delta z_t \\ - \beta_{14}(L)\,\Delta u_t - \beta_{15}(L)\,\Delta T1_t \\ - \gamma_{11} ecm_{t-r}^b + \beta_{18}(L)\,\Delta p_t + \epsilon_{1t}, \end{aligned} \tag{28}$$

where Δ is the difference operator, the $\alpha_{1j}(L)$ and $\beta_{1j}(L)$ are polynomials in the lag operator L:

$$\begin{aligned} \alpha_{1j}(L) &= \alpha_{1j,1}L + \cdots + \alpha_{1j,(r-1)}L^{r-1},\ j = 1,2, \\ \beta_{1j}(L) &= \beta_{1j,0} + \beta_{1j,1}L + \cdots + \beta_{1j,(r-1)}L^{r-1},\ j = 2,4,5,6. \end{aligned}$$

The $\beta-$polynomials are defined so that they can contain contemporaneous effects. The order r of the lag polynomials may of course vary between variables and is to be determined empirically. This specification is a generalization of the typical European wage curve, where the American version is derived by setting $\gamma_{11} = 0$—see Blanchard and Katz (1999).

Any increase in output above the optimal trend exerts a (lagged) positive pressure on prices, measured by the output gap_t, as in Phillips-curve inflation models—see Clarida et al. (1999). In addition, product price inflation interacts with wage growth and productivity gains and with changes in the payroll tax-rate, as well as with corrections from an

[10] Note that, due to the log-form, $\zeta = is/(1-is)$ where is the import share in private consumption.

earlier period's deviation from the equilibrium price (as a consequence of e.g., information lags, see Andersen (1994, Chapter 6.3)):

$$\Delta q_t - \alpha_{21,0}\Delta w_t = c_2 + \alpha_{22}\left(L\right)\Delta q_t + \alpha_{21}\left(L\right)\Delta w_t + \beta_{21}\left(L\right) gap_t \\ - \beta_{22}\left(L\right)\Delta z_t + \beta_{25}\left(L\right)\Delta T1_t - \gamma_{22} ecm^{f}_{t-r} + \epsilon_{2t}, \qquad (29)$$

where

$$\alpha_{2j}(L) = \alpha_{2j,1}L + \cdots + \alpha_{2j,(r-1)}L^{r-1},\ j = 1,2,$$
$$\beta_{2j}\left(L\right) = \beta_{2j,0} + \beta_{2j,1}L \cdots + \beta_{2j,(r-1)}L^{r-1},\ j = 1,2,5.$$

Solving equation (24) for Δq_t (i.e., the equation is differenced first), and then substituting out in equations (27), and (29), the theoretical model condenses to a wage-price model suitable for estimation and similar to the early equilibrium-correction formulation of Sargan (1980):

$$\begin{bmatrix} 1 & -a_{12,0} \\ -a_{21,0} & 1 \end{bmatrix} \begin{bmatrix} \Delta w \\ \Delta p \end{bmatrix}_t = \begin{bmatrix} \alpha_{11}(L) & -a_{12}(L) \\ -a_{21}\left(L\right) & \alpha_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta w \\ \Delta p \end{bmatrix}_t +$$
$$\begin{bmatrix} 0 & \beta_{12}\left(L\right) & -\zeta\frac{\alpha_{12}(L)}{1-\zeta} & -\beta_{14}\left(L\right) & -\beta_{15}\left(L\right) & -\eta\frac{\alpha_{12}(L)}{1-\zeta} \\ b_{21}\left(L\right) & -b_{22}\left(L\right) & \zeta\alpha_{22}(L) & 0 & b_{25}\left(L\right) & \eta\alpha_{22}(L) \end{bmatrix} \begin{bmatrix} gap \\ \Delta z \\ \Delta pi \\ \Delta u \\ \Delta T1 \\ \Delta T3 \end{bmatrix}_t \qquad (30)$$
$$-\begin{bmatrix} \gamma_{11} & 0 \\ 0 & \gamma_{22} \end{bmatrix} \times \begin{bmatrix} 1 & -\left(1+\zeta d_{12}\right) & -\delta_{13} & \zeta d_{12} & \delta_{15} & \delta_{16} & \eta d_{12} \\ -\left(1-\zeta\right) & 1 & \left(1-\zeta\right) & -\zeta & 0 & -\left(1-\zeta\right) & -\eta \end{bmatrix} \begin{bmatrix} w \\ p \\ z \\ pi \\ u \\ T1 \\ T3 \end{bmatrix}_{t-r}$$
$$+\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}_t,$$

where we have omitted the intercepts to save space, and have substituted the equilibrium correction terms using (25) and (26) above. The mapping from the theoretical parameters in (28) and (29) to the coefficients of the model (30) is given by:

$$\begin{aligned} a_{12,0} &= \frac{\alpha_{12,0}}{1-\zeta} + \beta_{18,0}, \\ a_{21,0} &= \left(1-\zeta\right)\alpha_{21,0}, \\ a_{12}\left(L\right) &= \frac{\alpha_{12}\left(L\right)}{1-\zeta} + \beta_{18}(L), \\ a_{21}\left(L\right) &= \left(1-\zeta\right)\alpha_{21}\left(L\right), \\ b_{2j}\left(L\right) &= \left(1-\zeta\right)\beta_{2j}\left(L\right),\ j = 1,2,5, \\ d_{12} &= \frac{\delta_{12}}{1-\zeta}, \\ e_1 &= \epsilon_1, \\ e_2 &= \left(1-\zeta\right)\epsilon_2. \end{aligned} \qquad (31)$$

The model (30) contains the different channels and sources of inflation discussed so far: Imported inflation Δpi_t, and several relevant domestic variables: the output gap, changes in the rate of unemployment, in productivity, and in tax rates. Finally the model includes deviations from the two cointegration equation associated with wage bargaining and price setting which have equilibrium correction coefficients γ_{11} and γ_{22} respectively. Consistency with assumed cointegration implies that the joint hypothesis of $\gamma_{11} = \gamma_{22} = 0$ can be rejected.

2.6.4 Economic interpretation of the steady state

The dynamic model in (30) can be re-written in terms of real wages $(w-p)_t$ and real exchange rates $(pi-p)_t$. Using a specification with first order dynamics, Bårdsen et al. (2005, Ch 6) discusses several different aspects of this model. Most importantly, the dynamic system is asymptotically stable under quite general assumptions about the parameters, including for example dynamic homogeneity in the two equilibrium correction equations. The steady state is conditional on any given rate of unemployment, which amounts to saying that our core supply side model does not tie down an equilibrium rate of unemployment. Instead, there is a stalemate in the dynamic "tug-of-war" between workers and firms that occurs for in principle, any given rate of unemployment, see Kolsrud and Nymoen (1998) and Bårdsen and Nymoen (2003) for proofs. Since there are no new unit root implied by the generalized dynamics in equation (30) above, the asymptotic stability holds also for this, extended, version of the model. We therefore have the following important results: The dynamics of the supply side is asymptotically stable in the usual sense that, if all stochastic shocks are switched off, then $(pi_t - q_t) \rightarrow rex_{ss}(t)$, and $(w_t + T1_t - q_t) = wq_{ss}(t)$ where $rex_{ss}(t)$ and $wq_{ss}(t)$ represent deterministic steady state growth paths of the real exchange rate and the producer real wage.

Generally, the steady-state growth paths depend on the steady state growth rate of import prices, and of the mean of the logarithm of the rate of unemployment, denoted u_{ss}, and the expected growth path of productivity $z(t)$. However, under the condition that $\delta_{13} = 1$, homogeneity of degree one with respect to productivity, which we have seen is implied theoretically by assuming bargaining power on the part of unions, $z(t)$ has a zero coefficient in the expression for rex_{ss}, which therefore is constant in the steady state. Moreover, assuming $\delta_{13} = 1$, the implied steady state wage share, $wq_{ss}(t) - z(t) = ws_{ss}$ which also is also a constant in steady state.

With $\delta_{13} = 1$, the implied steady-state inflation rate therefore follows immediately: Since $\Delta(pi_t - q_t) = 0$ in steady state, and $\Delta p_t = (1-\zeta)\,\Delta q_t + \zeta \Delta pi_t$, domestic inflation is equal to the constant steady state rate of imported inflation,

$$\Delta p_t = \Delta pi_t = \pi. \tag{32}$$

The above implicitly assumes an exogenous, and for simplicity, constant, nominal exchange rate. For the case of endogenous nominal exchange rate, as with a floating exchange rate regime, it might be noted that since

$$pi_t = v_t + p_t^*,$$

where v_t is the nominal exchange rate, and the index of import prices in foreign currency is denoted p_t^*, the stability of inflation requires stability of Δv_t. This condition can only be verified by the use of a more complete model representation of the economy, which is what we do when we consider the steady state of a complete econometric model in section 3.2 below. However, to anticipate events slightly, the complete model that we document below meets the requirement in the sense that $\Delta^2 v_t \rightarrow 0$ in the long-run. But our results also indicate that the π in (32) is affected by the rate of change in nominal exchange rate, which might be non-zero in an asymptotically stable steady state.

The supply-side determined steady state has a wider relevance as well. For example, what does the model tell about the dictum that the existence of a steady state inflation rate requires that the rate of unemployment follows the law of the natural rate or NAIRU? The version of this natural rate/NAIRU view of the supply side that fits most easily into our framework is the one succinctly expressed by Layard et al. (1994)

> 'Only if the real wage (W/P) desired by wage-setters is the same as that desired by price setters will inflation be stable. *And, the variable that brings about this consistency is the level of unemployment*'.[11]

Translated to our conceptual framework, this view corresponds to setting $ecm_t^b = ecm_t^f = 0$ in (22) and (23), with $\delta_{13} = 1$, and solving for the rate of unemployment that reconciles the two desired wage shares, call it u^w:[12]

$$u^w = \frac{m_w + m_q}{-\delta_{15}} + \frac{1 - \delta_{12}}{-\delta_{15}}(p - q) + \frac{1 - \delta_{16}}{-\delta_{15}} T1,$$

which can be expressed in terms of the real exchange rate $(p - pi)$, and the two tax rates as:

$$u^w = \frac{-(m_w + m_q)}{\delta_{15}} + \frac{1 - \delta_{12}}{\delta_{15}(1 - \zeta)} \zeta (p - pi) + \frac{1 - \delta_{12}}{\delta_{15}(1 - \zeta)} \eta T3 + \frac{1 - \delta_{16}}{-\delta_{15}} T1 \qquad (33)$$

This is one equation in two endogenous variables, u^w and the wedge $(p-pi)$, so it appears that there is a continuum of u^w values depending on the size of the wedge, in particular of the value of the real exchange rate. It is however custom to assume that the equilibrium value of the wedge is determined by the requirement that the current account is in balance in the long run. Having thus pinned down the long run wedge as a constant equilibrium real exchange rate $\overline{(p - pi)}$, it follows that NAIRU u^w is determined by (33). If the effect of the wedge on wage claims is not really a long run phenomenon then $\delta_{12} = 1$ and u^w is uniquely determined from (33), and there is no need for the extra condition about balanced trade in the long-run, see Layard et al. (2005, p. 33).

Compare this to the asymptotically stable equilibrium consisting of $u_t = u_{ss}$, $\Delta p_t = \pi$ and $w_t + T1 - q_t - z_t = ws_{ss}$. Clearly, inflation is stable, even though u_{ss} is determined 'from the outside', and is not determined by the wage-and price-setting equations of the model. Hence the (emphasized) second sentence in the above quotation has been disproved: It is not necessary that u_{ss} corresponds to the NAIRU u^w in equation (33) for inflation to be stable with a well defined value in steady state.

Figure 1 illustrates the different equilibria. Wage-setting and price setting curves correspond to (deterministic versions) of equation (22) and (23). The NAIRU u^w is given by the intersection of the curves, but the steady state rate of unemployment u_{ss} may be lower than u^w, the case shown in the graph, or higher. The figure further indicates (by a •) that the steady state wage share will reside at a point on the line segment A-B: Heuristically, this is a point where price setters are trying to attain a lower real wage by nominal price increases, at the same time at the wage bargain is delivering nominal wage increases that push real wage upwards.

Bårdsen et al. (2005, Ch 6) show which restrictions on the parameters of the system (30) that are necessary for $u_t \to u_{ss} = u^w$ to be an implication, so that the NAIRU corresponds to the stable steady state. In brief, the model must be restricted in such a way that the nominal wage and price setting adjustment equations become two conflicting dynamic equations for the real wage. Because of the openness of the economy, this is not achieved by imposing dynamic homogeneity. What is required is to purge the model (30) of all nominal rigidity, which seems to be unrealistic on the basis of both macro and micro evidence.

[11] Layard et al. (1994, p 18), authors' italics.

[12] Strictly, we take the expectation through in both equations.

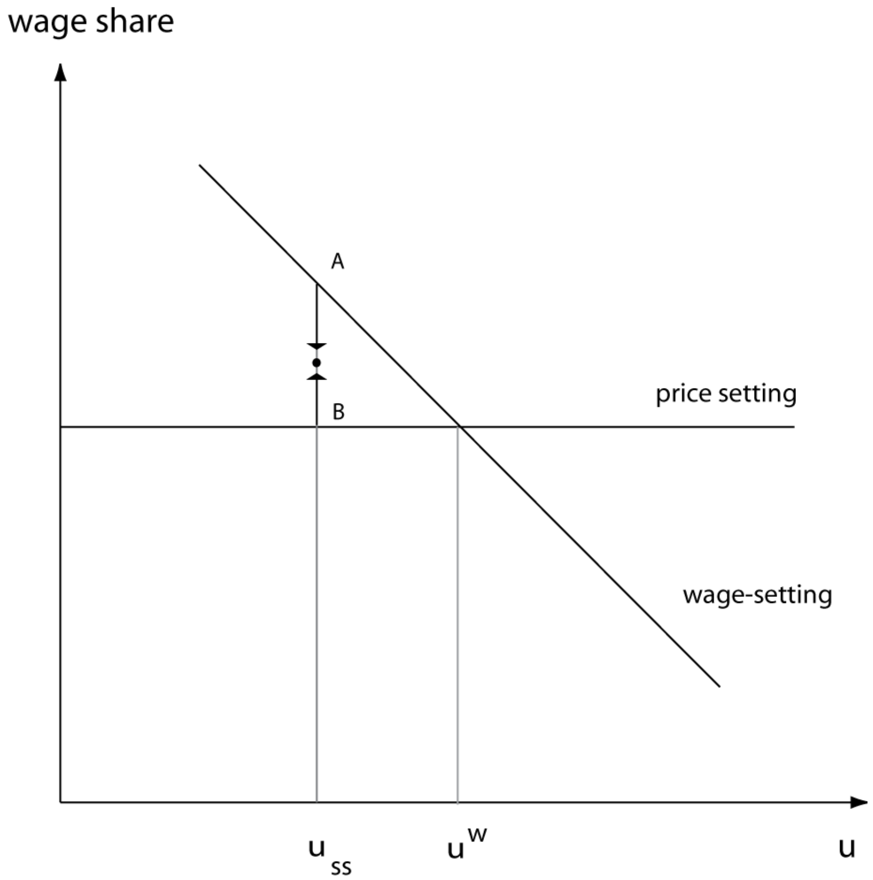

Figure 1: Real wage and unemployment determination, NAIRU and the steady state rate of unemployment u_{ss}.

We have seen that the Layard-Nickell version of the NAIRU concept corresponds to a set of restrictions on the dynamic model of wage and price setting. The same is true for the natural rate of unemployment associated with a vertical Phillips curve, which still represents the baseline model for the analyses of monetary policy. This is most easily seen by considering a version of (28) with first order dynamics and where we abstract form short-run effects of productivity, taxes and unemployment ($\beta_{12} = \beta_{14} = \beta_{15} = 0$). With first order dynamics we have:

$$\Delta w_t - \alpha_{12,0}\Delta q_t = c_1 - \gamma_{11} ecm^b_{t-1} + \beta_{18}\Delta p_t + \epsilon_{1t},$$

and using (22) we can then write the wage equation as:

$$\begin{aligned}\Delta w_t = k_w + \alpha_{12,0}\Delta q_t + \beta_{18}\Delta p_t - \mu_w u_{t-1} \\ - \gamma_{11}(w_{t-1} - q_{t-1}) + \gamma_{11}(1 - \delta_{12})(p_{t-1} - q_{t-1}) + \gamma_{11}\delta_{16}T1_{t-1} + \epsilon_{1t}\end{aligned} \tag{34}$$

where $k_w = c_1 + \gamma_{11}m_w$, and the parameter μ_w is defined in accordance with Kolsrud and Nymoen (1998) as:

$$\mu_w = \gamma_{11}\delta_{13} \text{ when } \gamma_{11} > 0 \text{ or } \mu_w = \varphi \text{ when } \gamma_{11} = 0. \tag{35}$$

The notation in (35) may seem cumbersome at first sight, but it is required to secure internal consistency: Note that if the nominal wage rate is adjusting towards the long run wage curve, $\gamma_{11} > 0$, the only logical value of for φ in (35) is zero, since u_{t-1} is already contained in the equation, with coefficient $\theta_w\varpi$. Conversely, if $\theta_w = 0$ so the the model of collective wage bargaining fails, it is nevertheless possible that there is a wage Phillips curve relationship, consistent with the assumed $I(0)$-ness of the rate of unemployment, hence $\mu_w = \varphi \geq 0$ in this case.

Subject to the restriction $\gamma_{11} = 0$, and assuming an asymptotically stable steady state inflation rate π, (34) can be solved for the Phillips-curve NAIRU u^{phil}:

$$u^{phil} = \frac{k_w}{\varphi} + \frac{(\alpha_{12,0} + \beta_{18} - 1)}{\varphi}\pi$$

which becomes a natural rate of unemployment, independent of inflation subject to dynamic homogeneity $\alpha_{12,0} + \beta_{18} = 1$.

