NATURAL RESOURCE MODELING
Volume 23, Number 3, August 2010

A BIOECONOMIC SHEEP–VEGETATION TRADE-OFF MODEL: AN ANALYSIS OF THE NORDIC SHEEP FARMING SYSTEM

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ABSTRACT. The paper studies the economy and ecology of sheep farming at the farm level and includes 2 different categories of the animals, ewes (adult females) and lambs. The model is analyzed in a Nordic economic and biological setting. During the outdoor grazing season, animals face limited grazing resources so that the weight gain of lambs is determined by the per-animal vegetation consumption. On the other hand, the number of grazing animals, lambs as well as ewes, determines the grazing pressure. This problem is studied under the assumption of a rational and well-informed farmer who aims to maximize profit in ecological equilibrium with zero animal and vegetation growth. We find that lamb-only slaughtering is optimal and that it is never beneficial for the farmer to keep livestock that overgraze pasture. It is also shown that higher meat prices and more profitable slaughtering make it economically rewarding for the farmer to keep more animals. A numerical illustration indicates that the optimal sheep farming decision may be more sensitive to changes in pasture quality and productivity than changes in economic conditions.

KEY WORDS: Sheep farming, grazing, stage model.

1. Introduction. In this paper, a bioeconomic sheep–vegetation trade-off model is analyzed. The main content of this trade-off is that high sheep densities yield high farm output in number of animals

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Received by the editors on 25th September, 2009. Accepted 18th May, 2010.

slaughtered. On the other hand, high sheep densities relative to pasture productivity are expected to cause a reduction in per-animal meat production and thus in income per animal. This problem has similarities with the standard predator–prey renewable natural resource problem (see, for example, Clark [1990]) where sheep are predators and vegetation is the prey. However, whereas the standard predator–prey problem is formulated within a biomass framework, the different *age categories* of the sheep are central in the following analysis. The study is conducted with a crucial distinction made between the outdoor grazing season (spring, summer, and fall) and the indoor winter feeding period. Lambs are born in late winter to early spring, just before the grazing season starts. This is the typical situation found in many strongly seasonal environments at northern latitudes such as in the Nordic countries and at high altitudes in continental Europe (e.g., mountainous areas in France and Spain). Sheep are the main livestock in animal husbandry in Norway, Iceland, the Faroe Islands, and Greenland, and in both Norway and Iceland most cultivated land is used for winter fodder production (58% and 95%, respectively; see, for example, Austrheim et al. [2008a]). Because winter grazing is practiced in the Faroe Islands, the present analysis essentially relates to the economic and biological setting found in Norway, Iceland, and Greenland.

Within this farming system, the individual farmer faces several decisions. The problem analyzed here is that of utilizing a given farm capacity (i.e., farm size) to provide the optimal number of animals to be fed and kept indoors during the winter season. A corollary of this problem is assessing the effect that summer grazing sheep density has on vegetation productivity and hence on per-animal meat production. The problem includes two categories, or stages, of sheep—lambs and ewes (adult females)—and is analyzed as an equilibrium harvest problem with zero animal and vegetation growth under the assumption that the farmer aims to do it "as well as possible," represented by current profit maximization. Analyzing the dynamic problem of maximizing present value profit is hence omitted from the present exposition. However, it is well known that the steady state of this problem coincides with our static problem except for the discount rent; that is, for zero discount rent, the solutions are similar.

There is extensive literature on the economics of livestock management (see, for example, Jarvis [1974], Kennedy [1986]), but most of this literature has little relevance to a farming system with a distinct seasonal subdivision between a winter indoor season and outdoor grazing. The problem of the typical cow–calf operator in the western United States has some similarities with the Nordic sheep farming system. However, one crucial difference here is that the length of the grazing season should be simultaneously determined together with the stocking level (see, e.g., Huffaker and Wilen [1991]). In contrast, the length of the grazing season is fixed because of climatic conditions in our problem.The spring lambing scheme is also taken for granted because of the climatic conditions. The animal growth model builds on that of Skonhoft [2008] but is extended to consider the constraint on animal weight growth from outdoor grazing conditions. Balancing the number and weight of animals is indeed seen as a crucial management problem in the Nordic countries (e.g., Ólafsdóttir and Júlíusson [2000], Mysterud and Austrheim [2005], Thomson et al. [2005]). The contribution of this paper is, from a theoretical point of view, to explain the stocking decision of an individual farmer and to explain how the balance between the number and weight of animals is influenced by various economic and ecological factors. The emphasis throughout is on analyzing and assessing the basic driving forces.

