Macroeconometric Modelling

4 General modelling and some examples

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From system... I

Standard procedure—see (**?**), Hendry (1995), Johansen (1995), Johansen (2006), Juselius (2007), Garratt, Lee, Pesaran, and Shin (2006) and Lütkepohl (2006) for detailed expositions. Here we follow the exposition in Lütkepohl (2005).
We are interested in analyzing the k-dimensional VAR(p) process of either $I(1)$ or $I(0)$ variables $\mathbf{y}_t$

$$\mathbf{y}_t = \boldsymbol{\nu} + \sum_{i=1}^{p} A_i \mathbf{y}_{t-i} + \mathbf{u}_t,$$

typically written in Equilibrium correction form (EqC) as

$$\Delta \mathbf{y}_t = \boldsymbol{\nu} + \Pi \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{y}_{t-i} + \mathbf{u}_t, \tag{1}$$

From system... II

with

$$\boldsymbol{\Gamma}_i = -\left(\mathbf{A}_{i+1} + \cdots + \mathbf{A}_p\right), \ i = 1, \ldots, p-1$$

The system is stable if

$$|-\boldsymbol{\Pi}| = \det\left(\mathbf{I}_k - \mathbf{A}_1 z - \cdots - \mathbf{A}_p z^p\right) \neq 0 \text{ for } |z| \leq 1.$$

If $-\boldsymbol{\Pi} = \left(\mathbf{I}_k - \mathbf{A}_1 z - \cdots - \mathbf{A}_p z^p\right)$ has a zero determinant with $(k-r)$ unit roots, the system still contains r steady-state relationships contained in the $(k \times r)$ matrix β defined by

$$\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'.$$

...to model I

From a discretized and linearized cointegrated VAR representation to a dynamic Simultaneous Model (SEM) in three steps:

1. Linearized and discretized approximation as a data-coherent statistical system representation in the form of a cointegrated VAR

$$\Delta \mathbf{y}_t = \boldsymbol{\nu} + \boldsymbol{\Pi} \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{y}_{t-i} + \mathbf{u}_t, \tag{2}$$

...to model II

2. Identify the steady state, by testing and imposing overidentifying restrictions on the cointegration space:

$$\Delta \mathbf{y}_t = \boldsymbol{\nu} + \boldsymbol{\alpha}^* \boldsymbol{\beta}^{*'} \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{y}_{t-i} + \mathbf{u}_t,$$

3. Identify the dynamics, by testing and imposing overidentifying restrictions on the dynamics:

$$\mathbf{A}_0 \Delta \mathbf{y}_t = \mathbf{A}_0 \boldsymbol{\nu} + \mathbf{A}_0 \boldsymbol{\alpha}^* \boldsymbol{\beta}^{*'} \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \mathbf{A}_0 \boldsymbol{\Gamma}_i \Delta \mathbf{y}_{t-i} + \mathbf{A}_0 \mathbf{u}_t,$$

...to model III

Comment:

- The steady-state parameters are *invariant* to the dating of the steady-state solution.
- However, the estimated short-run parameters, and therefore the interpretation of the model dynamics, *need not be*—see Bårdsen (1992), Bårdsen and Fisher (1999).

...to model IV

- This is easily seen by noting that the model can equivalently be written as

$$\mathbf{A}_0 \Delta \mathbf{y}_t = \mathbf{A}_0 \boldsymbol{\nu} + \mathbf{A}_0 \boldsymbol{\alpha}^* \boldsymbol{\beta}^{*'} \mathbf{y}_{t-p} + \sum_{i=1}^{p-1} \mathbf{A}_0 \mathbf{D}_i \Delta \mathbf{y}_{t-i} + \mathbf{A}_0 \mathbf{u}_t,$$

with

$$\mathbf{D}_i = -\left(\mathbf{I}_k - \mathbf{A}_1 - \cdots - \mathbf{A}_i\right), \ i = 1, \ldots, p-1.$$