However, the claim that u_t^{phil} represents an asymptotically stable stable solution must be stated with some care. As shown in e.g., Bårdsen and Nymoen (2003) $\gamma_{11} = 0$ is a necessary but not a sufficient condition. The sufficient conditions include $\gamma_{22} = 0$ in addition to $\gamma_{11} = 0$ and instead of equilibrium correction in wages and prices, dynamic stability requires equilibrium correction in the unemployment equation or in a functionally equivalent part of the model.

The result that the steady state level of unemployment is generally undetermined by the wage-price sub-model is a strong case for building larger systems of equations, even if the main objective is to model inflation, Conversely, in general no inconsistencies, or issues about overdetermination, arise from enlarging the wage/price setting equations with a separate equation for the rate of unemployment, where demand side variables may enter.

Looking ahead, in section 4.3 below, we show how the specification of the supply side, either as a Phillips curve model, PCM, or as incomplete competition model, ICM, given by equation (28) and (29) above, gains economic significance though the implications of the chosen specification for optimal interest rate setting.

2.6.5 Implementation in NAM

We have implemented the above model of the supply side in our quarterly model of the Norwegian economy called NAM (Norwegian Aggregated Model).[13]

The estimated versions of (30) are given in Section 3.1, equations (38) and (39). The equilibrium correction terms are defined consistently with the two long-run equations (25) and (26). For example, $\delta_{12} = 0$, $\zeta = 0.7$, $\delta_{15} = 0.1$ and $\delta_{16} = 1$ are taken as known parameters from the cointegration analysis documented in Bårdsen et al. (2005, Ch. 9.2). With this parameterization, the estimated equilibrium correction coefficients $\hat{\gamma}_{11}$ and $\hat{\gamma}_{22}$ are jointly and individually significant (the t-values are 8.6 and 3.8 for example).

The estimated short-run dynamics can also be interpreted in the light of the theoretical model (30). For example, the estimated wage equation (39) shows that $\hat{a}_{12}(1) = 1$, saying that dynamic homogeneity with respect to consumer price is a valid restriction on wage dynamics of the wage equation. Identification of the short-run wage price model is in terms of zero-restrictions on the GDP growth variable in the Δw_t equation, and on the change in the rate of unemployment in the Δp_t equation. There are over-identifying restrictions as well though.

[13] Norwegian Aggregate Model (NAM) is a model project which extends from the early econometric assessment of wage- and price-inflation in ?, further developed in ?, Bårdsen and Fisher (1999), and the monetary transmission model of ?. Earlier versions of the model are documented in ?, Bårdsen et al. (2003), and Bårdsen et al. (2005).

NAM is used for both research purposes and for teaching. The macroeconomic data is from the model databases of Statistics Norway (KVARTS model) and Norges Bank (FPAS database).

Specific versions of the model are currently operative for a) econometric forecasts of the Norwegian macroeconomy (NAM-EF) and b) model based analysis of financial stability in Norway (NAM-FS).

3 Building a model for monetary policy analysis

Monetary policy now plays a dominant role in stabilization policy in general and in managing inflation in particular. As economists have recognized for a long time, inflation is a many-faceted phenomenon. In particular, in open economies, a proper understanding of the inflation mechanism requires the construction of a model that separates the internal dynamics of the domestic wage price spiral from the factors that impinge upon it from outside. The complexity of the real-world inflation process also means that models which only include one or two dimensions typically fail to characterize the data. Our starting point is therefore that, at a minimum, foreign and domestic aspects of inflation have to modelled jointly, and that the inflationary impetus from the labour market—the battle of mark-ups between unions and monopolistic firms—needs to be represented in the model.

Last section ended with an example of econometric specification of a model of wage and price setting that defines an integral part of the NAM model. Earlier versions of this model have been used to analyze the issues raised by the introduction of model based monetary policy in Norway, see Bårdsen et al. (2002),Bårdsen et al. (2003), and Bårdsen et al. (2005). NAM is in use for forecasting as part of the Normetrics forecasting system.[14] A designated version is operational for stress testing by the financial stability division at the Central Bank of Norway.

3.1 The model and its transmission mechanisms

In the regime with inflation targeting, the policy instrument in the model is the money market interest rate, symbolized by R in Figure 2 (and throughout the paper), with the estimated reaction function reported in equation (46).[15]

The qualitative transmission mechanisms of the model, from the perspective of monetary policy analysis, are shown in Figure 2. The corresponding quantitative approximate transmission mechanisms are easily worked out with the stylized model version in Section 4.1. That simplified quantitative transmission mechanism is derived from the dynamic econometric model reported in Tables 1-2. We report the estimated macroeconometric relationships in equilibrium correction form, with cointegration coefficients imposed as known. The identities that complete the NAM model are not reported. To save space, seasonals and other dummies are also omitted from the equations in the two tables. The definitions of the variables in the equations are given in the Appendix.

Consider, for example, the analysis conducted in Section 4.2.1 of an increase in the interest rate R. The immediate and direct effect is an appreciation of the krone, measured as an increase in the exchange rate v, defined as kroner per unit foreign exchange. The multiplier is approximated in (60) as $\frac{\Delta v_t}{\Delta R_t} \approx -0.04$ while the complete equation is reported in (36)[16]. The decrease in v will affect domestic price and wage setting through decreased import prices pi, as reported in equations (37)-(39), which gives corresponding approximate partial multipliers as $\frac{\Delta pi_t}{\Delta v_t} \approx 0.9$ and $\frac{\Delta p_t}{\Delta pi_t} \approx 0.03$ from (61) and (62), Hence, at least for a period of time after the interest rate increase, the *exchange rate channel* will provide inflation dampening following an increase in the interest rate.

The exchange rate channel also affects wages and prices indirectly, through GDP y and unemployment u, reported in equations (44) and (41), respectively. The mechanisms are as follows. Due to sticky prices, the real exchange rate appreciates together with the nominal rate, causing decreased competitiveness, lower output, and higher unemployment—

[14] (http://folk.uio.no/rnymoen/normetrics_index.html)

[15] In practice, the policy instrument is the sight deposit rate set by the Central Bank, but since the sight deposit rate represents (banks') marginal funding cost, changes in the sight rate are transmitted to the money market rate immidiately.

[16] The size of the depreciation will depend upon the risk premium, whether expectations counteract or strengthen the initial effect of the interest rate cut, and so forth, cf. section **??**

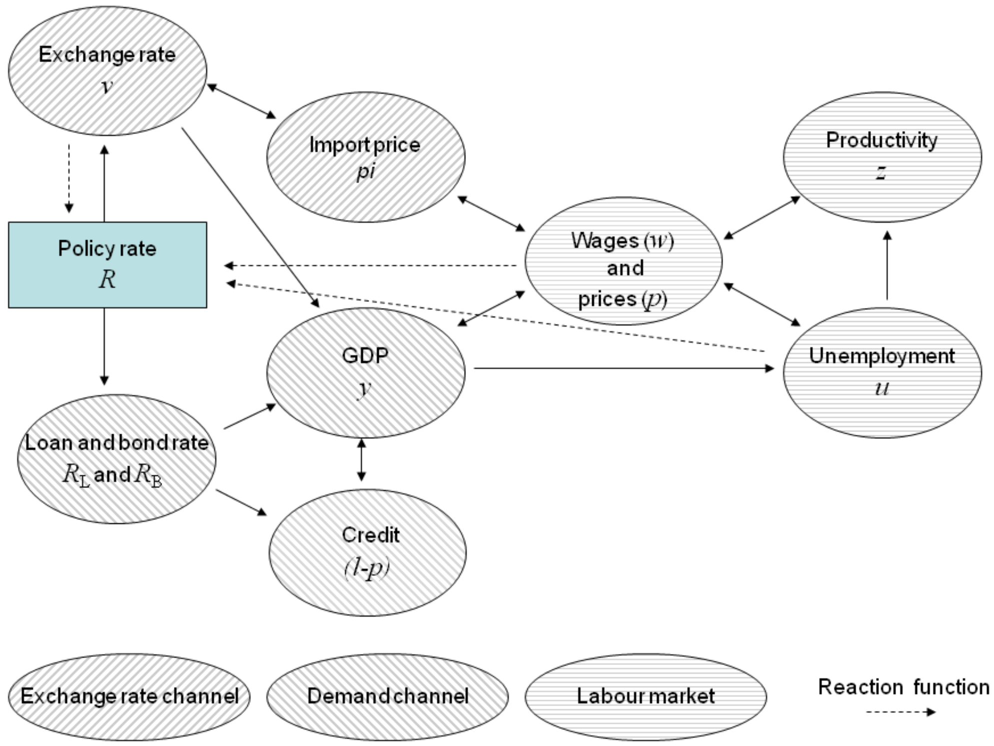

Figure 2: The transmission mechanisms in the model in tables 1 and 2.

Together with the interaction with productivity z in equation (40), this constitutes the *labour-market channel*. For example, the approximate partial real-wage response from a shock to unemployment is $\frac{\Delta(w-p)_t}{\Delta u_t} \approx -0.04$ from (63).

The interest rate effects on the real economy are first channelled through financial markets, where an increase in the money market rate leads to adjustment of the banks' interest rate R_L, and bond yield R_B, see (42)-(43). A rise in R_L affects GDP through an increased real interest rate.[17] This is the *demand channel* found in mainstream monetary policy models, see e.g., Ball (1999). In the model, there is also a second, *credit channel* whereby interest rates affect output: When interest rates are raised, the amount of available real credit is reduced, as documented in (45), which has a negative effect on output. The average partial multiplier is $\frac{\Delta y_t}{\Delta(l-p)_t} \approx 0.4$, using (68).

The transmission mechanism pictured in Figure 2 shows that the model contain both positive and negative feed-back effects from wage and price adjustments, to GDP and unemployment. Higher inflation means that the real interest rate continues to fall in the first periods after the initial cut in the nominal rate (positive feed-back). On the other hand, and again due to the raised rate of inflation, the real exchange rate will start to stabilize (negative feed-back).

In the figure, the focus is on the transmission mechanisms, which may give the impression that the development of wages and prices is mainly 'determined by' monetary policy. This is not the case since, for example, the important trend component in wages is related to productivity growth through wage bargaining—see (38)-(40). Having analyzed the transmission mechanism of the model, we now turn to the steady-state properties, pinned down by the overidentified cointegrated steady-state relationships of the model, which are discussed in the next section.

[17] Denoted ρ in below.

Table 1: The econometric model NAM. Standard errors are reported in parentheses below the coefficients. See appendix A for information about the statistics reported below each equation

The exchange rate

$$\Delta v_t = -\underset{(0.012)}{0.042}\left\{(v+p^*-p) - 0.12\left[(R-\pi)-(R^*-\pi^*)\right]-\mu_v\right\}_{t-1}$$

$$-\underset{(0.00693)}{0.0361}\; IT_t \times \Delta(R-R^*)_t - \underset{(0.015)}{0.036}\;\Delta^2 po_{t-1} \tag{36}$$

$OLS, \quad T = 1994\,(2) - 2007\,(2) = 56, \quad \hat{\sigma} = 1.6\%$
$F_{\mathsf{AR}(1-4)}(4,44) = 1.60[0.19] \quad F_{ARCH(1-4)}\,(4,40) = 0.99\,[0.42]$
$\chi^2_{normality}(2) = 0.07[0.97] \quad F_{\mathsf{x}}(6,41) = 0.59[0.80]$

where $\pi_t \equiv 100\frac{\Delta_4 P_t}{P_{t-4}}$ and $\pi^*_t \equiv 100\frac{\Delta_4 P^*_t}{P^*_{t-4}}$.

Import prices

$$\Delta pi_t = -\underset{(0.0806)}{0.431}\left[(pi - v - pi^*) - 0.55\,(p - v - p^*) - \mu_{pi}\right]_{t-1}$$

$$+\underset{(0.0718)}{0.429}\;\Delta v_t + \underset{(0.211)}{1.07}\;\Delta pi^*_t \tag{37}$$

$OLS, \quad T = 1990\,(1) - 2007\,(1) = 69, \quad \hat{\sigma} = 1.1\%$
$F_{AR(1-4)}(4,59) = 1.78[0.15] \quad F_{ARCH(1-4)}\,(4,55) = 0.99\,[0.42]$
$\chi^2_{Normality}(2) = 0.81[0.67] \quad F_{Het}(9,53) = 1.15[0.34]$

Prices, wages and productivity

$$\Delta p_t = -\underset{(0.006)}{0.052}\left[p_{t-3} - 0.7\,(w-z)_{t-1} - 0.3 pi_{t-1} - \mu_p\right] - \underset{(0.025)}{0.06}\;\Delta z_t$$

$$+\underset{(0.049)}{0.29}\;\Delta p_{t-2} + \underset{(0.011)}{0.024}\;\Delta pi_t + \underset{(0.0038)}{0.059}\;\Delta pe_t + \underset{(0.014)}{0.042}\;\Delta y_{t-1} \tag{38}$$

$$\Delta w_t = -\underset{(0.019)}{0.065}\left[(w_{t-1} - p_{t-2} - z_{t-1}) + 0.1 u_{t-4} - \mu_w\right] + \underset{(0.071)}{0.56}\;\Delta p_t + 0.43\Delta p_{t-1}$$

$$-\underset{(0.0053)}{0.039}\left(\Delta^2 u_{t-1} + \Delta u_{t-3}\right) + \underset{(0.023)}{0.73}\;\Delta T1_t \tag{39}$$

$$\Delta z_t = -\underset{(0.062)}{0.64}\left[z_{t-3} - 0.47\,(w-p)_{t-1} - 0.0029 Trend_t - 0.03 u_{t-2} - \mu_z\right]$$

$$+\underset{(0.05)}{0.24}\;\Delta\,(w-p)_t - \underset{(0.036)}{0.83}\;\Delta_2 z_{t-1} \tag{40}$$

$FIML, \quad T = 1979\,(3) - 2007\,(1) = 111, \quad \hat{\sigma}_P = 0.3\%, \quad \hat{\sigma}_W = 0.6\%, \quad \hat{\sigma}_Z = 1.2\%$
$F_{vec,\mathsf{AR}(1-5)}(45,250) = 1.07[0.36] \quad F_{vec,Het}(354,224) = 1.11[0.20]$
$\chi^2_{vec,Normality}(6) = 7.02[0.32]$

The rate of unemployment

$$\Delta u_t = -\underset{(0.019)}{0.078}\left\{u_{t-1} - 7.69\Delta\,(w-p)_{t-2} - 0.05\left[(R_L - \pi) - 100\Delta_4 y\right]_{t-2} - \mu_u\right\}$$

$$+\underset{(0.071)}{0.44}\;\Delta u_{t-1} + \underset{(0.059)}{0.49}\;\Delta u_{t-4} - \underset{(0.074)}{0.27}\;\Delta u_{t-5} \tag{41}$$

$OLS, \quad T = 1981\,(1) - 2007\,(2) = 106, \quad \hat{\sigma} = 5.0\%$
$F_{AR(1-5)}(5,92) = 1.04[0.25] \quad F_{ARCH(1-4)}\,(4,89) = 0.48\,[0.75]$
$\chi^2_{Normality}(2) = 0.50[0.78] \quad F_{Het}(13,83) = 1.44[0.16]$

Table 2: The econometric model NAM, continued

Interest rates

$$\Delta R_{L,t} = -\underset{(0.024)}{0.33}\left(R_L - 0.41R_B - 0.76R - \mu_{R_L}\right)_{t-1} + \underset{(0.023)}{0.58}\,\Delta R_t \tag{42}$$

$$\Delta R_{B,t} = -\underset{(0.061)}{0.17}\left(R_B - 0.43R - 0.57R_B^* - \mu_{R_B}\right)_{t-1} + \underset{(0.039)}{0.43}\,\Delta R_t + \underset{(0.067)}{0.97}\,\Delta R_{B,t}^* \tag{43}$$

$FIML,\quad T = 1993\,(2) - 2006\,(4) = 55,\quad \hat{\sigma}_{R_L} = 0.10,\quad \hat{\sigma}_{R_B} = 0.19$
$F_{vec,\mathsf{AR}(1-4)}(16,84) = 0.53[0.92]\quad F_{vec,Het}(54,90) = 1.30[0.13]$
$\chi^2_{vec,Normality}(4) = 4.61[0.33]$

GDP-output

$$\Delta y_t = -\underset{(0.041)}{0.21}\left[y_{t-2} - 0.9g_{t-1} - 0.16\,(v + p^* - p)_{t-1} + 0.06\,(R_L - \pi)_{t-1} - \mu_y\right]$$

$$-\underset{(0.091)}{0.74}\,\Delta y_{t-1} + \underset{(0.058)}{0.42}\,\Delta g_t + \underset{(0.11)}{0.67}\,\Delta\,(l-p)_{t-1} \tag{44}$$

$OLS,\quad T = 1986\,(2) - 2007\,(1) = 104,\quad \hat{\sigma} = 1.4\%$
$F_{AR(1-5)}(5,72) = 2.08[0.07]\quad F_{ARCH(1-4)}\,(4,69) = 0.30\,[0.87]$
$\chi^2_{Normality}(2) = 1.85[0.68]\quad F_{Het}(11,65) = 1.21[0.30]$

Credit

$$\Delta\,(l-p)_t = -\underset{(0.022)}{0.094}\left[(l-p)_{t-3} - 2.65y_{t-4} + 0.04(R_{L,} - i_B)_{t-4} - \mu_{l-p}\right]$$

$$+\underset{(0.043)}{0.15}\,\Delta_2 y_{t-2} + \underset{(0.087)}{0.24}\,\Delta^2\,(w-p)_t \tag{45}$$

$OLS,\quad T = 2000\,(1) - 2007\,(2) = 30,\quad \hat{\sigma} = 0.7\%$
$F_{AR(1-3)}(3,23) = 0.98[0.42]\quad F_{ARCH(1-3)}\,(3,20) = 0.32\,[0.81]$
$\chi^2_{Normality}(2) = 4.73[0.09]\quad F_{Het}(6,19) = 0.25[0.95]$

Money market interest rate

$$\Delta R_t = -\underset{(0.047)}{0.27}\left[R_{t-1} - 5.6 - 1.2\,(\pi_{Ct} - \bar{\pi}_C) + \left(U_{t-2} - \bar{U}\right) - 0.86\left(R_{t-2}^* - \bar{R}^*\right)\right]$$

$$+\underset{(0.088)}{0.51}\,\Delta_2 R_t^* - \underset{(0.013)}{0.051}\,\Delta_2\left(\frac{V \times P^*}{P}\right)_t \tag{46}$$

$OLS,\quad T = 1999\,(3) - 2007\,(2) = 32,\quad \hat{\sigma} = 0.19$
$F_{AR(1-3)}(3,22) = 1.62[0.21]\quad F_{ARCH(1-3)}\,(3,19) = 0.60\,[0.62]$
$\chi^2_{Normality}(2) = 0.82[0.67]\quad F_{Het}(12,12) = 0.59[0.81]$

where $\pi_{Ct} \equiv 100\frac{\Delta_4 P_{C,t}}{P_{C,t-4}}$; $\bar{\pi}_U = 2.5$ (the inflation target); $\bar{U} = 3.4$ (average unemployment rate); $\bar{R}^* = 3.5$ (average foreign short-run interest rate).