The paper is organized as follows. We first briefly present the Nordic sheep farming system in Section 2. Section 3 provides information about the sheep–vegetation interaction, and the simplified ecological model is presented. In Section 4, we closely examine this system in equilibrium with zero animal and vegetation growth. The revenue and cost functions are described in Section 5, and the stocking problem of the farmer is solved under the assumption of current profit maximization. Section 6 provides a numerical illustration, and Section 7 summarizes our findings.

2. The Nordic sheep farming system. The following analysis is related to economic and ecological conditions found in Norway, but these also exist in Iceland and Greenland. There are approximately 16,000 sheep farms in Norway, all family farms. Because there are around 2.1 million animals during the outdoor grazing season, the average farm size therefore only accounts for some 130 animals during the summer. Norwegian farms are located either close to mountain areas and other sparsely populated areas or along the coast, with a means to transport sheep to more distant alpine areas. The main product is meat, which accounts for about 80% of the average farmer's income.

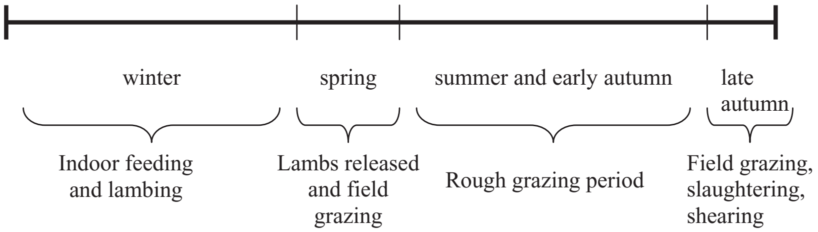

FIGURE 1. Seasonal subdivision in the Nordic sheep farming system.

The remainder comes from wool because sheep milk production is virtually nonexistent today (Nersten et al. [2003]). In Iceland, there are about 450,000 winterfed animals today. Meat is also the most important product from sheep farming here. In Greenland, the land for sheep grazing is much more restricted, and the population of ewes in 2007 was estimated at 25,000 (Austrheim et al. [2008a]).

Housing and indoor feeding are required throughout winter because of snow and harsh weather conditions (Figure 1). In Norway, winter feeding typically consists of hay grown on pastures close to farms (80%), with the addition of concentrate pellets provided by the industry (20%). Lambs are born from late winter to early spring, and in late spring and early summer the animals usually graze on fenced land close to the farm at low elevations, typically in the areas where winter food for the sheep is harvested during summer. When weather conditions permit (for reasons of plant phenological development), sheep are released into rough grazing areas in the valleys and mountains. In Norway, most sheep (about 75% of the total metabolic biomass) graze in the northern boreal and alpine region (Austrheim et al. [2008b]). The outdoor grazing season in mountain areas ends between late August and the middle of September and does not normally exceed 130 days. During the rough grazing period, flocks may be vulnerable to accidents and disease, and in some regions also to large predators. Aunsmo [1998] and Nersten et al. [2003] provide more details. After the grazing season, the animals are mustered and the wool is shorn. Slaughtering takes place immediately or after a period of grazing on the farmland (more details are provided in Austrheim et al. [2008a]). The seasonal subdivision is similar in Iceland and Greenland.

Because of an increase in the number of sheep combined with an abundance of low-quality fodder plants, there are signs of overgrazing in some alpine areas in Norway. However, in general, overgrazing is not a serious problem, and studies of productive and species-rich alpine environments show only modest effects of grazing on plant community patterns, at least in the short term (Austrheim et al. [2008a]). In Iceland, the situation is different because sheep numbers have decreased significantly during the past few decades as a result of overgrazing.