An application: Norwegian Aggregate Model-NAM

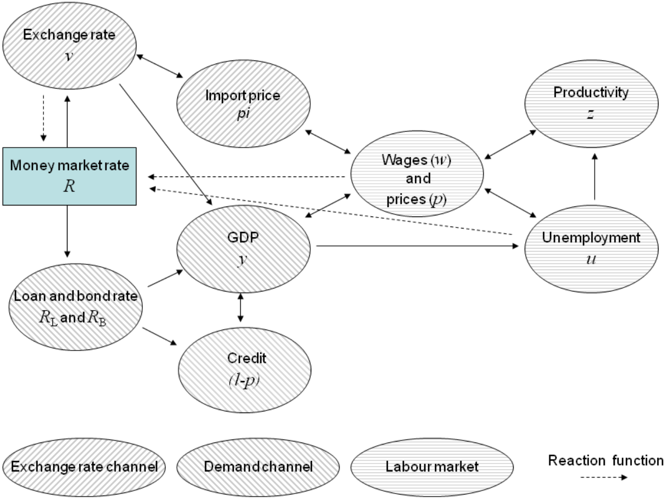
Properties

- Short-run growth framework
- Good fit
- Small and transparent

- Good forecasts?

An application: NAM I

Overview



An application: NAM I

Steady-state relationships

$$(v + p^* - p)_t = -0.12\left[(R - \pi) - (R^* - \pi^*)\right] + \mu_v \tag{3}$$

$$(pi - v - pi^*)_t - 0.55\,(p - v - p^*)_t = \mu_{pi} \tag{4}$$

$$p_t - 0.7(w - z)_t - (1 - 0.7)\,pi_t = \mu_p \tag{5}$$

$$(w - p - z)_t = -0.1u + \mu_w \tag{6}$$

$$z_t - 0.47\,(w - p)_t - 0.0029\,Trend_t = 0.03u + \mu_z \tag{7}$$

$$0 = u - 7.7\Delta\,(w - p) - 4.5\left[0.01\,(RL - \pi) - \Delta_4 y\right] - \mu_u \tag{8}$$

$$0 = RL - 0.41RB - 0.76R - \mu_{RL} \tag{9}$$

$$0 = RB - 0.43R - 0.57RB^* - \mu_{RB} \tag{10}$$

$$y_t - 0.9g_t - 0.16(v + p^* - p)_t = -0.06\,(RL - \pi) + \mu_y \tag{11}$$

$$(l - p)_t - 2.65y_t + 0.04(RL - RB)_t = \mu_{l-p} \tag{12}$$

An application: NAM

Stylized dynamic version

$$\Delta v_t = -0.04\Delta(R - R^*)_t - 0.04\left\{(v + p^* - p) - 0.12\left[(R - \pi) - (R^* - \pi^*)\right]\right\}_{t-1}$$

$$\Delta\left(pi - pi^* - v\right)_t = -0.1\Delta v_t - 0.43\left[(pi - pi^* - v) - 0.55\left(p - p^* - v\right)\right]_{t-1}$$

$$\Delta p_t = -0.09\Delta z_t + 0.03\Delta pi_t + 0.08\Delta pe_t + 0.06\Delta y_t - 0.07\left[p - 0.7(w - z) - 0.3pi\right]_{t-1}$$

$$\Delta\left(w - p\right)_t = -0.04\Delta u_t + 0.73\Delta T1_t - 0.07\left[(w - p - z) + 0.1u\right]_{t-1}$$

$$\Delta z_t = 0.09\Delta\left(w - p\right)_t - 0.24\left[z - 0.47\left(w - p\right) - 0.003\,Trend - 0.03u\right]_{t-1}$$

$$\Delta u_t = -0.23\left\{u - 7.65\Delta\left(w - p\right) - 4.46\left[0.01\left(R_L - \pi\right) - 4\Delta y\right]\right\}_{t-1}$$

$$\Delta R_{L,t} = 0.58\Delta R_t - 0.33\left(R_L - 0.41R_B - 0.76R\right)_{t-1}$$

$$\Delta\left(R_B - R_B^*\right)_t = 0.43\Delta R_t - 0.17\left(R_B - 0.43R - 0.57R_B^*\right)_{t-1}$$