3.2 Steady state

Equation (47)-(56) represent the model's implied long-run relationships. Cointegrated combinations of non-stationary variables are on the left sides of the equations, while stationary variables are evaluated at their mean values to the right.

$$(v + p^* - p)_t = -0.12\,[(R - \pi) - (R^* - \pi^*)] + \mu_v \tag{47}$$
$$(pi - v - pi^*)_t - 0.55\,(p - v - p^*)_t = \mu_{pi} \tag{48}$$
$$p_t - 0.7(w - z)_t - (1 - 0.7)\,pi_t = \mu_p \tag{49}$$
$$(w - p - z)_t = 0.1u + \mu_w \tag{50}$$
$$z_t - 0.47\,(w - p)_t - 0.0029Trend_t = 0.03u + \mu_z \tag{51}$$
$$0 = u - 7.7\Delta\,(w - p) - 4.5\,[0.01\,(RL - \pi) - \Delta_4 y] - \mu_u \tag{52}$$
$$0 = RL - 0.41RB - 0.76R - \mu_{RL} \tag{53}$$
$$0 = RB - 0.43R - 0.57RB^* - \mu_{RB} \tag{54}$$
$$y_t - 0.9g_t - 0.16(v + p^* - p)_t = -0.06\,(RL - \pi) + \mu_y \tag{55}$$
$$(l - p)_t - 2.65y_t + 0.04(RL - RB)_t = \mu_{l-p} \tag{56}$$

Equation (47) represents the equilibrium on the market for foreign exchange in a floating exchange rate regime. The central bank's foreign currency reserves is exogenous in the floating exchange rate regime that we are modelling, and therefore is not specified in (47). The nominal exchange rate v equilibrates the market in each time period, specifically also in the hypothetical steady-state represented in (47). The relationship follows from the definition of the risk premium, rp:

$$rp_t = R_t - R_t^* - (v_{t+1}^e - v_t).$$

where $(v_{t+1}^e - v_t)$ denotes expected depreciation. In real terms, using the definition $rex = v + p^* - p$, the relationship can be written as

$$\left(v_{t+1}^e + p_{t+1}^{*e} - p_{t+1}^e\right) - (v_t + p_t^* - p_t) = \left[R_t - \left(p_{t+1}^e - p_t\right)\right] - \left[R_t^* - \left(p_{t+1}^{*e} - p_t^*\right)\right] - rp_t$$
$$\Delta rex_t^e = \left(R_t - \pi_{t+1}^e\right) - \left(R_t^* - \pi_{t+1}^{*e}\right) - rp_t.$$

If expected depreciation of the real exchange rate is assumed to react to deviations from the equilibrium real exchange $\overline{rex}$,

$$\Delta rex_t^e = \alpha\,(rex_t - \overline{rex})$$

the solution for the realized real exchange rate becomes

$$rex_t = \frac{1}{\alpha}\left[\left(R_t - \pi_{t+1}^e\right) - \left(R_t^* - \pi_{t+1}^{*e}\right)\right] + \overline{rex} - \frac{1}{\alpha}rp_t.$$

Finally, replacing expected with realized inflation and assuming a constant risk premium, the steady-state real exchange rate relationship becomes

$$rex = -.12\,[(R - \pi) - (R^* - \pi^*)] + \mu_v$$

The sign of α shows whether expectations are regressive ($\alpha < 0$) or extrapolitive ($\alpha > 0$), so in our case, since real interest rates are in percentage points, α is given as

$$\frac{1}{\alpha} = -.12 \times 100,$$

implying that expected depreciation is regressive with approximately 8 percent adjustment per period:

$$\Delta rex_t^e = -0.083\,(rex_t - \overline{rex})\,.$$

The long-run pass-through from the exchange rate and foreign prices onto import prices in domestic currency pi is represented by equation (48). It is a homogenous function of v and foreign producer prices pi^*, but the import price also increases if the real exchange rate (in terms of consumer prices) appreciates. This is due to pricing-to-markets in import price setting. Equation (48) is written in a way that shows the long-run relationship between the two operational definitions of the real-exchange rate. Hence, a one percent real appreciation in terms of consumer prices is associated with only 0.55 percent appreciation in terms of import prices.

The equations for wage formation and domestic price setting, in steady-state form, are given by (49)–(50) and have already been as discussed in section 2.6 above. Equation (51) models output per hour of labour input, or productivity z. Productivity is positively influenced by both the real wage $(w - p)$ and the unemployment rate u—corresponding to efficiency wage models—as well as neutral technological progress approximated by a linear trend.

The steady-state rate of unemployment in equation (52) is seen to depend on the growth of real wages (which is constant in the steady state), and the difference between the real-interest rate and the real GDP growth rate—see also Hendry (2001b), which can potentially change, and will then induce a change in the long-run mean of the rate of unemployment.

The four last equations represent the steady-state relationships for the market interest rates for loans R_L and bonds R_B, (53) and (54), followed by the equations for GDP, (55), and domestic credit (56). Government expenditure is seen to be important in the GDP equation, but aggregate demand is also influenced by the market for foreign exchange through the real exchange rate $(v + p^* - p)$ (dubbed *rex* above) and to the domestic real-interest rate. According to equations (55), and (56), secular growth in domestic real credit growth is conditioned by the growth of the real economy, not the other way around, even though in the dynamic model credit growth is an explanatory variable for GDP growth, cf. (44) and (45).

From the set of long-run relationships, it is straight forward to derive the steady state of the model. Differentiation of (47), gives inflation as

$$\Delta p = \Delta p^* + \Delta v, \tag{57}$$

and steady-state wage growth is then

$$\Delta w = \Delta p^* + \Delta z + \Delta v. \tag{58}$$

while import inflation follows from the price equation (49) and becomes

$$\Delta pi = \Delta p^* + \Delta v. \tag{59}$$

Note that these relationships represent the same qualitative conclusion that we obtained from analysis of the theoretical supply side model, compare equation (32) for example, which is therefore seen to generalize to the full set of economic steady state relationships.

In the light of the above, it also becomes clear that the economic long-run relationship (48) represents no separate restriction on the long-run relationship between growth rates, as it is a relationship between the marginal means of the two real exchange rates. The rest of the model can be solved for the steady state rate of productivity, and for the steady state unemployment level using (58), (51) and (52). Finally the GDP growth rate and for rate of growth in credit then follow from (47) and (56).

The above steady-state properties are derived for an exogenous policy interest rate R. The nature of solution is not changed if we instead specify either an estimated interest rate reaction function, on the basis of data, or a response function based on theories of optimal policy rules. But the behaviour of the dynamics will be affected, and the policy analysis, and the level of predicted long-run inflation will depend on how the interest rate is modelled.

We agree with Hendry and Mizon (2000) that there are reasons for being pragmatic about how the policy instrument is 'treated' in a macroeconometric model. For some purposes it is relevant to treat the instrument as exogenous, like in the analysis of the steady state (this sub-section) and its stability (next sub section). That analysis will answer whether there is a fundamental tendency of dynamic instability in the model that instrument use will have to counteract in order to avoid the economy taking an unstable course. Or, conversely, whether there is sufficient stability represented by the modelled relationships. In that case the challenges to instrument setting are more linked to timing and to meeting a specific inflation target for example, than to 'securing overall nominal or real dynamic stability.

In the following sections we will use both the open and the closed version of the NAM model, with respect to the policy instrument. In the next section, and the first section on policy use (section 4.2.1) we will use the open model with exogenous R_t. Later, for example when evaluating tracking performance in section 4.2.2, we will use a version with a econometrically modelled interest rate reaction function which is documented in Table 2. To discuss optimal policy in section 4.3, we will use a version with a theoretically derived interest rate response function.

3.3 Stability of the steady state

In an important paper, Frisch (1936) anticipated the day when it would become common among economists to define (and measure) 'normal' or natural values of economic variables by the values of the variables in a stationary state. The steady-state defined by the long-run model above correspond to such natural values of the model's endogenous variables. But it is impractical to derive all the "natural values" using algebra, even in such a simple system as ours. Moreover, the question about the stability of the steady state, e.g., whether the steady-state is globally asymptotically stable—or perhaps stability is only a saddle-path property—can only be addressed by numerical simulation of the dynamic system of equations.

We therefore follow Frisch's' suggestion and simulate the dynamic NAM model. Figure 3 shows the "natural values" for inflation (Δp_t above), the rate of unemployment, wage inflation, GDP growth, growth in import prices and the rate of currency depreciation. The first solution period is 2007(4) and the last solution period is 2035(4). The simulation period is chosen so long because we want to get a clear impression about whether these variables approach constant ("natural") values or not, and whether the effect of initial conditions die out, as they should if the solution is globally asymptotically stable. The simulation is stochastic. The solid lines represent the mean of the 1000 replications, and the 95% prediction intervals represented by the distance between the two dashed lines accommodate only residual uncertainty, not coefficient uncertainty.

In order to match the theoretical steady-state above as closely as possible, the solution in Figure 3 is based on a constant domestic money market interest rate ($R_t = R_{2007(3)}$). The exogenous $I(1)$ variables have also been given constant growth rates for the length of the simulation period, for example, foreign inflation is fixed at 2%. The impression is clearly that NAM has a asymptotically stable steady state, even under the assumption of a constant interest rate (the money market is by equilibrium by endogenous money supply in the counter factual 'regime' defined by the constancy of the interest rate). The annual rate of inflation is seen to stabilize at a level just above 2.5%, and the natural rate of unemployment (in the Frisch sense) appears to be 3%. Clearly, in this scenario an inflation rate of 2.5% is attainable with very moderate instrument use. The seasonality of unemployment, modelled by dummies, is clearly visible and represent no problem in terms

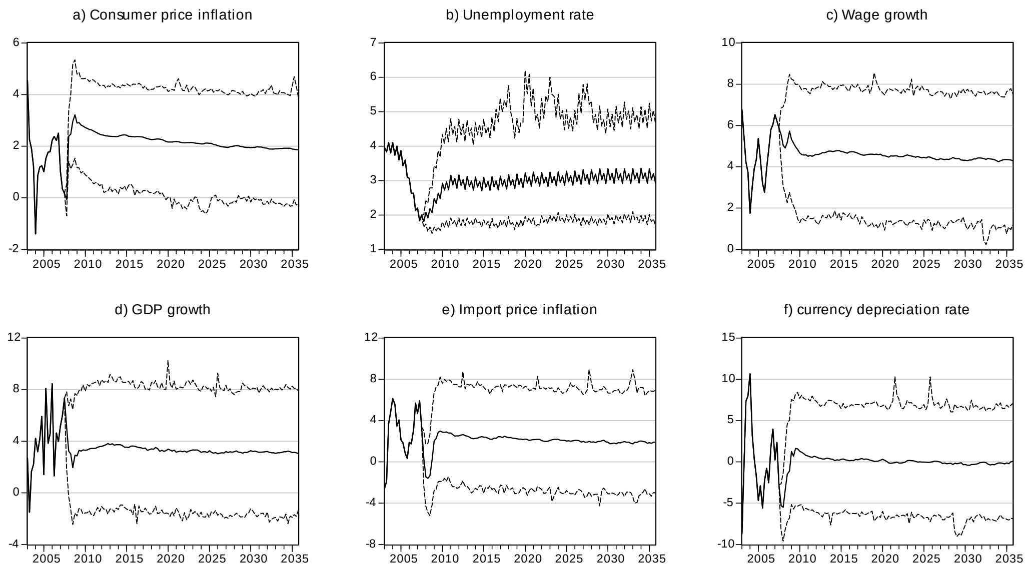

Figure 3: Stochastic dynamic simulation of NAM for the period 2007(4)-2035(4). The money market interest rate is kept constant at 2007(3) level for the length of the simulation period. The distance between the dotted lined indicate 95% prediction intervals See the text for explanation.

of stability.

4 Macroeconometric models as tools for policy analysis

In this last section we discuss several aspects of model usage that are of relevance for policy analysis. We begin with a practical problem, namely that congruent modelling, or a high degree of 'data coherence' to cite the influential Pagan (2003) report on monetary policy modelling, give rise to dynamic specifications that are too complicated too be of any help when the task is to explain the basic policy channels and lags between instruments and target. Nevertheless, section 4.1 shows that the need for simplicity in communication is not an argument for compromising empirical validity at the modelling stage, since it is always possible and convenient to work from the empirically valid and complicated model, to a simple and stylized model that contains the essential dynamics of the full model. In the other sub-sections, we use dynamic simulation, the main tool of model usage, to elucidate the strength of the policy instrument, to check that the model solution generates the properties of the actual data, and whether simulation over a long forecast horizon gives the steady-state we expect from the theoretical analysis above. We also discuss optimal policy response, forecast properties, and strategies to reduce the damage that structural breaks have on forecasts from equilibrium correction models. Finally in this section, we raise the more fundamental question of testing non-nested hypothesis about the supply side of the economy, our example will be the New Keynesian Phillips curve against the model of wage and price adjustment that have been presented above.

4.1 Tractability: Stylized representations

There is marked difference between the intricate and complex dynamics often found in empirical models — at least if they model the data—and the very simplified dynamics typically found in theoretical models. The purpose of this section is to enhance the understanding of the properties of a model through the use of stylized representations. A dynamic model, for example,

$$\Delta y_t = 2 - 0.4\Delta y_{t-1} - 0.6\Delta y_{t-2} \\ + 0.2\Delta x_t - 0.5\Delta x_{t-1} + 3\Delta x_{t-2} - 1\Delta x_{t-3} \\ - 0.5\left(y_{t-3} - 4x_{t-4}\right) + v_t,$$

can be approximated by a stylized model with simplified dynamics, in this example:

$$\Delta y_t = 1 + 0.85\Delta x_t - 0.25\left(y - 4x\right)_{t-1}.$$

This is achieved by using the mean of the dynamics of the variables.[18]

In the same way as above, we let lower case of the variables denote natural logarithms, so $\Delta z_t \approx \frac{Z_t - Z_{t-1}}{Z_{t-1}} = g_{z_t}$. If we assume that on average, the growth rates are constant—the variables could be "random walks with drift"—the expected values of the growth rates are constants:

$$E\Delta y_t = g_y \forall t$$
$$E\Delta x_t = g_x \forall t.$$

If the variables also are cointegrated, the expectation of the linear combination in the equilibrium correction term is also constant, so

$$E\left(y_{t-3} - 4x_{t-4}\right) = E\left(y_{t-1} - 4x_{t-1}\right) = \mu\forall t.$$

Under these assumptions, the mean dynamics of the model becomes:

$$\begin{aligned} E\Delta y_t &= 2 - 0.4E\Delta y_{t-1} - 0.6E\Delta y_{t-2} \\ &\quad + 0.2E\Delta x_t - 0.5E\Delta x_{t-1} + 3E\Delta x_{t-2} - 1E\Delta x_{t-3} \\ &\quad - 0.5E\left(y_{t-3} - 4x_{t-4}\right) + Ev_t \\ g_y\left(1 + 0.4 + 0.6\right) &= 2 + \left(0.2 - 0.5 + 3 - 1\right)g_x - 0.5\mu \\ g_y &= \frac{2}{2} + \frac{1.7}{2}g_x - \frac{0.5}{2}\mu \\ g_y &= 1 + 0.85g_x - 0.25\mu. \end{aligned}$$

We can therefore write the mean-approximated, or stylized, dynamic model as

$$\Delta y_t = 1 + 0.85\Delta x_t - 0.25\left(y - 4x\right)_{t-1}.$$

To illustrate, the dynamic behaviour of the model and its mean approximation are shown below. The upper panel shows the dynamic, or period, responses in y_t to a unit change in x_{t-i}. The latter panel shows the cumulative, or interim, response. The graphs illustrates how the cyclical behaviour—due to complex roots—is averaged out in the stylized representation.

Note that all that is done is to exploit so-called growth coefficients—see Patterson, K. D. and Ryding, J. (1984) and Patterson (1987). The steady-state growth

$$g_y = 4g_x$$

implies the steady-state mean μ as

$$\begin{aligned} \left(4 - 0.85\right)g_x &= 1 - 0.25\mu \\ \mu &= 4 - 12.6g_x, \end{aligned}$$

[18] Although the derivations are presented for a single equation with exogenous regressors, for ease of exposition, the techniques are of course the same for systems.