3. Ecological model. There is a dynamic relationship between large herbivores and the plants on which they forage (e.g., Hobbs [1996], Augustine and McNaughton [1998], Danell et al. [2006]). This is because grazing affects the quantity and quality of vegetation, which in turn affects the growth of the herbivores (Choquenot [1991], Simard et al. [2008]). Experimental studies show lower body mass growth of lambs at high sheep density (80 sheep per km^2) as compared with low sheep density (25 sheep per km^2; Mysterud and Austrheim [2005]). Removal of plant tissue affects individual fitness (e.g., plant growth) directly and may cause biomass reduction of preferred plant species (i.e., fodder plants; see, for example, Bråthen and Oksanen [2001], Eskelinen and Oksanen [2006]). Indirect effects, which operate by changing the competitive balance with other species, may be even more important for the development of the vegetation community. In particular, in low productivity ecosystems such as the one considered here, heavy grazing may favor heavily defended, nonpalatable plant species to the detriment of palatable species (Austrheim et al. [2007]). Invasion of such species affects the strength of density-dependent effects on the weight growth of sheep in the long term. The farmer may thus increase current stock numbers at the expense of reduced growth in subsequent years. Indeed, with increasing density of sheep on pasture, a higher proportion of low-quality plants (Kausrud et al. [2006]) and vegetation types (Mobæk et al. [2008]) may result. In ruminants, even slight changes in plant quality can reduce body growth rates substantially because they contain fewer nutrients per bite as well as increase rumination time (White [1983]). However, moderate grazing is expected to facilitate plant biomass production in productive habitats and thus the fodder availability for moderate grazing as compared with no grazing (McNaughton [1979]).

In the simplified sheep–vegetation model to be formulated, we assume a single plant species, or composite homogeneous vegetation, expressed as vegetation quantity and measured in tons of vegetation biomass. This composite vegetation biomass is consumed by sheep during the outdoor grazing season and regenerates through a natural growth process. The model is formulated at a discrete time with a seasonal subdivision between the outdoors grazing period (spring, summer, and fall) and indoor winter feeding period (Figure 1). The sheep population is structured (e.g., Caswell [2001]) as ewes and lambs. As already indicated, lambs are born in late winter to early spring, just before the grazing season begins. Lambs not slaughtered enter the adult population after the slaughtering period (i.e., September–October). All male lambs are assumed to be slaughtered because very few (or none when artificial insemination is practiced) are kept for breeding. Therefore, only female adults are considered. Fertility is assumed to be fixed, a reasonable assumption because farmers provide extra feed to buffer environmental effects (e.g., in a poor year there is high density relative to food resources in the pasture). Natural mortality differs between adults and lambs and is considered fixed and density independent. All natural mortality is assumed to occur during the grazing season. Demographic data on sheep are available in Mysterud et al. [2002].

The number of adult females in year $(t+1)$ after the slaughter consists of the previous year's adults and female lambs that have survived natural mortality and have not been slaughtered. This is written as $X_{t+1} = Y_t s_Y(1-h_{Y,t}) + X_t s_X(1-h_{X,t})$, where Y_t is the number of female lambs; s_X and s_Y are the natural survival fractions of adult females and lambs, respectively; and $h_{X,t}$ and $h_{Y,t}$ are the fractions slaughtered. With the fecundity rate b (lambs per adult female) and ψ as the fraction of female lambs recruited (ψ is usually close to 0.5), $Y_t = \psi b X_t$ yields the number of female lambs. Therefore, when ignoring the possibility of additional animals from outside, the ewe population growth is governed by

$$X_{t+1} = \psi b X_t s_Y(1-h_{Y,t}) + X_t s_X(1-h_{X,t}). \tag{1}$$

Vegetation growth consists of natural growth and consumption by grazing sheep, and follows the Noy-Meir [1975] model in which *per-animal* vegetation consumption increases with vegetation availability. It is assumed that the number of grazing animals influences the vegetation consumption while the amount consumed in turn determines the weight gain of the animals during the grazing season. See also, for example, Huffaker and Wilen [1991]. The food intake of the ewes may be greater than that of the lambs, but it is for simplicity supposed that all animals influence vegetation consumption in a similar manner. In addition to consumption, vegetation regenerates through a natural growth process represented by a single-peaked value function. Vegetation growth may then be written as[1]

$$V_{t+1} - V_t = f(V_t) - g(V_t)(1 + b)X_t, \tag{2}$$

where $g(V_t)$ is the sheep's per-capita consumption function and $f(V_t)$ yields the natural growth function. In the numerical analysis and in the theoretical reasoning, we consider a consumption function specified as $g(V_t) = kV_t/(V_t + c)$, where $k > 0$ is the maximum vegetation biomass intake per animal and $c > 0$ determines the shape of the consumption pattern. Natural growth is described by the standard logistic function $f(V_t) = rV_t(1 - V_t/Q)$, with $r > 0$ as the maximum specific vegetation growth rate (vegetation productivity) and $Q > 0$ as the carrying capacity.