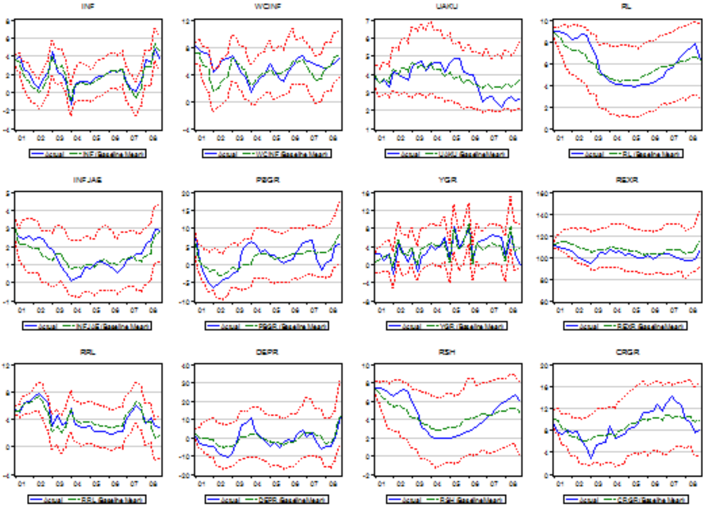
$$\Delta y_t = 0.16\Delta g_t + 0.38\Delta\left(I - p\right)_t - 0.12\left[y - 0.9g_{t-1} - 0.16(v + p^* - p) + 0.06\left(R_L - \pi\right)\right]_{t-1}$$

$$\Delta\left(I - p\right)_t = 0.3\Delta y_t - 0.09\left[(I - p) - 2.65y + 0.04(R_{L,} - R_B)\right]_{t-1}$$

$$\Delta R_t = -0.27\left[R - 5.6 - 1.2\left(\pi_C - \bar{\pi}_C\right) + \left(U - \bar{U}\right) - 0.86\left(R^* - \bar{R}^*\right)\right]_{t-1}$$