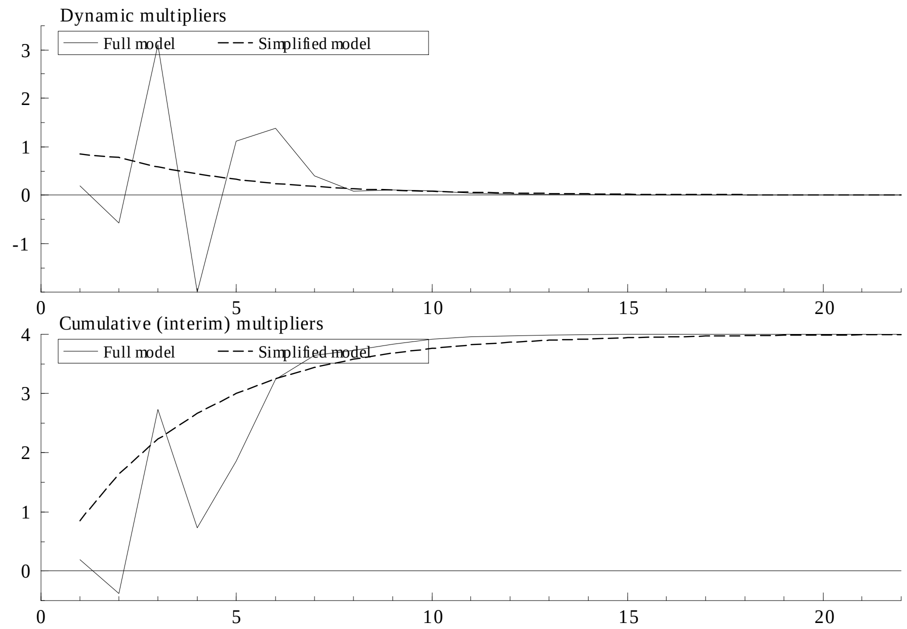

Figure 4: The dynamic responses of the example model and its mean approximation.

so the steady-state relationship between the variables is

$$y_t = (4 - 12.6g_x) + 4x_t,$$

which will hold both for the complete as well as the stylized dynamic representation of the model.

Using this approach, the full econometric model reported in Section 3.1 can be given the following stylized representation

$$\Delta v_t = -0.04\Delta(R - R^*)_t - 0.04\left\{(v + p^* - p) - 0.12\left[(R - \pi) - (R^* - \pi^*)\right]\right\}_{t-1} \quad (60)$$

$$\Delta\left(pi - pi^* - v\right)_t = -0.1\Delta v_t - 0.43\left[(pi - pi^* - v) - 0.55\left(p - p^* - v\right)\right]_{t-1} \quad (61)$$

$$\Delta p_t = -0.09\Delta z_t + 0.03\Delta pi_t + 0.08\Delta pe_t + 0.06\Delta y_t - 0.07\left[p - 0.7(w - z) - 0.3pi\right]_{t-1} \quad (62)$$

$$\Delta\left(w - p\right)_t = -0.04\Delta u_t + 0.73\Delta T1_t - 0.07\left[(w - p - z) + 0.1u\right]_{t-1} \quad (63)$$

$$\Delta z_t = 0.09\Delta\left(w - p\right)_t - 0.24\left[z - 0.47\left(w - p\right) - 0.003Trend - 0.03u\right]_{t-1} \quad (64)$$

$$\Delta u_t = -0.23\left\{u - 7.65\Delta\left(w - p\right) - 4.46\left[0.01\left(R_L - \pi\right) - 4\Delta y\right]\right\}_{t-1} \quad (65)$$

$$\Delta R_{L,t} = 0.58\Delta R_t - 0.33\left(R_L - 0.41R_B - 0.76R\right)_{t-1} \quad (66)$$

$$\Delta\left(R_B - R_B^*\right)_t = 0.43\Delta R_t - 0.17\left(R_B - 0.43R - 0.57R_B^*\right)_{t-1} \quad (67)$$

$$\Delta y_t = 0.16\Delta g_t + 0.38\Delta\left(l - p\right)_t - 0.12\left[y - 0.9g_{t-1} - 0.16(v + p^* - p) + 0.06\left(R_L - \pi\right)\right]_{t-1} \quad (68)$$

$$\Delta\left(l - p\right)_t = 0.3\Delta y_t - 0.09\left[(l - p) - 2.65y + 0.04(R_{L,} - R_B)\right]_{t-1} \quad (69)$$

where the constants are omitted for ease of exposition. This representation reproduces the same steady-state as the full model, but with stylized dynamics. The averaged transmission mechanisms can be traced through the interrelationships of mean dynamic effects of shocks to the model—in contrast to the steady-state effects described above. See the discussion in Section 3.1 for an example.

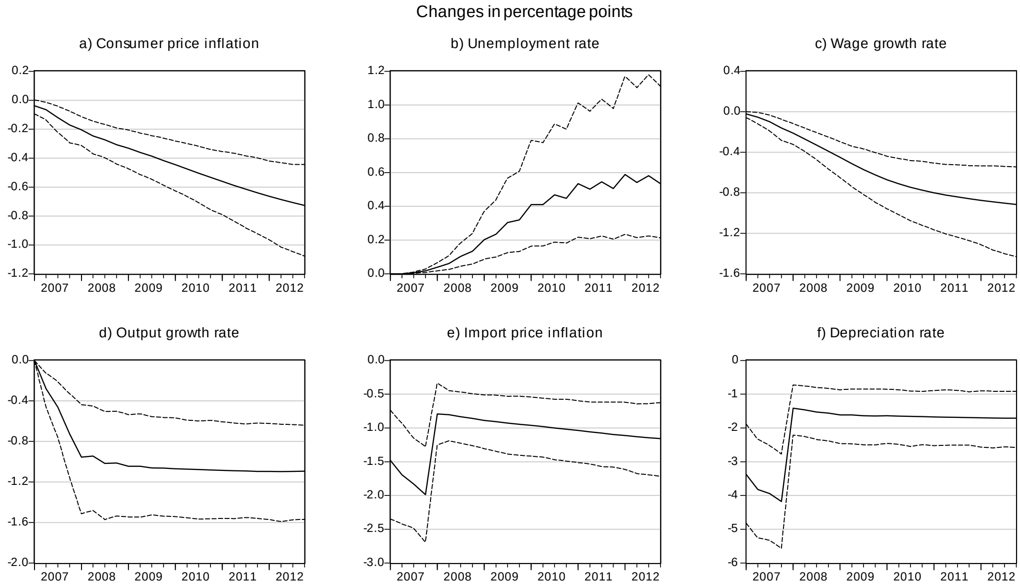

Figure 5: Dynamic multipliers from a permanent increase in the money market interest rate by 100 basis points, 2007(1)-2012(4). The multipliers are shown as deviations from the baseline simulation in figure 3. The distance between the two dotted lines represent the 95 % confidence intervals.

4.2 Shock analysis: Dynamic simulations

It seems obvious that a model for policy analysis should be empirically adequate to a high degree, and that care must be taken not to compromise this property for other desirable properties. However, in the same as way as with stability and invariance, "data fit" is a relative concept. Models that are outperformed in terms of fit might still be useful for policy analyses because they are highly relevant for the purpose, as noted by e.g., Pesaran and Smith (1985). This subsection will discuss relative model adequacy—also compared to optimal monetary policy.

4.2.1 How strong is the policy instrument?

As a background to policy analysis it is important to obtain a quantitative view of the transmission mechanisms, specifically to see whether the policy instrument has a sizeable effect on the variables that are subject to central bank targeting (in formal or more informal ways). The open version of the model, with exogenous interest rate (R_t), is then relevant, since among the parameters of the model are the dynamic multipliers of the endogenous variables with respect to the policy instrument.

Since the main channel of interest rate transmission is through the exchange rate, output and the level of unemployment, the interest rate is actually quite effective in counteracting demand shocks. However, shocks on the supply side, for example in wage setting (e.g. permanently increased wage claims), or in foreign inflation, can be more difficult to curb by anything but huge increases in the interest rate. Hence, stabilization of inflation after a supply shock may represent a formidable challenge to monetary policy.

The recent monetary history of Norway has demonstrated the relevance of this analysis: Since mid 2003 core inflation rate has been consistently below the target of 2.5%, for long periods less than 1%, despite determined interest rates cuts in 2003, and no interest rate increases before July 2005, and then only gradually and in small steps.

The interest rate multiplier on inflation is shown in panel a) of Figure 5. Panel f) shows that the effect first goes through the *exchange rate channel*: the impact multiplier is a 3 percent appreciation, and in steady-state there is a appreciation of 1.2 percent, thus illustrating equation (57). Panel e) shows that import price growth is affected in the same way as the exchange rate, see equation (59), but that the impact effect is smaller. Panels b), c) and d) show the *labour market* and *demand channels*. Wage growth is reduced, as predicted by (58), leaving real wage growth unaffected in steady state, as this is determined by productivity growth. Unemployment increases, since real interest rates increases, while the negative GDP growth follows from increasing real interest rates and appreciating real exchange rates.

4.2.2 Fitting the facts

In this section we document how well the econometric model NAM explains the evolution of important endogenous variables over the 17 quarter period from 2003(3)-2007(3).

The solutions are conditional upon the actual values of the non-modelled variables.[19] Experience has shown that particularly important explanatory variables are foreign interest rates (R^* and R_B^*), and consumer (p^*) and producer prices (pi^*). Domestic government expenditure (g) and electricity prices (pe), are also very important for the overall fit of the model. Finally, oil prices (in USD) is a highly relevant explanatory variable, mainly through the market for foreign exchange.

The interest rate has been the instrument of monetary policy during the solution period. We therefore use the (more) closed version of the model, where the domestic money market interest rate is estimated as a function of the core inflation gap and the unemployment rate gap in equation (46).

Figure 6 shows that NAM generates the features of the macroeconomic development. Inflation in panel a), the rate of unemployment in panel b), GDP growth in panel d), and the money market interest rate in panel g) are very well explained by the model solutions. The graph of actual and simulated nominal currency depreciation in panel i) shows that movements in the exchange rate is also well explained, brought about by the interest rate differential and equilibrium correction with respect to the real exchange rate—which is shown in panel f).

4.2.3 Shock analysis with dynamic multipliers

Next, we investigate how the economy, according to the model, is likely to respond to shocks. We use the same model version as in section 4.2.2. Amongst the many shocks of interest to a small open economy we here consider a negative foreign price shock. The deflationary 5% shock occurs in 2007(1) and Figure 7, panels a)-f), reports the dynamic multipliers for inflation, wage increases, the rate of unemployment, GDP growth, the increases in the import price index, and the rate of currency depreciation. Since all variables in the graph are measures in percentage points, the multipliers shown are absolute deviations from the baseline. In the same way as in Figure 5, parameter uncertainty are indicated by the dotted lines, representing 95% confidence intervals.

Since it is a temporary shock to foreign inflation, there is no reason to expect a permanently lower rate of domestic inflation. Panels a) and c) of Figure 7 confirms that presumption, but also shows that the first multipliers of price and wage inflation are negative and significant. Part of the adjustment to a lower nominal path of the economy involve higher unemployment rate and lower GDP growth, cf. panel b) and d), which is explained by the initial appreciation of the real exchange rate. The nominal depreciation in panel f), due to a lower domestic interest rate (not shown) is not enough to offset the loss of competitiveness, initiated by the deflationary price shock.

[19] A full listing of variables is given in the appendix.

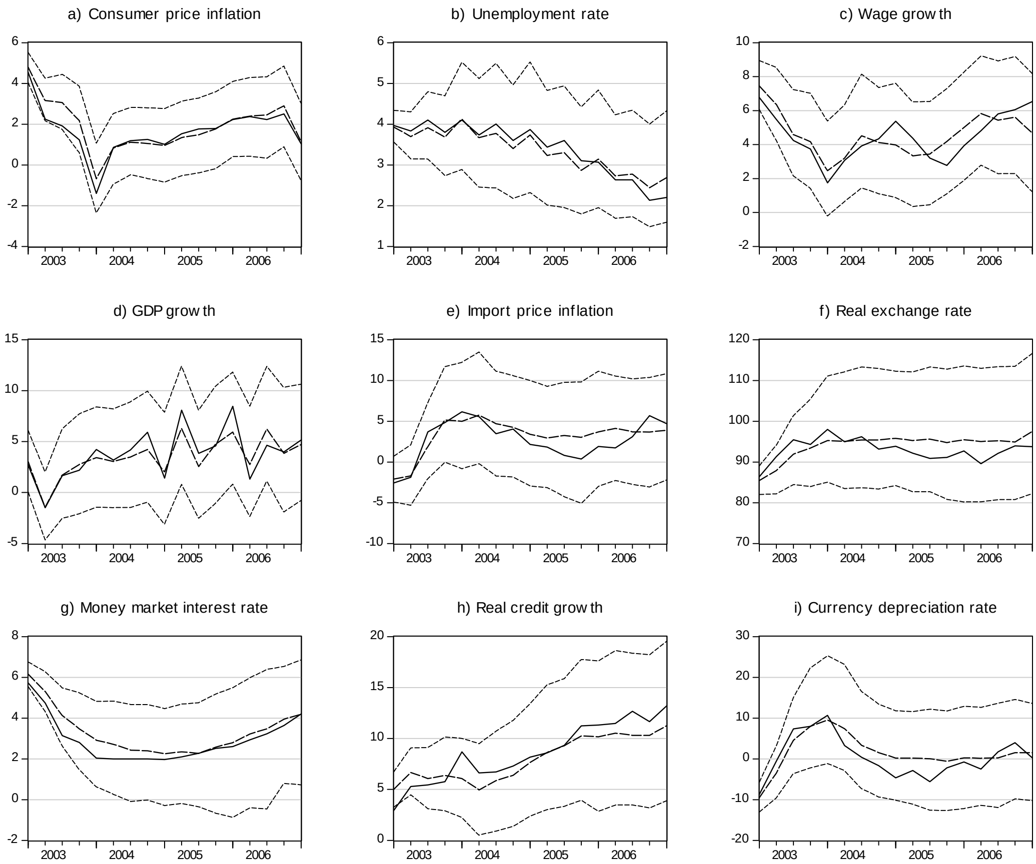

Figure 6: Dynamic simulation of NAM 2003(1)-2007(1). Actual values are indicated by solid lines, and model solution by dashed lines. The distance between the two dotted lines represents 95 % prediction intervals. The units on all the vertical axes are percent, and time is along the horizontal axes.

The nominal rigidities in the model, which transforms the nominal shock into a change in the real exchange rate,.is an important property in explaining the multipliers. The rigidities are not due to 'incredibly long' adjustment lags in price and wage setting: Domestic inflation, for example. is seen to return to its baseline path after $2\frac{1}{2}$ years. Instead, there is a second wave of effects due to the interaction between product and labour markets. Hence, the experiment demonstrates rather well the important theoretical point made in 2.6.4, namely that the steady-state inflation rate does not imply a unique equilibrium rate of unemployment, since the rate of inflation reaches its steady state long before the multipliers of the rate of unemployment have died out.

4.3 Aspects of optimal policy: The impact of model specification on optimal monetary policy

As noted above, the version of NAM with an econometrically modelled interest rate makes no claims of representing optimal interest rate setting under inflation targeting, or optimal policy response to a shock. In stead, the multipliers of the last paragraph should be interpreted as counterfactuals: They show the response that would occur if the interest rate reacted *as if* it followed the econometrically estimated interest rate equation.

However, in a recent paper Akram and Nymoen (2008) show how optimal monetary policy can be implemented in NAM, and how the predicted economic outcome depends on the specification of the supply side of the model. For that purpose, they replace the

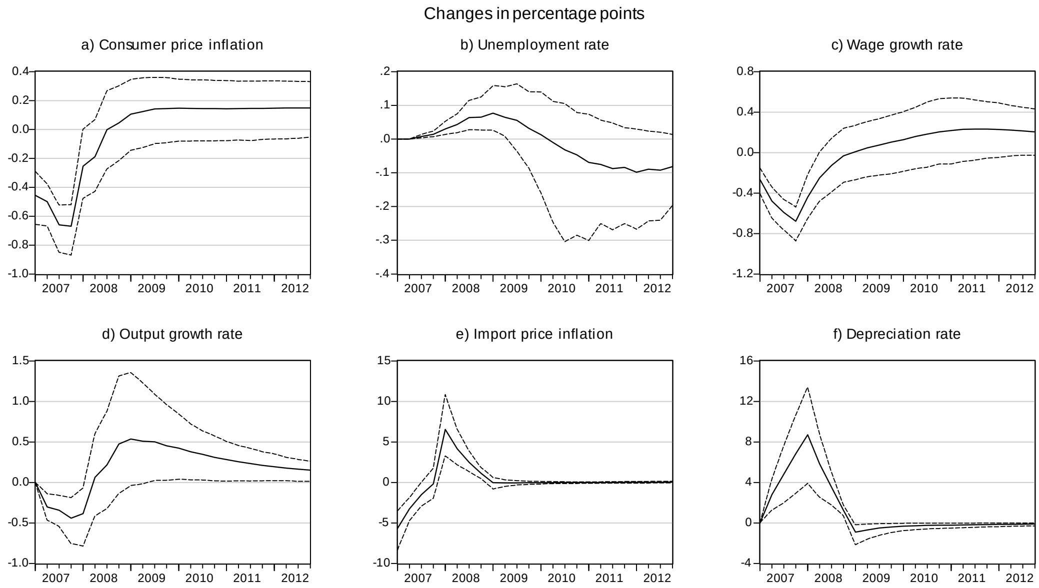

Figure 7: Dynamic multipliers from a permanent reduction in exogenous (foreign) prices by 5%, 2007(1)-2012(4). The distance between the two dotted lines represent the 95 % confidence intervals. The unit on the vertical axes are percentage points in all panels.

econometrically modelled interest rate equation with a theoretically derived interest rate rule due to Akram (2007):

$$R_{t+m} = R_0 + (1 - \varrho_H)\frac{\beta_\varepsilon}{(1-\phi)}\varepsilon_t + \varrho_H(R_{t+m-1} - R_0),\ m = 0, 1, 2..., H. \tag{70}$$

This rule defines an interest rate path corresponding to a specific policy horizon H. The response coefficient $\beta_{\varepsilon,H} \equiv (1 - \varrho_H)\frac{\beta_\varepsilon}{(1-\phi)}$ determines by how much the interest rate must deviate initially from the neutral rate R_0 to counteract inflationary effects of a shock ε_t. A high value of the smoothing parameter ϱ_H can be associated with a strategy of gradualism in interest rate setting. Thus two preference parameters $\beta_{\varepsilon,H}$ and ϱ_H depend on the policy horizon, H. The last parameter in the theoretical interest rate equation is ϕ, which represents the (objective) degree of persistence in the inflation shock.