The weight gain of the lambs during the grazing season coincides with the weight at the end of the season; that is, the slaughter weight (kg per animal). It is assumed proportional to per-animal vegetation consumption

$$w_{Y,t} = qg(V_t), \tag{3}$$

where the parameter $0 < q < 1$ translates grazing biomass into meat biomass. For the specified consumption function, the lamb slaughter weight is an increasing, concave function of vegetation quantity; that is, better grazing conditions increase the per-animal weight but to a decreasing degree. For the adults, there is generally no weight change during the grazing season on productive pastures while there may be some loss in low productivity areas (Mysterud and Austrheim [2005]). However, as a reasonably good approximation, we simply neglect any

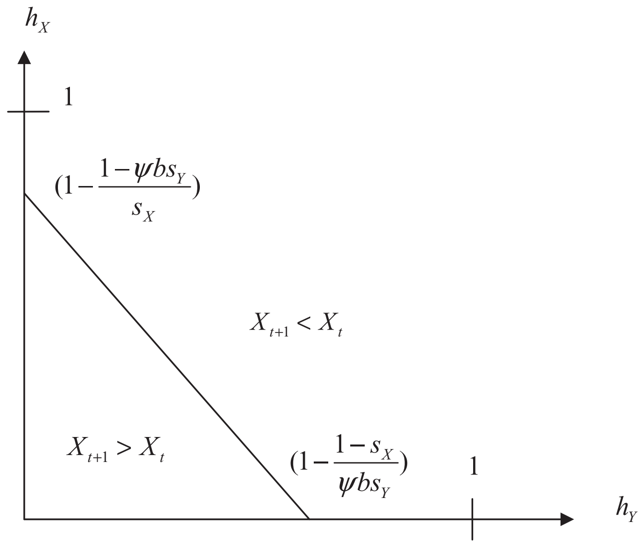

FIGURE 2. Equilibrium (constant animal population) harvesting relationship (equation (1′)). h_Y , female lamb slaughtering fraction; h_X , ewe (adult female) slaughtering fraction.

possible connection between the amount of vegetation and weight, and the adult slaughter weight is fixed and determined outside the model

$$w_{X,t} = w_X . \tag{4}$$

4. Ecological equilibrium. As mentioned, the stocking decision of the farmer is analyzed in ecological equilibrium, i.e., when vegetation and animal growth equal zero. Because the population growth equation (1) is linear for number of animals, there are infinite combinations of harvesting fractions that sustain a stable population. Therefore, for a constant number of animals $X_{t+1} = X_t = X$, we have

$$X = \psi bXs_Y(1 - h_Y) + Xs_X(1 - h_X), \tag{1′}$$

or simply $1 = \psi bs_Y(1 - h_Y) + s_X(1 - h_X)$ when $X > 0$ (see Figure 2). This intersects with the h_X axis at $[1 - (1 - \psi bs_Y)/s_X]$, which may be above or below 1. Therefore, the highest adult slaughter rate compatible with zero animal growth is $\min\{1, [1-(1-\psi b s_Y)/s_X]\}$. For all realistic parameter values, it is below 1 (see numerical section), and this is assumed to hold in the subsequent analysis. It intersects with the h_Y axis at $[1-(1-s_X)/\psi b s_Y] < 1$ and is hence the highest lamb-slaughtering rate compatible with equilibrium.

The equilibrium vegetation growth condition $V_{t+1} = V_t = V$ next yields

$$f(V) = g(V)(1+b)X. \tag{2'}$$

Depending on the slope of the natural growth function $f(V) = rV(1-V/Q)$ and the sheep consumption curve $g(V)(1+b)X = (kV/(V+c))(1+b)X$ (see above), there may be one or two equilibria (see also Noy-Meir [1975]). A necessary and sufficient condition for a unique equilibrium is that the consumption curve intersects with the natural growth function from below and where more animals, *ceteris paribus*, means less vegetation biomass. In the opposite case, there are two interior equilibria. However, the lower vegetation level equilibrium, for a given number of animals, is not stable and not considered. Therefore, these functions are scaled such that the consumption curve intersects with the natural vegetation growth curve from below; that is, $f'(V) < g'(V)(1+b)X$ holds at the unique (interior) equilibrium (cf. Figure 3). For the given specific functional forms, the sheep–vegetation equilibrium is $r(1-V/Q) = [k/(V+c)](1+b)X$ and is defined for $0 < X < rc/k(1+b)$ and $0 < V < Q$. Within these intervals, vegetation quantity is hence a decreasing function of the stocking rate.