An application: NAM

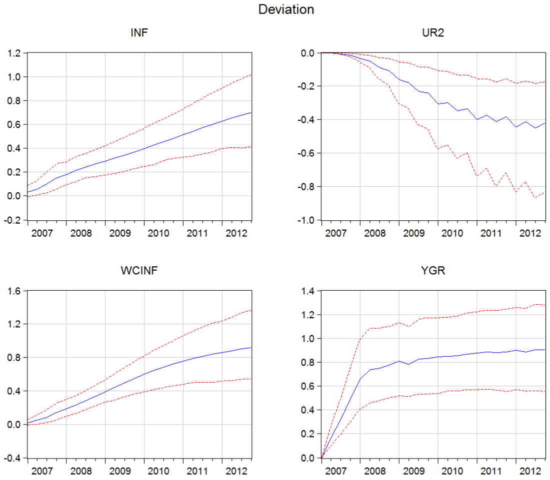
Dynamic simulation



An application: NAM

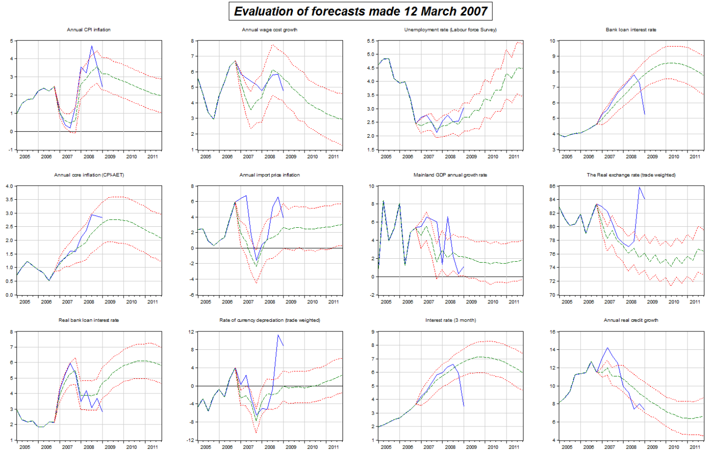
Policy analysis

100 basis points permanent reduction in the short rate from 2007



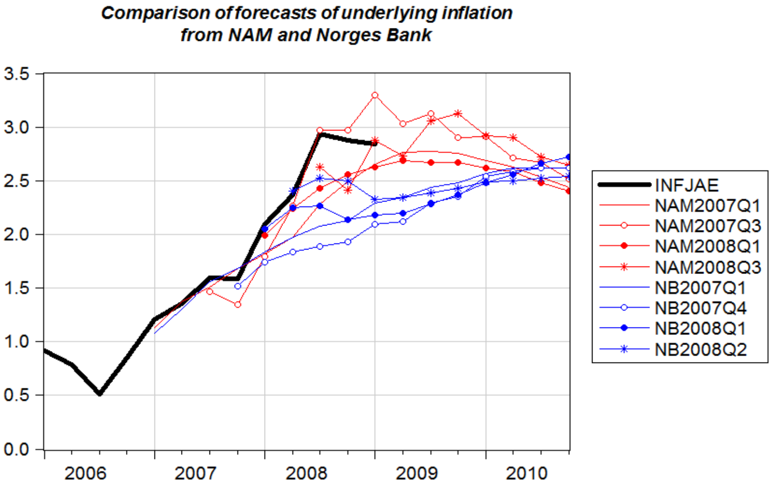
An application: NAM

Forecasts from march 2007: evaluated 02 June, 2009



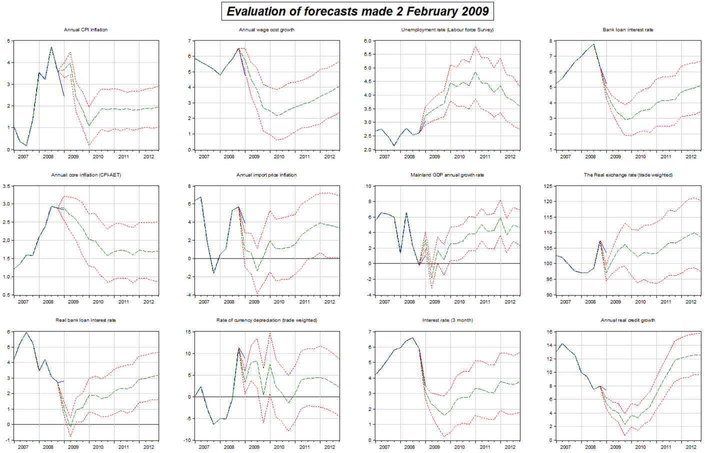
An application: NAM

Forecast comparison: NAM and Norges Bank



An application: NAM

Forecasts: 2 february 2009



An example: How to make a textbook model

Let us start with the stylized representation of the full-scale model

$$\Delta v_t = -0.04\Delta(R - R^*)_t - 0.04\left\{(v + p^* - p) - 0.12\left[(R - \pi) - (R^* - \pi^*)\right]\right\}_{t-1}$$
$$\Delta\left(pi - pi^* - v\right)_t = -0.1\Delta v_t - 0.43\left[(pi - pi^* - v) - 0.55\left(p - p^* - v\right)\right]_{t-1}$$
$$\Delta p_t = -0.09\Delta z_t + 0.03\Delta pi_t + 0.08\Delta pe_t + 0.06\Delta y_t - 0.07\left[p - 0.7(w - z) - 0.3pi\right]_{t-1}$$
$$\Delta\left(w - p\right)_t = -0.04\Delta u_t + 0.73\Delta T1_t - 0.07\left[(w - p - z) + 0.1u\right]_{t-1}$$
$$\Delta z_t = 0.09\Delta\left(w - p\right)_t - 0.24\left[z - 0.47\left(w - p\right) - 0.003\,Trend - 0.03u\right]_{t-1}$$
$$\Delta u_t = -0.23\left\{u - 7.65\Delta\left(w - p\right) - 4.46\left[0.01\left(R_L - \pi\right) - 4\Delta y\right]\right\}_{t-1}$$
$$\Delta R_{L,t} = 0.58\Delta R_t - 0.33\left(R_L - 0.41R_B - 0.76R\right)_{t-1}$$
$$\Delta\left(R_B - R_B^*\right)_t = 0.43\Delta R_t - 0.17\left(R_B - 0.43R - 0.57R_B^*\right)_{t-1}$$
$$\Delta y_t = 0.16\Delta g_t + 0.38\Delta\left(I - p\right)_t - 0.12\left[y - 0.9g_{t-1} - 0.16(v + p^* - p) + 0.06\left(R_L - \pi\right)\right]_{t-1}$$
$$\Delta\left(I - p\right)_t = 0.3\Delta y_t - 0.09\left[(I - p) - 2.65y + 0.04(R_{L,} - R_B)\right]_{t-1}$$
$$\Delta R_t = -0.27\left[R - 5.6 - 1.2\left(\pi_C - \bar{\pi}_C\right) + \left(U - \bar{U}\right) - 0.86\left(R^* - \bar{R}^*\right)\right]_{t-1}$$