In addition to the preferences about policy horizon and gradualism captured by (70), the optimal interest rate response is influenced by the user's choice of macroeconomic model. Akram and Nymoen focus on the labour market channel, since this part of the transmission mechanism has the been the focus of most model controversy. In one model version, used to derive optimal policy, the incumbent model of the supply side, with equilibrium correction in both wage and price setting is used. As explained in section 2.6 above this model is called the incomplete model of wage and price setting and is referred to as ICM in Figure 8. The two other version are: The model with wage-and price Phillips curves, PCM in the figure, and a version with a vertical Phillips curve, PCMr in the figure. As noted above, in section 2.6.4, among these three specifications, it is the PCMr that represents the consensus view in modern monetary economics.

Akram and Nymoen shows that econometric encompassing tests favour the ICM model of the supply side, but also that the PCM and the PCMr appears to be well specified on their own terms, for example the residual properties of the two Phillips curve models do not signal any problems. Hence there is a question about whether the outcome of the encompassing test has any practical (or "economic"), significance, or whether this test of model adequacy only has an academic interest. The analysis suggest that the econometric test result contains valuable information.

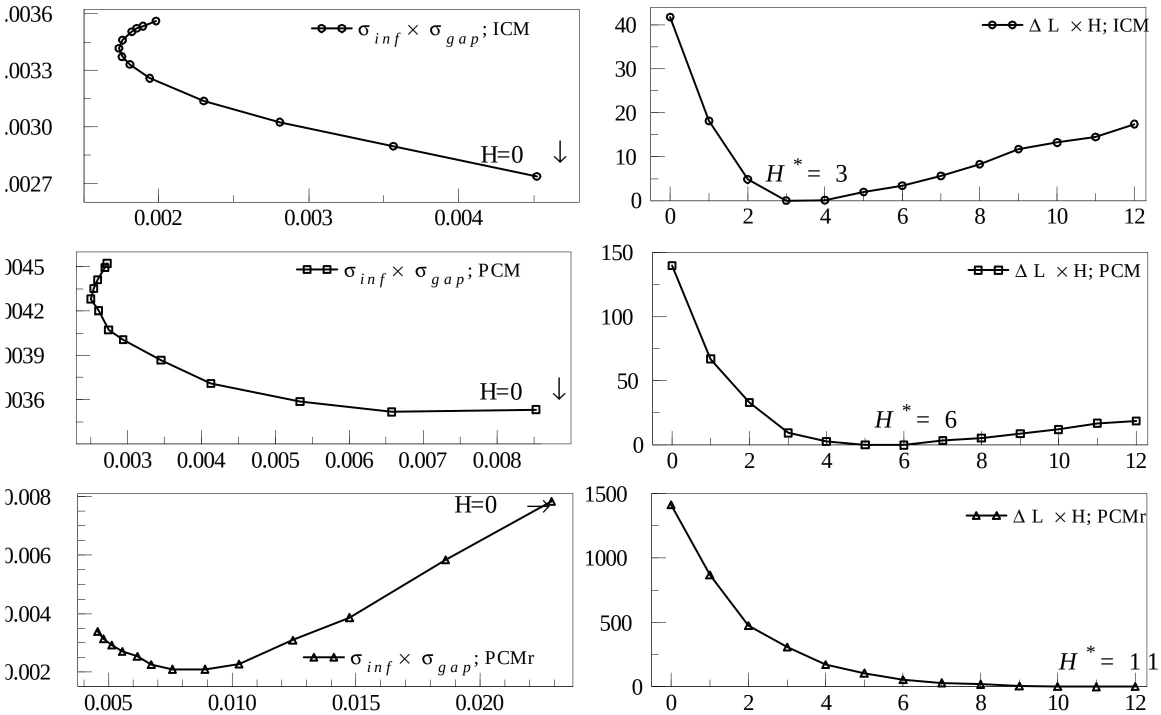

Figure 8: Economic performance and optimal policy suggested by three specifications of the supply side in the face of a supply shock. Left column: Trade-offs between standard deviations of inflation gap, σ_{inf}, and output gap, σ_{gap} (horizontal axis) associated with different (policy) horizon-specific rules in response to the demand shock. The trade-offs are plotted for rules associated with policy horizons (H) in the range of 0–12 quarters. The trade-offs associated with different horizons follow each other, where that one for H=0 is indicated. Right column: Values of the relative loss function denoted ΔL (in %), at the different policy horizons (horizontal axis). Akram and Nymoen (2008)

Consider for example Figure 8 which presents the economic performance of (optimal and sub-optimal) policies employed in response to the supply shock. The left column of the figure shows that there is a trade-off between price and output stabilization for different ranges of policy horizons. Specifically, in the case of ICM and PCM there is a trade-off in the range of 0 to 8 quarters. Policy horizons that are longer than 8 quarters appear inefficient as both price and output stabilization can be improved by shortening the policy horizon. The opposite is the case for PCMr. In this case, the trade-off curve is associated with policy horizons that are longer than 6 quarters, while policy horizons shorter than 6 seem inefficient. In the right column of the figure, it transpires that the three models recommend substantially different policy horizons. Even though the efficiency frontiers for ICM and PCM are defined by almost the same policy horizon, the optimal horizon is 3 quarters conditional on ICM, but 6 quarters in the case of PCM. In the case of PCMr the policy horizon is 11 quarters.

Based on the above and several other simulation experiments, Akram and Nymoen find that econometric differences bear heavily on (model-based) policy recommendations and are thus not merely of academic interest. Their analysis quite strongly suggests that differences in model specifications and even in parameter values across models can lead to widely different policy implications. Interestingly, it appears that imposing a set of parameter restrictions may have stronger influence on policy implications than choosing a different functional form of the model. Monetary policy based on a model that turns out to be an invalid characterization of the economy and its transmission mechanism may lead to substantial losses in terms of economic performance, even when policy is guided by gradualism, for example in the form of a long policy horizon.

4.4 Theory evaluation: The New Keynesian Phillips curve

Macro economics is an evolving science. New hypotheses and theories are put forward, sometimes with far-reaching consequences: for policy, teaching, and of course also for model building. A macroeconomic model project that lasts for some time is therefore bound to face the need to adopt to new theoretical developments. However, new ideas in macro economics are usually partial, and can claim superiority with respect to existing ones only by replacing old *ceteris paribus* clauses by new ones. In a macroeconometric modelling context it therefore makes sense to test the new theories before they are implemented. If the model is in operational use, for policy recommendations for example, this step may be as much of a virtue as a necessity, since unverified changes in policy response and forecasting may critically damage beliefs in the relevance of model analysis.

In this section we discusses some approaches to how the model of the supply side presented in section 2.6 can be tested against a new and important development represented by the New Keynesian Phillips curve.

As showed in Bårdsen et al. (2005, Ch.4-6) three important and much used models of inflation and unemployment are consistent with the view that wages and prices are equilibrium correcting $I(1)$ variables, while the rate of unemployment is $I(0)$. They are: the incomplete competition model in equilibrium correction form, the standard open economy wage Phillips curve and the wage Phillips curve with homogeneity restrictions, in other words ICM, PCM and PCMr of section 4.3. *Qua* equilibrium correction models, these theories can be readily be identified as special cases of a VAR.

Cointegration is thus a common feature of the three mentioned economic models. They can be identified with their different theories about the main adjustment mechanisms at work. In the case of the WCM, wages equilibrium correct with respect to deviations from the wage level predicted by the theoretical bargaining model. In the case of the PCM, by definition, wages do not equilibrium correct with respect to lagged wages, so in this model equilibrium correction has to be *indirect* and via the reaction of the rate of unemployment, see Bårdsen and Nymoen (2008). In section 4.3 these properties were shown to be relevant for the assessment of different supply-side models for policy analysis.

We will show below that the same insight also applies to the New-Keynesian Phillips curve, meaning that its equilibrium correction implications can be tested against the ICM or (any version) of conventional Phillips curves. However, we first give a brief summary of the current empirical status of the New Keynesian Phillips curve.

4.4.1 The New Keynesian Phillips curve

The New Keynesian Phillips Curve, hereafter NPC, has become regarded by many as the new standard model of the supply side in the macro models used for monetary policy analysis. This position is due to its theoretical underpinnings, laid out in Clarida et al. (1999), and to the supportive empirical results in the studies of Gali and Gertler (1999), henceforth GG) on US data, and Gali et al. (2001) (henceforth GGL) on euro-area data.

The hybrid NPC is given as

$$\pi_t = \underset{\geq 0}{a^f} E_t[\pi_{t+1}] + \underset{\geq 0}{a_b^b} \pi_{t-1} + \underset{\geq 0}{b} s_t, \tag{71}$$

where π_t is the rate of inflation, $E_t[\pi_{t+1}]$ is expected inflation one period ahead and s_t is a time series of firms' real marginal costs. When the "pure" NPC is (71) with $a^b = 0$, and represents the case where all firms firms (that are aggregated over) form rational expectations. Both the pure and the hybrid form are usually presented as "exact", i.e., without an error term. When $E_t[\pi_{t+1}]$ is replaced by π_{t+1} for estimation, a moving average error term is implied. This has motivated "robust" estimation with for example pre-whitening switched on in GMM, and leading papers downplay the relevance of congruency for the evaluation of the NPC.

After assessing some of the critique that have been directed towards the NPC, Gali et al. (2005) assert that the NPC, in particular the dominance of forward-looking behaviour, is robust to choice of estimation procedure and to possible specification bias. They conclude that the following three results are proven characteristics of NPC for all data sets:

1. The two null hypotheses $a^f = 0$ and $a^b = 0$ are rejected both individually and jointly.

2. The coefficient on expected inflation exceeds the coefficient on lagged inflation substantially. The hypothesis of $a^f + a^b = 1$ is typically not rejected at conventional levels of significance, although the estimated sum is usually a little less than one numerically.

3. When real marginal costs are proxied by the log of the wage-share, the coefficient b is positive and significantly different from zero at conventional levels of significance.

As mentioned, critics of the NPC have challenged the robustness of all three, but with different emphasis and from different perspectives. The inference procedures and estimation techniques used by GG and GGL(2001) have been criticized by Rudd and Whelan (2005, 2007), and others, but GGL (2005) show that their initial Results 1 and 2. remain robust to these objections.

When it comes to # 3, GGL (2005) overlook that several researchers have been unable to confirm their view that the wage-share is a robust explanatory variable in the NPC. Bårdsen et al. (2004b) showed that the significance of the wage share in the GGL (2001) model is fragile, as it depends on the exact implementation of the GMM estimation method used, thus refuting that result 3 is a robust feature of NPC estimated on euro-area data.

Fanelli (2007) using a vector autoregressive regression model on the euro-area data set, find that the NPC is a poor explanatory model. On the US data, Mavroeidis (2006) have shown that real marginal costs appears to be an irrelevant determinant of inflation, confirming the view in Fuhrer (2006) about the difficulty of developing a sizeable coefficient on the forcing variable in the US NPC. Already the studies cited represent evidence that refutes the claim that 3 is robust. Instead it is to be expected that depending on the operational definition of real marginal costs, the estimation method and the sample, the numerical and statistical significance of b will vary across different studies.

Of course, result 3 is just as important as 1. and 2. for the status of the NPC as an adequate model, so if that part of the model is non-structural, it might be that be that 1. and 2. have another explanation than the intended, which is that there is a good match between the NPC and the true inflation process. Bårdsen et al. (2004b) (euro area) and Bjørnstad and Nymoen (2006) (OECD panel data) demonstrated hat the significance of a^f can be explained by a linear combination of better forcing variables which resides among the overidentifying instruments. Their presence is revealed by the significance of the Sargan (1964) specification test. Importantly, the re-specified model in the two mentioned studies lend themselves directly to interpretation either as conventional Phillips curves, or as an equilibrium correction price equation consistent with the theory of monopolistic competition in product market and a certain element of coordination in wage bargaining, see Sargan (1980), Nymoen (1991), Bårdsen et al. (2005, Ch 4-6). Hence, the NPC fail to parsimoniously encompass these models.

4.4.2 The equilibrium correction implications of the NPC

The original NPC makes no reference to open economy issues. Batini et al. (2005) have shown that the main theoretical content of the NPC generalizes, but that consistent estimation of the parameters a^f, a^b and b requires that the model is augmented by variables which explain inflation in the open economy case. Hence, the open economy NPC is

$$\Delta p_t = \underset{\geq 0}{a^f} \Delta p^e_{t+1} + \underset{\geq 0}{a^b} \Delta p_{t-1} + \underset{\geq 0}{b} s_t + c\, x_t, \tag{72}$$

where x_t, in most cases a vector, contains the open-economy variables, and c denotes the corresponding coefficient vector. The change in the real import price, $\Delta(pi_t - p_t)$ in our notation, is the single most important open economy augmentation of the NPC. The results in Batini et al. (2005) are, broadly speaking, in line with GG's and GGL's properties 1.-3. above, but as noted above, those properties are not robust when tested against the existing UK model in Bårdsen et al. (1998).

We follow GG and measure s_t by the log of the labour share:

$$s_t = ulc_t - q_t, \tag{73}$$

where ulc denotes unit labour costs (in logs) and q is the log of the price level on domestic goods and services, compare section 2.6 above. Next, define the aggregate price level as

$$p_t = \zeta q_t + (1 - \zeta)\, pi_t. \tag{74}$$

with $(1 - \zeta)$ as the import share. If we solve for q_t, insert in (73) and re-write, we obtain the following equation for the wage-share:

$$s_t = -\frac{1}{\gamma}\left[p_{t-1} - \gamma\; ulc_{t-1} - (1-\gamma)\, pi_{t-1}\right] + \Delta ulc_t - \frac{1}{\gamma}\Delta p_t + \frac{1-\gamma}{\gamma}\Delta pi_t. \tag{75}$$

We can then re-write the open economy NPC as

$$\begin{aligned}\Delta p_t = &\frac{a^f}{\left(1+\frac{b}{\gamma}\right)}\Delta p^e_{t+1} + \frac{a^b}{\left(1+\frac{b}{\gamma}\right)}\Delta p_{t-1} - \frac{b}{(\gamma+b)}\left[p_{t-1} - \gamma\; ulc_{t-1} - (1-\gamma)\, pi_{t-1}\right] \\ &+ \frac{\gamma\; b}{(\gamma+b)}\Delta ulc_t + \frac{b\,(1-\gamma)}{(\gamma+b)}\Delta pi_t + \frac{\gamma\; c}{(\gamma+b)}\, x_t,\end{aligned}$$

or

$$\begin{aligned}\Delta p_t = &\alpha^f \Delta p^e_{t+1} + \alpha^b \Delta p_{t-1} + \beta(ulc_{t-1} - p_{t-1}) - \beta\,(1-\gamma)\,(ulc_{t-1} - pi_{t-1}) \\ &+ \beta\; \gamma\; \Delta ulc_t + \beta\,(1-\gamma)\,\Delta pi_t + \psi x_t,\end{aligned} \tag{76}$$

where we have defined α^f, α^b, β and ψ as new coefficients for simplification.

Equation (76) brings out that the NPC implies an equilibrium correction price equation which is very similar to the 'incumbent' model in NAM, cf. equation (30) in 2.6. However there are two notable differences. First, and foremost, the forward-looking term Δp^e_{t+1} with an expected high coefficient α^f. The incumbent model of course implicitly restricts this coefficient to zero. Second, in the NPC, there are parameter restrictions on coefficients or the following variables Δulc_t, Δpi_t , $(ulc_{t-1} - p_{t-1})$ and $(ulc_{t-1} - pi_{t-1})$, in fact they are functions of the underlying parameters b and γ.

It follows that for the purpose of testing the NPC we can start with a an equilibrium correction model with a lead in the inflation term:

$$\begin{aligned}\Delta p_t = &\alpha^f \Delta p^e_{t+1} + \alpha^b \Delta p_{t-1} + \beta_1(ulc_{t-1} - p_{t-1}) + \beta_2(ulc_{t-1} - pi_{t-1}) \\ &+ \beta_3 \Delta ulc_t + \beta_4 \Delta pi_t + \psi x_t.\end{aligned} \tag{77}$$

and test the following hypotheses: H_0^a: $\alpha^f = 0$, H_0^b: $\beta_3 = \beta_1 + \beta_2$ and H_0^c: $\beta_4 = -\beta_2$. Rejection of H_0^a together with non-rejection of H_0^b and H_0^c constitute evidence that support the NPC, while non-rejection of H_0^a is telling evidence against the NPC.

As noted above, NPC models are usually specified with the rate of change in the real import price as one of the elements in x_t. Equation (77) is consistent with that interpretation, the only caveat applies to β_4 and H_0^b, since $\beta_4 = -\beta_2$ no longer follows logically from the NPC. This is because β_4 is a composite parameter also when the NPC is the valid model.