5. Revenue and costs. We disregard income from wool production, and meat sales are the only revenue component for the farmer. Because slaughtering takes place after natural mortality, the number of ewes and female lambs removed are $X_t s_X h_{X,t}$ and $\psi b X_t s_Y h_{Y,t}$, respectively. As mentioned above, the entire male lamb subpopulation $(1-\psi)bX_t s_Y$ is slaughtered. The number of animals removed is then $H_t = bX_t s_Y(\psi h_{Y,t} + 1 - \psi) + X_t s_X h_{X,t}$. With p_X as the net (of slaughtering costs) ewe slaughtering price (NOK per kg) and p_Y the lamb net slaughtering price, both assumed to be fixed and independent

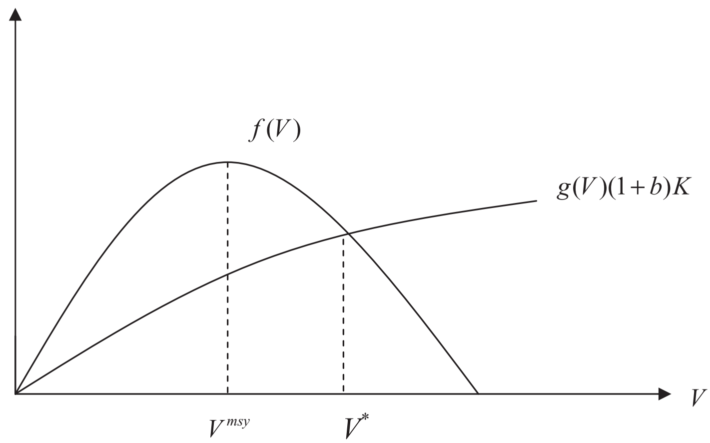

FIGURE 3. The natural vegetation growth–consumption relationship with a unique vegetation equilibrium. Equilibrium indicated when binding farm capacity, $X^* = K$.

of the number of animals supplied at the farm level, the current meat income of the farmer is given by $R_t = [p_Y w_{Y,t} b X_t s_Y (\psi h_{Y,t} + 1 - \psi) + p_X w_{X,t} X_t s_X h_{X,t}]$.

The cost structure differs sharply between the outdoor grazing season and the indoor feeding season, and the indoor costs are substanially higher. Throughout this analysis, we assume a given farm capacity. Therefore, the costs of buildings, machinery, and so forth are fixed (see also below). The indoor season variable costs include labor (typically an opportunity cost), electricity, and veterinary costs in addition to fodder. These vary with the given length of the indoor season (Section 1). For simplicity and without loss of any generality, the cost is assumed to increase linearly with the size of the winter population, $C_t = \alpha X_t$ with $\alpha > 0$.

As indicated, during the grazing period the sheep may graze on communally owned lands ("commons") or private land. Within the Nordic sheep farming system, such land may be available cost free, or the farmer may pay a *fixed* yearly rental (Austrheim et al. [2008a]). There may be some transportation and maintenance costs, but such costs are neglected because they are generally rather low. The total yearly variable cost is hence simply assumed to be the indoor season cost. Therefore, when inserting for equations (3) and (4) and ignoring discounting within the year, the current profit of the farmer is described by

$$\pi_t = R_t - C_t = p_Y qg(V_t)bX_t s_Y(\psi h_{Y,t} + 1 - \psi) + p_X w_X X_t s_X h_{X,t} - \alpha X_t. \tag{5}$$

As mentioned, a given farm capacity is assumed. However, although not allowing for investment in farm capacity, it is taken into account as a possible constraint.[2] Capacity is related to the number of animals kept during the winter and is

$$X_t \leq K. \tag{6}$$

The capacity may be binding or not. When binding, we obviously find that the current profit of the last animal to be kept during the winter is positive. However, when it is not binding, the marginal profit of the last animal is also positive. This is due to the shadow cost of vegetation consumption. The subsequent analysis explains this in more detail.