An example: How to make a textbook model I

If we make some simplifying assumptions, like:

1. closed economy
2. no public sector
3. one interest rate
4. no debt
5. disregard energy
6. disregard unemployment
7. and make productivity a stochastic trend

An example: How to make a textbook model II

...we have a textbook model—still a bit too rich—but close

$$\Delta p_t = a_{12}\Delta y_t - c_{11}\left[p - (w - z) - \mu_1\right]_{t-1}$$
$$\Delta y_t = -c_{22}\left[y + \beta_{23}(R - \Delta p) - z - \mu_2\right]_{t-1}$$
$$\Delta R_t = -c_{33}\left[R_{t-1} - a_{31}\left(\Delta p_t - \overline{\Delta p}\right) - a_{32}\left(\Delta y_t - \overline{\Delta y}\right) - \mu_3\right]$$
$$\Delta(w - p - z)_t = -c_{44}(w - p - z + \mu_1)_{t-1}$$
$$\Delta z_t = \mu_5$$

NB: note that R is $I(0)$ and endogenous. What problems will this cause?

Calibration for simulation I

$$\Delta p_t = .1\Delta y_t - .2\left[p - (w - z) - .2\right]_{t-1}$$
$$\Delta y_t = -.1\left[y + .5\left(R - \Delta p\right) - z - 1\right]_{t-1}$$
$$\Delta R_t = -.1\left[R_{t-1} - 1.5\left(\Delta p_t - 0.02\right) - 0.5\left(\Delta y_t - 0.04\right) - 0.05\right]$$
$$\Delta\left(w - p - z\right)_t = -.2\left(w - p - z + .2\right)_{t-1}$$
$$\Delta z_t = .04$$

Simultaneous equations form of model

$$
\begin{aligned}
\Delta p_t - a_{12}\Delta y_t &= c_{11}\mu_1 - c_{11}\left[p - (w - z)\right]_{t-1} \\
\Delta y_t &= c_{22}\mu_2 - c_{22}\left(y + \beta_{23}R - z\right)_{t-1} \\
&\quad + c_{22}\beta_{23}\Delta p_{t-1} \\
\Delta R_t - c_{33}a_{31}\Delta p_t - c_{33}a_{32}\Delta y_t &= -c_{33}\left(a_{31}\overline{\Delta p} + c_{33}\overline{\Delta y} - \mu_3\right) \\
&\quad - c_{33}R_{t-1} \\
\Delta\left(w - p - z\right)_t &= -c_{44}\mu_1 - c_{44}\left(w - p - z\right)_{t-1} \\
\Delta z_t &= \mu_5
\end{aligned}
$$

Calibrated model on matrix form

Ready for simulation

$$\underbrace{\begin{pmatrix} 1 & -.1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -.1\times 1.5 & -.1\times .5 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}}_{\mathbf{A}_0} \underbrace{\begin{pmatrix} \Delta p \\ \Delta y \\ \Delta R \\ \Delta w \\ \Delta z \end{pmatrix}_t}_{\mathbf{y}_t} = \underbrace{\begin{pmatrix} .2\times .2 \\ .1\times 1 \\ -.1\times(1.5\times .02+.5\times .04-.05) \\ -.1\times .2 \\ .04 \end{pmatrix}}_{\mathbf{A}_0\boldsymbol{\nu}}$$

$$-\underbrace{\begin{pmatrix} .2 & 0 & 0 \\ 0 & .1 & 0 \\ 0 & 0 & .1 \\ -.1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\mathbf{A}_0\boldsymbol{\alpha}^*} \underbrace{\begin{pmatrix} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & .5 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}}_{\beta^{*\prime}} \underbrace{\begin{pmatrix} p \\ y \\ R \\ w \\ z \end{pmatrix}_{t-1}}_{\mathbf{y}_{t-1}} + \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ .1\times .5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}}_{\mathbf{A}_0`_1} \begin{pmatrix} \Delta p \\ \Delta y \\ \Delta R \\ \Delta w \\ \Delta z \end{pmatrix}_{t-1}$$

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