4.4.3 Testing the equilibrium correction implications of the NPC

Consider the hybrid NPC of the form (72) where we allow for two lags of inflation as well as three deterministic seasonals. This is because NAM makes use of seasonally unadjusted quarterly data. The wage-share variable s_t is treated as an endogenous variable, but we also include electricity prices (Δpe_t) and import prices (Δpi_t) in the x_t vector of exogenous variables. As already noted inflation is measured by the consumer price index. IV estimation gives:

$$
\begin{aligned}
\Delta p_t \;=\; & \underset{(0.107)}{0.3659}\ \Delta p_{t+1} + \underset{(0.0228)}{0.04122}\ s_t + \underset{(0.0772)}{0.08759}\ \Delta p_{t-1} \\
& + \underset{(0.0653)}{0.2676}\ \Delta p_{t-2} + \underset{(0.00597)}{0.06385}\ \Delta pe_t + \underset{(0.0169)}{0.04024}\ \Delta pi_t \\
& + constant and\ seasonals \\
& \text{IV, } T = 111\ (1979(3)\text{ - }2007(1)) \\
& \chi^2_S(9) = 11.664[0.2329]
\end{aligned}
\tag{78}
$$

The results shows a significant coefficient of the forward term of the same magnitude as the sum of the coefficient of the two lagged inflation rates. The wage-share s_t has the correct positive sign and is significant at a 10% level. By and large, these results are in line with the typical NPC results 1-3 discussed above.

The instruments include $ulc_{t-1} - p_{t-1}$ and $ulc_{t-1} - pi_{t-1}$, as explained above, together with lags of productivity growth, lagged electricity price growth, the same wage dummy as in NAM and lagged unemployment. The Sargan (1964) test of instrument validity ($\chi^2_S(n)$) is insignificant. However, residual misspecification tests reveal that (78) is not a congruent model, since substantial heteroscedasticity, autocorrelation and non-normality. In order to obtain a more congruent NPC model we may use the dummy saturation technique in Autometrics, see Doornik (2008).

$$
\begin{aligned}
\Delta p_t \;=\; & \underset{(0.102)}{0.622}\ \Delta p_{t+1} + \underset{(0.013)}{0.02509}\ s_t + \underset{(0.0533)}{0.2614}\ \Delta p_{t-2} \\
& + \underset{(0.00675)}{0.0549}\ \Delta pe_{tt} + constant\ and\ 11\ dummies \\
& \text{IV, } T = 111\ (1979(3)\text{ - }2007(1)) \\
& \chi^2_s(15) = 19.442[0.1944]
\end{aligned}
\tag{79}
$$

The 11 dummies include both seasonals and break dummies, showing that the NPC equation is not a completely time invariant structural model. However, abstracting from that problem equation (79) is seen to adhere even closer to the stylized facts of the NPC than equation (78).

As explained above, we want to test the hypothesis that (79) encompasses the price equation of the incumbent model. To do that, we first include $ulc_{t-1} - p_{t-1}$ and $ulc_{t-1} - pi_{t-1}$ as explanatory variables to obtain an empirical version of the embedding equation (77) above. We then do a general-to-specific search by means of *autometrics* in PcGive.

The preferred model is reported as equation (80):

$$\begin{aligned}
\Delta p_t \;=\;& \underset{(0.069)}{0.003849}\ \Delta p_{t+1} + \underset{(0.0259)}{0.08832}\ \Delta ulc_t + \underset{(0.0577)}{0.1515}\ \Delta p_{t-2} \\
&+ \underset{(0.0143)}{0.09883}\ (ulc_{t-1} - p_{t-1}) - \underset{(0.00478)}{0.01943}\ (ulc_{t-1} - pi_{t-1}) + \underset{(0.00518)}{0.05164}\ \Delta pe_t \\
&+ Constant + 4\ dummies \\
&\text{IV, } T = 111 \text{ (1979(3) - 2007(1))} \\
&\chi^2_S(16) = 25.434[0.0625]
\end{aligned} \tag{80}$$

It is seen that the estimated forward coefficient $\hat{\alpha}^f$ is practically zero in (80) compared to 0.62 in the NPC in (79). Hence the H_0^c: $\alpha^f = 0$ cannot be rejected statistically at any meaningful level of significance.

4.5 Forecasting for monetary policy

A hallmark of modern and flexible inflation targeting is that the operational target variable is the forecasted rate of inflation, see i.e., Svensson (1997). One argument for this choice of target is to be ahead of events, rather than to react after actual inflation has deviated from target. In this way one may hope to achieve the target by a minimum of costs to the real economy in terms of e.g., unwanted output fluctuations or large fluctuations in the exchange rate. However, any inflation forecast is uncertain, and might induce wrong use of policy. Hence, a broad set of issues related to inflation forecasting is of interest for those concerned with the operation and assessment of monetary policy.

A favourable starting point for inflation targeting is when it can be asserted that the central bank's forecasting model is a good approximation to the inflation process in the economy. In this case, forecast uncertainty can be represented by conventional forecast confidence intervals, or by the fan-charts used by today's best practice inflation targeters.[20] The point of the probabilistic forecasts is to convey to the public that the forecasted inflation numbers only will coincide with the actual future rate of inflation on average, and that neighbouring inflation rates are almost as probable. By the same token, conditional on the forecasting model's representation of uncertainty, still other inflation rates are seen to be wholly improbable realizations of the future.

However, the idea about model correctness and stationarity of macroeconomic processes is challenged by the high incidence of failures in economic forecasting, see e.g., Hendry (2001a). A characteristic of a forecast failure is that forecast errors turn out to be larger, and more systematic, than what is allowed if the model is correct in the first place. In other words, realizations which the forecasts depict as highly unlikely (e.g., outside the confidence interval computed from the uncertainties due to parameter estimation and lack of fit) have a tendency to materialize too frequently. Hence, as a description of real-life forecasting situations, an assumption about model correctness is untenable and represents a fragile foundation for forecast based interest rate setting, see Bårdsen et al. (2003).

4.5.1 Assumptions about the forecasting situation

In modern monetary policy the forecasted rate of inflation is the intermediate target. It is then of interest to clarify as closely as possible what are the realistic properties of the forecast. Anticipated forecast properties are closely linked to the assumptions we make about the forecasting situation. An useful classification, see Clements and Hendry (1999, Ch 1), is:

[20] See e.g., Ericsson (2001) for a accessible discussion of forecast uncertainty, and its presentation in published forecasts.

A The forecasting model coincides with the true inflation process except for stochastic error terms. The parameters of the model are known constants over the forecasting period.

B As in A, but the parameters have to be estimated.

C As in B, but we cannot expect the parameters to remain constant over the forecasting period—structural changes are likely to occur.

D We do not know how well the forecasting model correspond to the inflation mechanism in the forecast period.

A is an idealized description of the assumptions of macroeconomic forecasting. There is still the incumbency of inherent uncertainty represented by the stochastic disturbances—even under A. Situation B represents the situation theoretical expositions of inflation targeting conjure up, see Svensson (1999). The properties of situation A will still hold—even though the inherent uncertainty will increase. If B represents the premise for actual inflation targeting, there would be no forecast failures, defined as a significant deterioration of forecast performance relative to in-sample behaviour, see e.g. Hendry (2006). The within-sample-fit of section 4.2.2 above corresponds to situation B, subject to the assumption that NAM is a congruent model of the aggregate Norwegian economy.

The limited relevance of situation B for inflation targeting becomes clear once we recognize that in practice we do not know what kind of shocks that will hit the economy during the forecast period. We generally refer to such changes as *regime shifts,* and their underlying causes include changes in technology and political decisions, and more generally, "the complexity and instability of human behaviour" (Elster (2007, p. 467)). A forecast failure effectively invalidates any claim about a "correct" forecasting mechanism. Upon finding a forecast failure, the issue is therefore whether the misspecification was detectable or not, at the time of preparing the forecast. It is quite possible that a model which has been thoroughly tested for misspecification within sample nevertheless forecasts badly, which is a situation that may occur in situation C.

As discussed by Clements and Hendry (1999), a dominant source of forecast failure is regime shifts in the forecast period, i.e., *after* the preparation of the forecasts. Since there is no way of anticipating them, it is unavoidable that *after forecast* breaks damage forecasts from time to time. For example, when assessing inflation targeting over a period of years, we anticipate that the forecasters have done markedly worse than they expected at the time of preparing their forecasts, simply because there is no way of anticipating structural breaks before they occur. The task is then to be able to detect the nature of the regime shift as quickly as possible, in order to avoid repeated unnecessary forecast failure.

However, experience tells us that forecast failures are sometimes due to shocks and parameter changes that have taken place prior to the preparation of the forecast, but which have remained undetected by the forecasters. Failing to detect a *before forecast* structural break might be due to low power of statistical tests of parameter instability. However, the power is actually quite high for the kind of breaks that are most damaging to model forecasts, see Hendry (2000). There are also practical circumstances that complicate and delay the detection of regime shifts. For example, there is usually uncertainty about the quality of the provisional data for the period that initialize the forecasts, making it difficult to assess the significance of a structural change or shock.

Hence both *after* and *before* forecast structural breaks are realistic aspects of real life forecasting situations that deserve the attention of inflation targeters. In particular, one should seek forecasting models and tools which help cultivate an adaptive forecasting process. The literature on forecasting and model evaluation provide several guidelines, see e.g., Hendry (2001a) and Granger (1999).

Situation D brings us to the realistic situation, namely one of uncertainty and discord regarding what kind of model that approximates represents reality, in other words the issues of model specification and model evaluation. In section 4.3 we saw that in policy analysis there is a clear gain from using the congruent model, and avoiding to base policy recommendations on an misspecified model of the supply side. In forecasting, the link between model misspecification and forecast failure is not always as straight-forward as one would first believe. The complicating factor is again non-stationarity, regime-shifts and structural change. For example, a time series model in terms of the change in the rate of inflation adapts quickly to changes in equilibria—a location shift— and is therefore robust to *before* forecast structural breaks of this type, even though it clearly a misspecified model of the data generation process over the historical data period, see Clements and Hendry (1999, Ch 5). Therefore a double differenced device (DDD) can deliver near unbiased estimated when a location-shift have occurred prior to the preparation of the forecast. Conversely, a forecasting model of the equilibrium correction type is less adaptable. Indeed, following an equilibrium shift, EqCM forecasts tend to move in the opposite to the data, thereby causing forecast failure, cf. Hendry (2006). Eitrheim et al. (1999) showed that this new theory of forecasting has practical relevance for understanding the properties of forecasts from an medium sized forecasting model of the Norwegian economy, in particular the more adaptive nature of DDD to several historical examples of structural breaks.

The new theory of forecasting that we build on does not deliver a *carte blanche* for using non-congruent models for prediction, though. DDD and near equivalent forecasting devices are robust for one particular reason: they do not equilibrium correct, and are therefore insulated from changes in the parameters that are most pernicious for forecasting. Replacing a congruent and adequate EqCM with another, less adequate, EqCM model for forecasting is not a good idea. The non-congruent EqCM is also subject to forecast failure on its own premises, and without location shifts, it will forecast worse than the congruent EqCM. In this respect there is a cost to compromising model adequacy also in forecasting. In terms of the contesting supply-side models of section 4.3 this is illustrated by Bårdsen et al. (2002) who show that although the PCM is robust to some of the location shifts that can damage forecasting from the ICM, the cost of high forecast variance and bias due to misspecified equilibrium correction dominates.

4.5.2 Real-time forecast performance

As mentioned above, NAM is part of the Normetrics system of models which was initiated in 2005. Model based forecasts for the Norwegian economy are produced in January, March, June and September each year and published on the web. So far, these model based forecasts have performed relatively well compared to competing forecasts. As an example Figure 9 shows forecasts for core inflation in 2006. Because of inflation targeting, this variable is the among the most thoroughly analyzed variable by both professional forecasters both in the private and government sector.

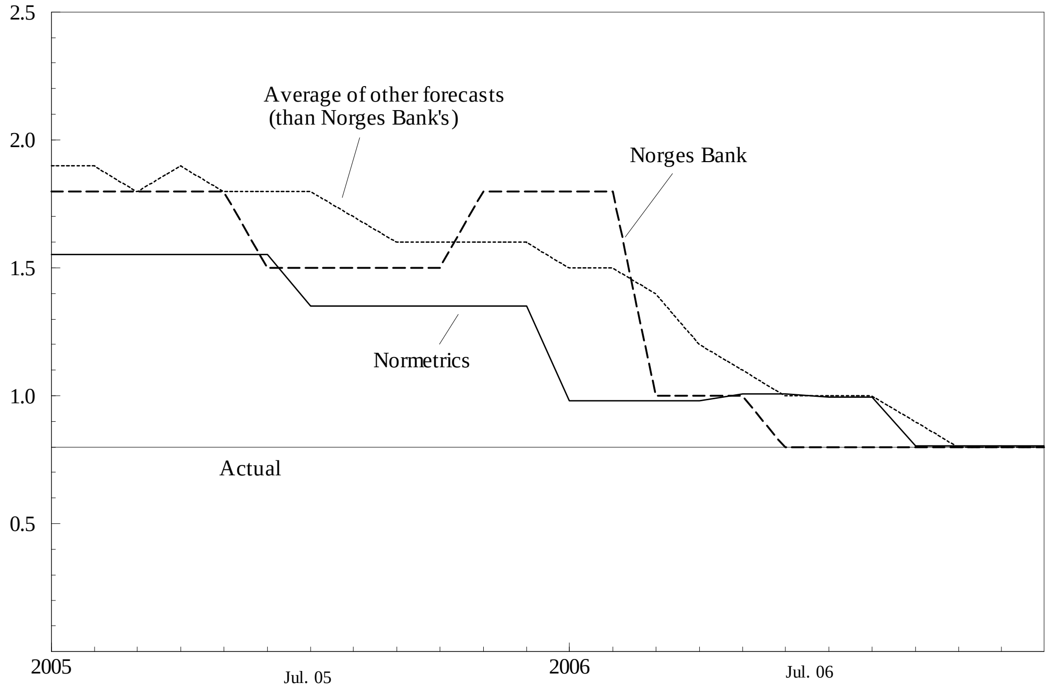

Figure 9: Forecasts for annual core inflation in 2006, published at different times. Percent. Monthy figures, Jan 05 - Dec 06. Source: Norges Bank, Monetary Policy Report 1/07. and http://folk.uio.no/rnymoen/Normetrics.

Figure 9 shows Norges Bank's projections together with the average of other professional Norwegian forecasters. The third graph shows the sequence of forecasts from the two Normetrics forecasting models, AIF and NAM.[21] The figure shows that Normetrics forecasts were never any worse than the average forecasts, and most vintages of Normetrics forecasts are considerably better. All Normetrics forecasts produced in 2005 also improve on Norges Bank's forecasts (June 2005 is the exception). In the period Nov. 05-Feb. 06 the gap between Normetrics and Norges Bank actually widens—as Norges Bank's forecast were adjusted upward, away from what eventually became the actual inflation rate of 0.8%. However, the forecast in Monetary Policy Report 2/06 is accurate, while the Normetrics forecast stays at 1% until Sept. 06.
Since equilibrium-correction is ubiquitous in macroeconometric models one can safely assume that some of the "other forecasts" are based on EqCMs. Specifically, Norges Bank's forecast are based on the rational expectations solution of an equilibrium correction model with lead in variables. Hence, all forecasts in the Figure 9 can have been damaged by any location shifts that took place in 2006—-they were *after forecast* structural breaks. In particular the forecast that were published (early) in 2005 had a large exposure to forecast failure. For the same reason, the forecast errors are reduced as more "2006-information" is conditioned upon. In that perspective we can interpret the figure as evidence that the Normetrics forecasts are more adaptive than the other equilibrium correction forecasting mechanisms covered by the graph.

The accuracy of the model based forecasts in Figure 9 are less impressive when compared to forecasts from simple DDD, though. For example, at the start of 2005 a forecast based on the double difference of the log of the CPI-AET index would predict a 2006

[21] Automatized econometric Inflation Forecasts which have been published twize a year, starting in July 2004. The forecasts are automatized, with a minimum of intervention after the econometric specification of the forecasting mechanism is completed. Therefore these econometrically based forecasts have been dubbed Automatized Inflation Forecasts forecasts (AIF for short).

inflation rate of 0.3%, which is a more accurate forecast than any of the econometric or professional forecasts that were produced in 2005. The DDD forecast is not improved on by Normetrics before January 2006, and not by Norges Bank before the Monetary Policy Report 1/06 (from March). However, by this time also the DDD forecast could also be have been updated, most simply by replacing the 0.3% by the actual rate of inflation in 2005 which was 1.0%. This forecast is practically identical to Normetrics forecasts from before September 2006, and it is not beaten by Norges Banks' forecast before MPR 2/06.

The year 2006 is not the only one that the Normetrics forecasts compete well with the Central Bank's forecasts for the monetary policy target variable. Juel et al. (2008) note that for a relatively long period of time, the automated forecasts from an empirically validated inflation model, dubbed AIF above, have been better than Norges Bank's forecasts. Figure 10 illustrates the point.