6. The optimal sheep–vegetation trade-off. The farmer is assumed to be "rational" and well informed with the goal of maximizing current profit (5) subject to the animal equilibrium condition (1′) and the vegetation equilibrium condition (2′), together with the farm capacity constraint (6). When omitting the time subscript, the Lagrangian of this problem reads

$$\begin{aligned} L = {} & p_Y qg(V)bXs_Y(\psi h_Y + 1 - \psi) + p_X w_X X s_X h_X - \alpha X \\ & - \lambda[X - \psi bXs_Y(1 - h_Y) - Xs_X(1 - h_X)] \\ & - \mu[g(V)(1 + b)X - f(V)] - \eta(X - K), \end{aligned}$$

where $\lambda > 0$ is the animal resource shadow price, $\mu > 0$ is the vegetation resource shadow price, and $\eta \geq 0$ is the farm capacity constraint shadow price. The first-order conditions of this problem with $X > 0$

and $V > 0$ and both harvest mortalities below 1 (see above) are

(7) $$\partial L/\partial h_Y = p_Y qg(V) - \lambda \leq 0; \quad 0 \leq h_Y < 1,$$

(8) $$\partial L/\partial h_X = p_X w_X - \lambda \leq 0; \quad 0 \leq h_X < 1,$$

(9) $$\partial L/\partial X = p_Y qg(V)bs_Y(\psi h_Y + 1 - \psi) + p_X w_X s_X h_X - \alpha - \mu g(V)(1 + b) - \eta = 0,$$

and

(10)
$$\partial L/\partial V = p_Y qg'(V)bXs_Y(\psi h_Y + 1 - \psi) + \mu[f'(V) - g'(V)(1 + b)X] = 0.$$

The interpretation of control condition (7) is that lamb slaughtering should take place up to the point where the marginal meat income (NOK per animal) is equal to, or below, the animal resource shadow price. Following the Kuhn–Tucker theorem, it holds as an equation when the removal of this subpopulation is optimal. The adult control condition (8) has the same interpretation. The animal stock equation (9) states that the number of ewes (adult females) should be maintained so that the value of an additional animal on the margin equals the marginal cost of doing so plus the marginal grazing cost evaluated as the shadow price and the shadow price of the farm capacity constraint. Finally, the vegetation condition (10) states that the marginal benefit of more vegetation through higher lamb weight should equal its marginal cost given by the difference between marginal vegetation growth and marginal consumption, evaluated as its shadow price. Because the vegetation consumption curve is assumed to intersect with the natural growth function from below so that $f'(V) - g'(V)(1 + b)X < 0$ (see above), the condition for a stable vegetation equilibrium (for a given number of animals) implies a positive vegetation shadow price, $\mu > 0$. Therefore, as expected, the profit of adding one more animal to the stock (equation (9)) is always strictly positive in an optimal program, $p_Y qg(V)bs_Y(\psi h_Y + 1 - \psi) + p_X w_X s_X h_X - \alpha > 0$.

One striking point of the solution of the model is that the control conditions (7) and (8) cannot generally be satisfied simultaneously as equations. Because of demand conditions, the meat price (NOK per kg) is higher for lambs, $p_Y > p_X$. On the other hand, irrespective of the grazing conditions, the per-animal weight is higher for the ewes, $w_X > qg(V)$. However, the meat price difference dominates the weight difference, and the lamb slaughter price (NOK per animal) is above that of the ewe slaughter price. $p_Y qg(V) > p_X w_X$ is therefore assumed to hold for all possible vegetation quantities (more details in the numerical section). Consequently, and because the lamb equilibrium slaughtering mortality is below 1 (see Figure 2), condition (7) must hold as an equation while (8) holds as an inequality. Slaughtering only lambs is hence optimal, and the animal shadow price is given by $\lambda^* = p_Y qg(V^*)$ (superscript "$*$" indicates optimal values). A corollary of this result is that lamb slaughtering should take place at the highest level compatible with the sheep population equilibrium, cf. equation ($1'$) and Figure 2.