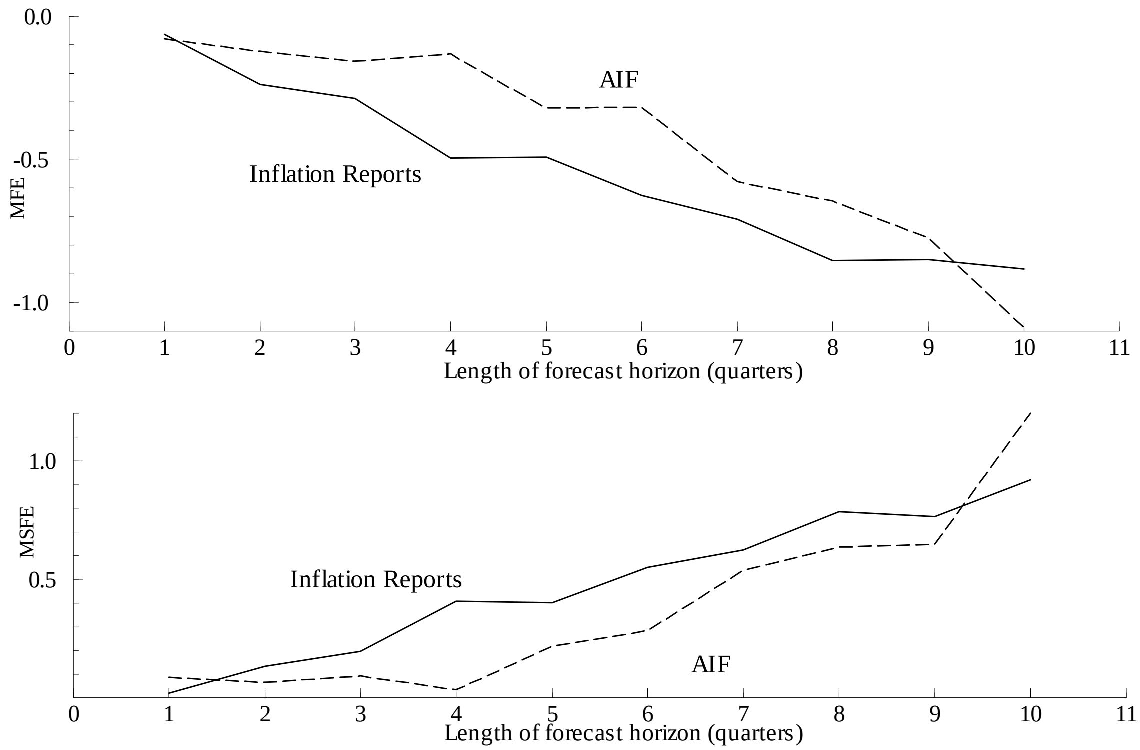

Figure 10: First panel: Mean forecast errors, MFE, for core inflation (annual rate of change in CPI-ATE). Inflation report forecasts and AIF forecasts. Second panel: Mean squared forecast errors, MSFE. Source: Inflation Report/Monetary Policy Report 2/04 - 2/07 and AIFs published at http://folk.uio.no/rnymoen/forecast_air_index.html.

The figure can be used to assess the ex-ante forecasts based on forecast errors for the period 2004q2-2007q3. The forecasted variable is the annual rate of CPI-AET inflation. The graphs in the upper panel in the figure show the mean forecast errors, MFE. A negative MFE means that the inflation forecasts are on average higher than the actual inflation rates in the period. The biases of the econometric model (AIF) forecasts are small for forecast horizons 1, 2, 3 and 4: the MFEs are less than 0.25 percentage points. Norges Bank's inflation forecasts from the Inflation/Monetary Policy Reports are more biased than AIF for horizons 2-8 quarters ahead. The biases of AIF become markedly bigger for forecasts length 7-10 quarters, and are not much different from the bias of forecasts produced by Norges Bank. The second panel of the figure shows the mean squared forecast errors, MSFE, to which large forecast errors contribute more than small errors. This measure gives more or less the same picture at the mean forecast errors.

4.5.3 Ex post forecast evaluation and robustification.

The discussion of Figure 9 illustrates the general point that although EqCMs forecast well when a process is difference-stationary, they are non-robust if there are non-stationarities due to location shifts in the forecast period. In this particular case, the mean of the rate of inflation seems to have changed, in 2004 or earlier, and the DDD is a more adaptable forecasting mechanism than the EqCMs in the case of the *before forecast* structural break. The main benefit of DDDs for model based forecasting, where one wants to retain the causal information of the model, is that DDDs can help the forecaster to robustify the EqCM forecasts by intercept corrections. The cost associated with DDDs is of course that the forecasts are more 'noisy' than EqCM forecasts, hence the forecast-error variances associated with robust forecasts can become large, like when the first difference of an autoregressive process doubles the 1-step forecast variance. In a model with one or two endogenous variables this cost may not be much of a issue, but in a multi-equation forecasting setting there may be a problem of extracting 'signal from noise' in practice. In the rest of this section we illustrate these issues by considering system forecasts.

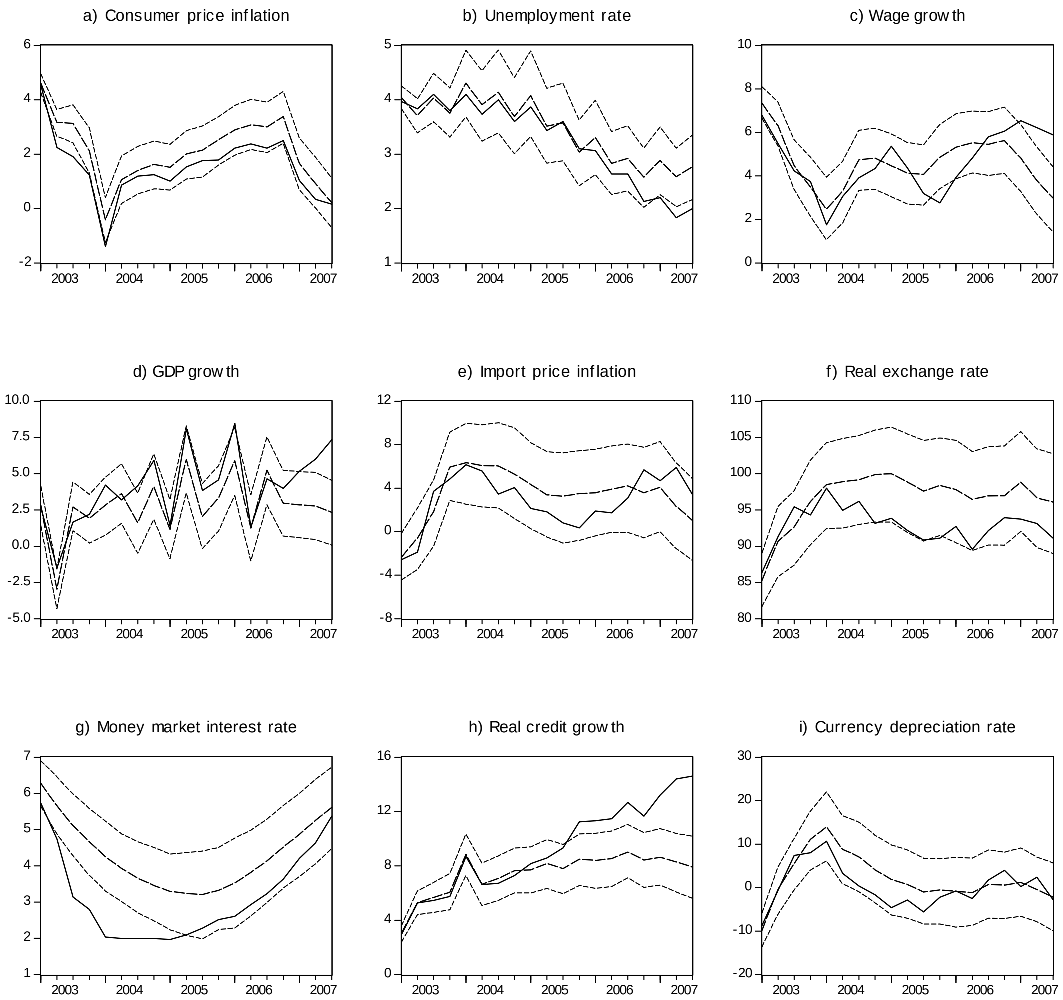

Figure 11: Dynamic NAM forecasts 2003(1)-2007(3), with end-of-sample for estimation of parameters in 2002(4). Actual values are shown by solid lines, and forecasts by dashed lines. The distance between the two dotted lines represents 70 % prediction intervals.

Figure 11 shows dynamic NAM forecasts for the period 2003(1)-2007(3).[22] The sample period for the estimation ends in 2002(4). Unlike the inflation forecasts in Figure 9, which are real-time ex ante forecasts, we now consider *ex post* forecasts which are conditioned by the true values of the non-modelled variables. Hence, the forecasts are not influenced by location shifts in the non-modelled variables (like foreign CPI inflation for example), but they are subject to location shifts that are due to changes in the means of the estimated cointegration relationships, or the autonomous growth rates (captured by intercepts after subtraction of equilibrium correction means).

Figure 11 serves as the reference case for discussion of the degree of adaptation of NAM forecasts, and for the properties of more robust forecasting devices that we investigate for comparison. There are no less than five possible instances of location shifts in the forecast of these 9 variables. First, there is a systematic overprediction of the rate of inflation (we use CPI inflation here) over the length of the forecast horizon. In the light of the 70% prediction interval, inflation overprediction is significant (forecast failure) in 2003. Second, unemployment is overpredicted and GDP growth in underpredicted for the 3 quarters of 2007. Third, the short-term interest rate is significantly overpredicted in 2003(2)-2004(4). Fourth, real credit growth is underpredicted (significantly) from 2005(3). Finally, the real exchange rate is systematically overpredicted from 2004(4) and onwards—the real appreciation of the Norwegian krone is not captured by the forecasts.

In the following we show how well the NAM forecasts adapt to these location shifts when we condition on 2004(4) and then 2006(4). In each dynamic forecast the coefficient estimate is updated. Figure 12 shows that the 2005(1)-2007(3) forecasts for inflation and, in particular, for the interest rate the real exchange rate have improved relative to Figure 11. For the other variables there is little change, and if anything the forecast failures for the rate of unemployment and for GDP growth stand out more clearly than before (also showing that there is a knock-on effect of the high employment forecast on wage inflation). The explanation may be that NAM is unable to adapt, or that the location shifts of the variables are of the *after forecast* also in this forecast.

[22] In this section, we use the model version used for the October 2007 NAM forecasts, see http://folk.uio.no/rnymoen/namforecast_okt07.pdf

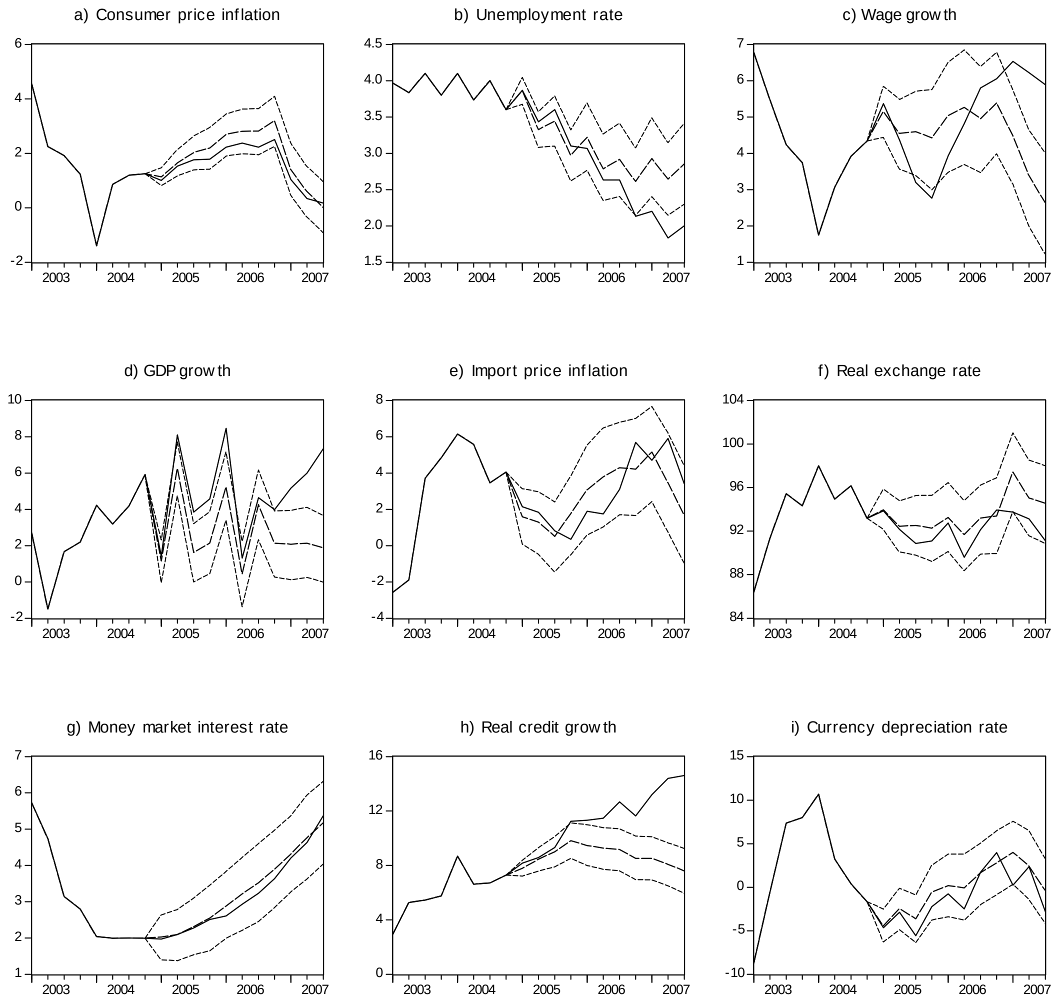

Figure 12: Dynamic NAM forecasts 2005(1)-2007(3), with end-of-sample for estimation of parameters in 2004(4). Actual values are shown by solid lines, and forecasts by dashed lines. The distance between the two dotted lines represents 70 % prediction intervals

Figure 13 focuses on the last three "problem variables" in terms of forecast failure. The rate of unemployment is now much better forecasted in 2007(1)-2007(3), which is a sign of adaptation to a location shift which is of the *before forecast* category in this forecast, where 2006(4) is in the information set. For GDP and real credit growth the forecast failures persist. In the present version of NAM, the high growth rates of 2007 can in part be explained be the effects of very high oil prices on demand. In fact, that modelling device was used in Figure 6, showing the goodness-of-fit, but clearly would not be known or of any help to a forecaster preparing forecast for 2007 late in the year of 2006.

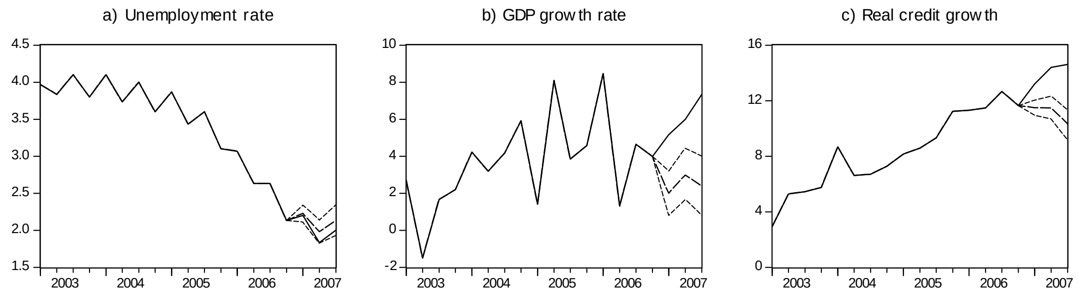

Figure 13: Dynamic NAM forecasts 2007(1)-2007(4), with end-of-sample for estimation of parameters in 2006(4). Actual values are shown by solid lines, and forecasts by dashed lines. The distance between the two dotted lines represents 70 % prediction intervals

As mentioned above, using differencing (DDD above) to forecast provides more robust forecast when non-stationarities are due to location-shifts. As discussed by Hendry (2006), a differenced version of the EqCM may be interpreted as an augmented DDD forecasting rule. We therefore consider forecasts from the differenced NAM model, and refer to these as dEqCM forecasts. In the differenced forecasting systems, some of the causal information embedded in NAM is retained at the same time as the dEqCM has no constants neither in the form of means of cointegration relationships or in the form of separate intercept terms, see Hendry (2006). Hence forecasts from the dEqCM does not equilibrium correct, thereby reducing the risks attached to EqCM forecasts.