An important result of this equilibrium stocking problem thus boils down to a simple principle, and single-stage slaughtering only results because the harvest benefit is linear in both harvest controls. This result has similarities with the well-known finding of Reed [1980], who studied the maximum sustainable yield problem of a fishery. On the other hand, the reason for slaughtering at the highest level compatible with ecological equilibrium follows from the lack of any density-dependent effects in the animal growth equation (1). The fact that there is an animal–vegetation interaction and that vegetation growth is density dependent does not affect this. At the same time, this means that the optimal slaughter rate, in contrast to the result in most bioeconomic models, depends on biological conditions (fertility and mortality) only. Therefore, the optimal equilibrium slaughtering rates are $h_X^* = 0$ and $h_Y^* = 1 - (1 - s_X)/\psi b s_Y$. This is stated as

Result 1. *Slaughtering is contingent upon the per-animal meat value only. Slaughtering only lambs is optimal, and this should take place at the highest level compatible with population equilibrium determined by (sheep) biological factors alone.*

The stock conditions (9) and (10) are considered next, and we distinguish between two cases: a binding and nonbinding farm capacity constraint. Suppose first that the capacity constraint (6) is binding; that is,

$X^* = K$ and $\eta^* > 0$. This may intuitively occur if the capacity is small, the farm profitability is high (e.g., a high lamb meat price), or both (see also below). The vegetation utilization is then determined through the equilibrium condition (2′), which is $f(V^*) = g(V^*)(1 + b)K$. Therefore, neither economic factors nor sheep biological factors, except the fertility parameter b, influence the vegetation quantity V^*. Not surprisingly, a higher farm capacity, when binding, and more animals means reduced V^*.

The optimal number of animals removed, consisting of lambs only (female and male), is found through $H^* = bX^*s_Y(\psi h_Y^* + 1 - \psi) + X^*s_X h_X^* = K(bs_Y - (1 - s_X))$ and is determined by farm capacity and sheep biological factors alone, but where the sex composition plays no role. Furthermore, in this case when the farm capacity binds, the capacity directly determines the lamb slaughter weight through the vegetation equilibrium condition, $w_Y^* = qg(V^*)$. The sheep–vegetation trade-off is then straightforward as higher farm capacity and a larger number of animals reduce the equilibrium vegetation quantity and hence the per-animal slaughter weight. On the other hand, the capacity effect on farm output (in kg meat) $w_Y^* H^* = qg(V^*)K(bs_Y - (1 - s_X))$ seems ambiguous. However, by taking the total differential, $d(w_Y^* H^*) = q[bs_Y - (1 - s_X)](g'KdV^* + gdK)$, and then combining it with the differential of the vegetation equilibrium condition $f'dV^* = (1 + b)(g'KdV^* + gdK)$, we find $d(w_Y^* H^*) = q[bs_Y - (1 - s_X)][f'/(1 + b)]dV^*$. Therefore, higher capacity and hence lower vegetation quantity mean higher output if the vegetation consumption curve intersects the vegetation natural growth function on the right-hand side of its peak value, $f' < 0$ and $V^* > V^{msy}$. In the opposite case, output and hence farm revenue $R = p_Y qg(V^*)K(bs_Y - (1 - s_X))$ fall with a higher capacity K. Because the cost αK at the same time increases, the farm profit clearly decreases as well. However, because higher farm capacity, if binding, implies higher profit following the logic of the optimization, it is evident that farm capacity cannot be binding in this last case. The optimal solution is therefore characterized by $V^* > V^{msy}$. This is stated as

Result 2. *Higher farm capacity, when binding, reduces per-animal slaughter weight but increases farm output.*

We next consider the situation with high farm capacity, low profitability, or both so that capacity is no longer binding; that is, $X^* < K$ and $\eta^* = 0$. The first-order condition (9), as already mentioned, indicates higher profit on the margin. The marginal animals hence also contribute to increased revenue and farm output. Therefore, as explained above, the stocking rate can never exceed one consistent with $V^* \leq V^{msy}$. If a vegetation quantity lower than V^{msy} is defined as "overgrazing" (but see Mysterud [2006]), we may state:

Result 3. *Irrespective of ecological and economic conditions, it is never beneficial for the farmer to keep animal stock that overgraze the pasture.*