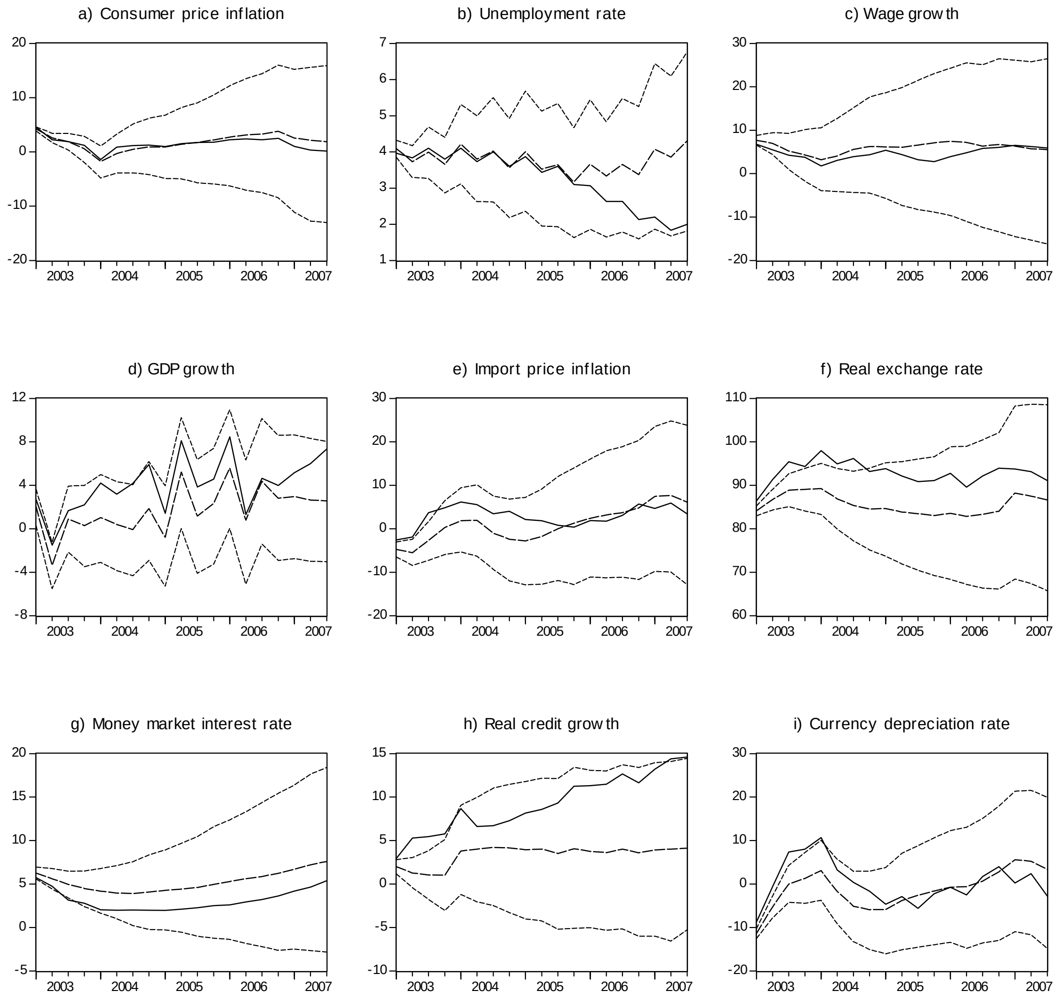

Figure 14: Dynamic dEqCM forecasts 2003(1)-2007(3), with end-of-sample for estimation of parameters in 2002(4). Actual values are shown by solid lines, and forecasts by dashed lines. The distance between the two dotted lines represents 70 % prediction intervals

Figure 14 shows the dEqCM forecast for the 2003(1)-2007(3) period. Comparison with the NAM forecasts in Figure 11 show that the increase in forecast variance is not a small cost in this case—note the difference in scaling as well. The absolute forecast errors appear to be much worse than the NAM forecast errors as well. For example, unemployment is predicted by the dEqCM to increase over the forecast horizon, and credit growth is underpredicted for the length of the horizon.

Figure 15 shows the dEqCM forecasts when we condition on information including 2004(4). Compared to the NAM forecasts that condition on the same information, see Figure 12 there is little to be gain in these forecasts. We note that the dEqCM interest rate forecast has adapted, but the same happened with the NAM forecasts. The dEqCM forecasts are still uninformative about the behaviour of unemployment and credit growth over the 2006(1)-2007(3) period.

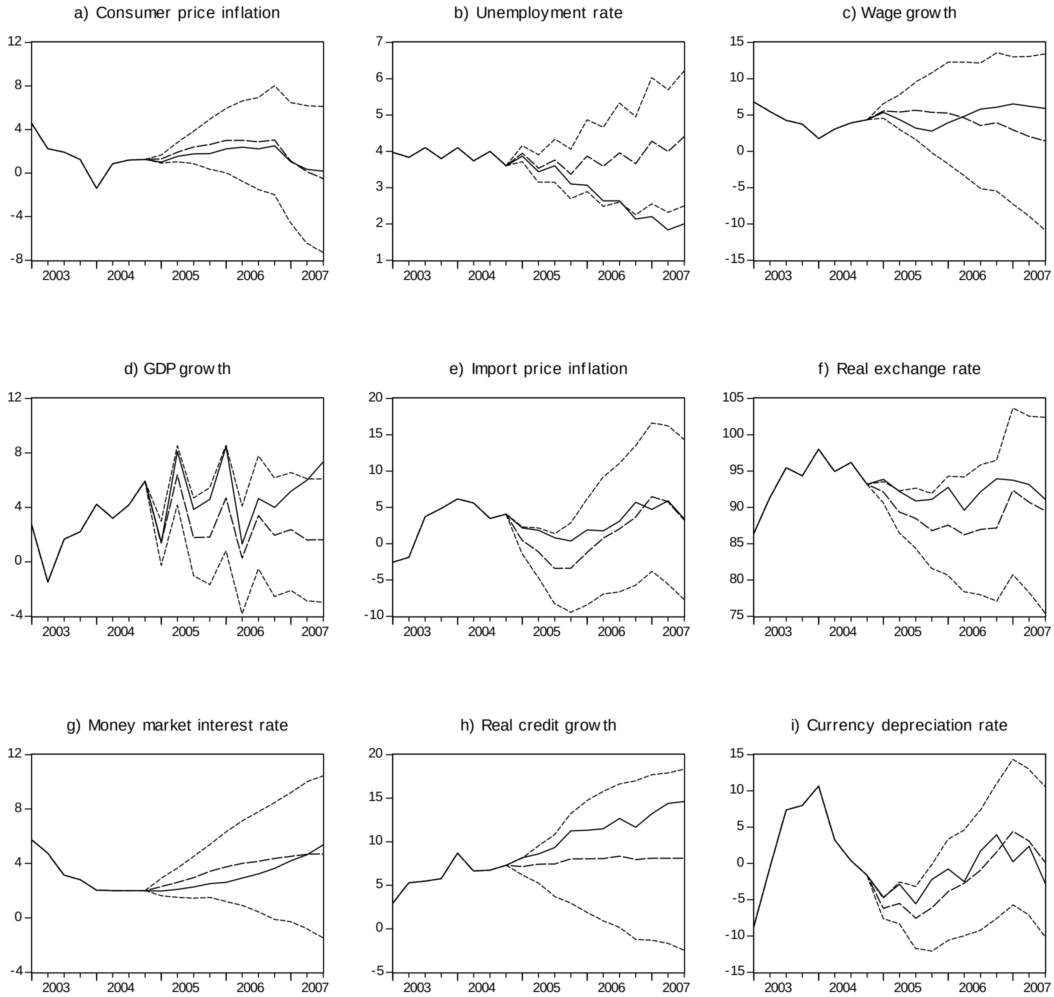

Figure 15: Dynamic dEqCM forecasts 2005(1)-2007(3), with end-of-sample for estimation of parameters in 2004(4). Actual values are shown by solid lines, and forecasts by dashed lines. The distance between the two dotted lines represents 70 % prediction intervals

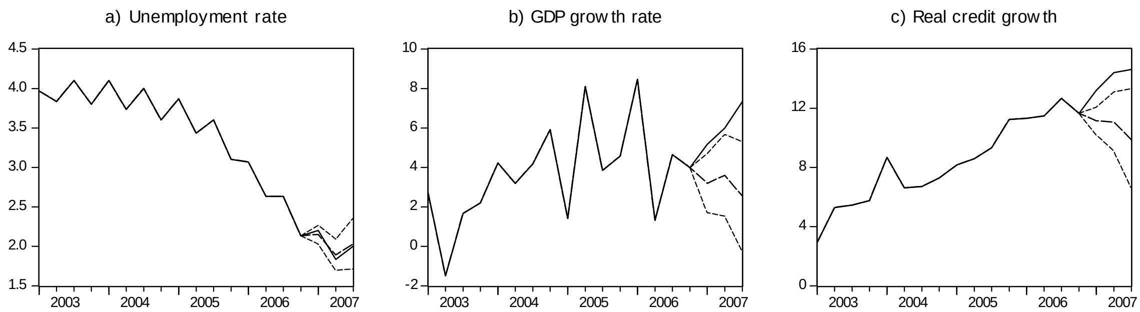

Figure 16: Dynamic dEqCM forecasts 2007(1)-2007(4), with end-of-sample for estimation of parameters in 2006(4) Actual values are shown by solid lines, and forecasts by dashed lines. The distance between the two dotted lines represents 70 % prediction intervals

Figure 16 indicates that for the three quarters of 2007, the dEqCM forecast for the rate of unemployment is better than the (already quite good) NAM forecasts. However, for GDP growth and credit growth, the dEqCM still does not adjust to the location shift.

These results suggest that in practice, a more discretionary approach may be called for. For example, instead of taking away all the equilibrium correction by differencing, one may concentrate on the subset of equations which have failed because of location shifts in recent forecasting rounds, since that will induce lack of adaptation also in the overall forecasting picture. To illustrate the possible benefit from such an approach, Figure 17 shows the 2007 forecasts for a dEqCM where only the equilibrium variables of the aggregate demand and for the credit equation have been "differenced away", to avoid confusion with the dEqCM used above, we refer to this forecasting model as partial-dEqCM. Figure 17 shows the one-step forecast, since any differences in adaptability is then easier to see.

Panels a) and b) show the NAM forecasts. The forecasts for 2007(1) are the same as in Figure 16 for these two variables, but 2007(2) and 2007(2) are different since the NAM forecasts are now conditional on first and second quarter information for exogenous and predetermined variables (the coefficients are not updated). The lack of adaptation to the location shifts of the growth rate of the actual series is apparent. Panel c) and d) shows the corresponding partial-dEqCM forecasts. For GDP growth there is less underprediction already in 2007(1), and in 2007(2) the forecast has adapted. Panel c) shows a marked improvement in adaptation also in the credit growth rate, although not before the second quarter of 2007(2).

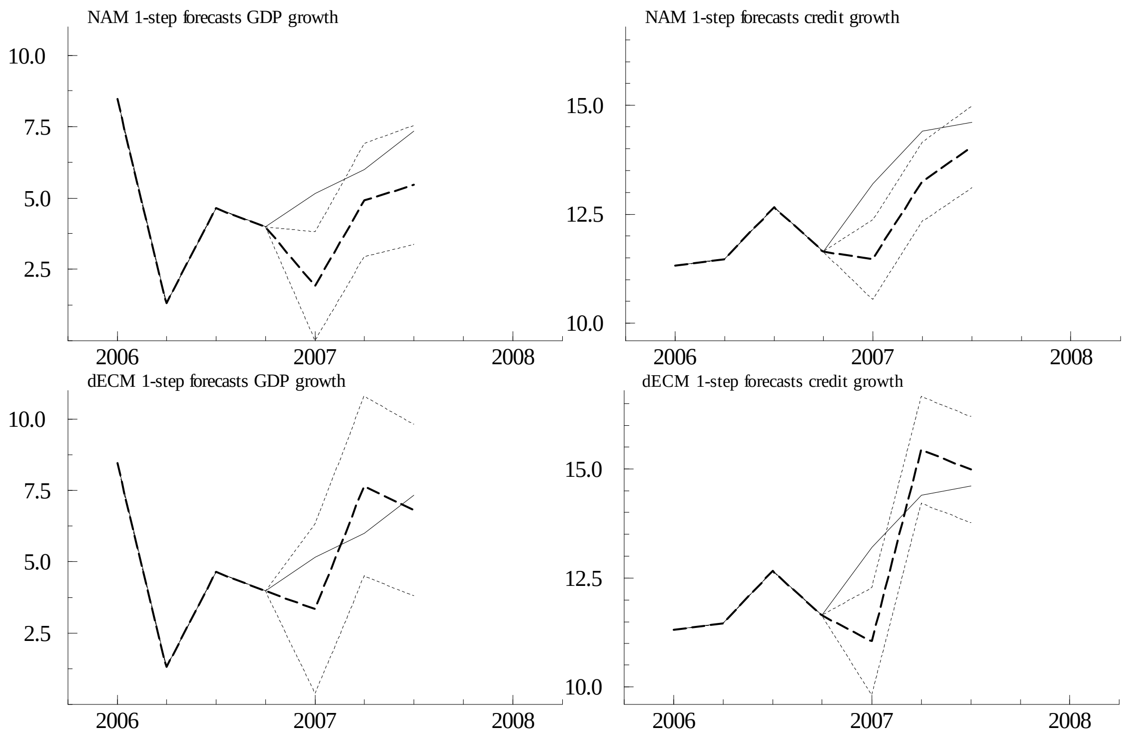

Figure 17: One-step forecast from NAM and a partial dEqCM. The forecasts are the dashed lines. The distance between the dotted lines represents 70 % prediction intervals

5 Conclusion

This chapter has presented a case for continuing to use macroeconometric models for policy analysis, including such analyses that relies on instrument use to attain a certain forecast for a target variable like inflation. Paradoxically, perhaps, the main line of argument starts from the recognition that accurate forecasting is a near impossibility in macroeconomics, because of the inherent non-stationarity of the economic time series included in the model

. Non-stationarity takes different forms, with different implications for macroeconometric modelling and forecasting, and above we have distinguished between unit roots and non-stationarity due to structural breaks. We have demonstrated that macroeconometric models can be developed theoretically and empirically in a way that is consistent with a unit root assumption. At the modelling and estimation stage, non-stationarity due to unit roots can principle be handled by cointegration methods, and, given that approach, unit roots are unlikely to be a source of forecast failures. On the contrary, a correct unit root assumption can help in concentrating on predictable functions of variables, like growth rates and linear combination of target variables.

Non-stationarity due to structural breaks in functional relationships of the economy seem to represent the real challenge to macroeconometric modelling. Structural breaks in the forecast period are particularly harmful since they are untraceable and will make the model forecasts go toward pre-break steady-state relationships. When the sample period is extended, structural breaks represent valuable sample information that provide power to tests of economic hypotheses, and in that perspective structural breaks are seen to be not entirely negative since there can be progress in macroeconometric modelling through forecast failure.

It follows from our approach that we agree with those who conclude that guidance from economic theory is important in the forecasting process, cf. **?**. But theory and modern econometrics do not provide immunity to new structural breaks. Nevertheless, the case for macroeconometric system of equations models may to a large extent depend of their ability to forecast relatively well compared to competing forecasting methods. Making models sufficiently adaptive to a structural break once the evidence is there that it has occurred is a necessary step in that project.

It is possible to look to other forecasting disciplines for inspiration. Meteorologists developed their forecasting theory, models and routines between the two world wars. Those forecast were based on a deterministic model of the dynamics of the atmosphere. Then, in the mid-sixties, the understanding of the dynamics were completely altered with the development and acceptance of chaos theory, with the logical consequence that accurate weather forecast is impossible. Thirty years after the arrival of the meteorologists 'impossibility theory' for forecasting, the weather forecasts are undeniably more accurate than ever. Hence, the weather is predictable even 10 days ahead, despite the chaos represented by the underlying forecasting models. It is perhaps unlikely that we will witness something similar in economics, but there are nevertheless parallels. Meteorologists have precise theory and almost continuous updating of initial conditions. Policy oriented modelling may have to live with idealized or partial theories, and with variables that are measured at relatively long time intervals, and which are influenced by measurement errors. Yet, as a discipline we have developed methods and strategies that are quite good at making the most of 'small data amounts'. Hence while meteorologists can rely on the theoretically specified model of weather dynamics, the interaction between economic theory, econometric theory, good model selection procedures, and diagnostic testing have together greatly improved our capability of modelling the macro economy, thus providing models that can aid policy decisions.

A Data definitions and equation statistics

A.1 Variables

The model employs seasonally unadjusted data. Unless another source is given, all data are taken from FPAS, the database of Norges Bank.

The model is developed and estimated with *Oxmetrics 5* (www.oxmetrics.net) and then reestimated and simulated with *Eviews 6* (www.eviews.com).

V Trade weighted nominal value of the krone based on import-shares of trading countries.

G Government sector consumption expenditure, fixed 1991 prices. Mill. NOK.

L Nominal credit volume. Mill. NOK.

R Money market interest rate (3 month Euro-krone interest rate).

R^* ECU interest rate. For the period 1967(1)-1986(3): Effective interest rate on foreign bonds, NOK-basket weighted. For the period 1986(4)-1996(4): ECU weighted effective rate on foreign bonds.

R_L Average interest rate on bank loans.

R_B Yield on 6 years government bond, quarterly average.

R_B^* Yield on long term foreign bonds. NOK basket weighted.

P Consumer price index (CPI).

P_C "Core" consumer price index.

P^* Consumer prices abroad in foreign currency.

P_E CPI, electricity, fuel and lubricants.

P_O USD oil price, per barrel Brent-Blend.

P_I Price deflator of total imports.

P_I^* Producer price index, trading partners.

Y Total value added at market prices in the mainland economy (defined as total economy minus North-Sea oil and gas production and international shipping). Fixed base year (1991) prices. Mill. NOK.

Z Mainland economy value added per man hour at factor costs.

$T1$ Payroll tax rate, mainland economy.

U Registered rate of unemployment.

W Nominal hourly wage COSTS in the mainland economy. NOK.

In addition, there is a step dummy, accounting for the introduction of inflation targeting.

$$IT = 0 \text{ until } 2001(1), 1 \text{ from } 2001(2)$$

A.2 Notation for estimation and mispecification tests

In Table 1 and 2, the estimation method, which is either ordinary least squares (OLS), or full information maximum likelihood (FIML) is indicated in the first line below each equation, along with the sample size (number of quarterly observations) which is denoted by T, and the residual standard error ($\hat{\sigma}$). For equations estimated with OLS, statistics for residual autocorrelation and ARCH form heteroscedasticity are reported in the second line below the equation. As indicated by the notation, these two statistics are F-distributed under their respective null hypotheses. For example, $F_{AR(1-4)}(4, 44)$ in the exchange rate equation denotes the F distributed test statistics with 4 and 44 degrees of freedom for the null hypothesis of no autocorrelation against the alternative of 4th order autoregressive autocorrelation. In the third line below the estimated OLS equations, we report the Chi-square distributed test of residual normality, and the F distributed test of heteroscedasticity due to squares of the regressors. For the equations estimated with FIML, systems versions of the misspecification tests are reported and are indicated by the extra subscript *vec*, as $F_{vec,\mathsf{AR}(1-4)}$. The numbers in brackets are p-values for the respective null hypotheses These, as well as the other standard diagnostics tests, are explained in Doornik and Hendry (2007a) (single equation diagnostics), or Doornik and Hendry (2007b) (system and simultaneous equations diagnostics).

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