This result, which is similar to the maximum sustainable economic yield policy of a fishery (e.g., Clark [1990]), can also be demonstrated as follows. If we first insert the optimal slaughter rates $h_X^* = 0$ and $h_Y^* = 1 - (1 - s_X)/\psi b s_Y$ into the profit function (5), the result is $\pi = p_Y q[bs_Y - (1 - s_X)]g(V)X - \alpha X$. Replacing X with the vegetation equilibrium condition (2′) and differentiating, we next find after some small rearrangements $d\pi/dV = [1/(1+b)]\{p_Y q[bs_Y - (1 - s_X)] f' - (\alpha/g^2)(f'g - fg')\}$. $d\pi/dV = 0$ is then characterized by $\{p_Y q [bs_Y - (1 - s_X)]g(V^*) - \alpha\}f'(V^*)g(V^*) = -\alpha f(V^*)g'(V^*)$. Because the left-hand side is positive and $g'(V) > 0$, this condition holds only when $f'(V^*) < 0$ or $V^* > V^{msy}$. Therefore, as stated above, a stocking rate that overgrazes the pasture is not economically beneficial for the farmer. For the specific vegetation natural growth function $f(V) = rV(1 - V/Q)$ (see above), $V^* > V^{msy} = Q/2$ thus indicates the optimal vegetation condition. Overgrazing according to this growth function is then synonymous with $V \leq Q/2$.[3]

We may also expect to find $\partial V^*/\partial\alpha > 0$, or equivalently, $\partial V^*/\partial p_Y < 0$. Not surprisingly, it can be shown that these results hold because of the second order condition for a maximum.[4] As $\partial V^*/\partial p_Y < 0$ implies $\partial X^*/\partial p_Y > 0$ through the vegetation equilibrium condition (2′), we may also state

Result 4. *A higher slaughter price yields a larger flock size and lower vegetation quantity.*

This result contrasts with standard bioeconomic harvesting theory (Clark [1990]). The working of the price effect here is, however, different from the standard model as there is no stock-dependent harvesting, or slaughter, cost included. Therefore, a higher price p_Y simply means that it becomes less expensive to keep animals during the indoor feeding season, which motivates the farmer to increase the number of animals. Because the fraction of animals slaughtered h_Y^* is determined by biological factors alone, we also find that it is beneficial for the farmer to increase the number of animals slaughtered and increase the farm output (in kg) for a higher lamb meat price. As more animals are added and greater pasture utilization means a lower per animal slaughter weight (see above), increased meat supply is hence met through a higher rate of removal of animals dominating the reduced per-animal (lamb) slaughter weight.

When α becomes small and negligible, it is also seen from the above condition of $d\pi/dV = 0$ that profit maximization implies $f'(V^*) = 0$ and $V^* = V^{msy}$. Our specific vegetation natural growth function, $V^* = Q/2$, inserted into the vegetation equilibrium condition (2′) $rV(1 - V/Q) = [kV/(V + c)](1 + b)X$ yields $X^* = r(Q + 2c)/4k(1 + b)$. Therefore, for these specific functional forms, this is the highest possible stocking rate under the present assumption of a well-informed, profit-maximizing farmer. Note that no economic parameters are included here, and the fertility parameter b is the only sheep biological parameter included.

It may also be of interest to assess how vegetation productivity affects the stocking decision and pasture utilization. With our specification of the vegetation natural growth function, $f(V) = rV(1 - V/Q)$, the intrinsic growth rate parameter r steers productivity. For this specific functional form, however, we find that it does not influence optimal vegetation utilization. This is observed by studying the above expression $d\pi/dV = [1/(1 + b)]\{p_Y q[bs_Y - (1 - s_X)]f' - (\alpha/g^2)(f'g - fg')\}$, where r is omitted when characterizing $d\pi/dV = 0$ because both f' and f include this productivity parameter as a multiplicative term. On the other hand, through the vegetation equilibrium condition $f(V^*) = g(V^*)(1 + b)X^*$, or $rV^*(1 - V^*/Q) = [(k/(V^* + c)](1 + b)X^*$, we find that higher productivity yields more animals. Therefore, again following the logic of our rational and well-informed farmer, different pasture productivity *ceteris paribus* translates into unchanged pasture utilization but different farm sizes in number of animals. This is stated as:

Result 5. *With higher vegetation productivity, it is beneficial for the farmer to keep more animals when the farm capacity constraint is not binding. When the natural growth of vegetation is described by a logistic growth function, higher vegetation productivity does not influence optimal pasture utilization.*

The above and other results can be confirmed more directly by inserting the specific functional forms of the vegetation consumption curve and the natural growth function into the above expression, $d\pi/dV = 0$. When solving for vegetation quantity, we find $V^* = (Q/2)\frac{p_Y qk[bs_Y - (1-s_X)] - \alpha(1-c/Q)}{p_Y qk[bs_Y - (1-s_X)] - \alpha}$. Next, inserting the vegetation equilibrium condition (2′), it is possible to find an explicit expression for the sheep stocking rate as well.

7. Numerical illustration

7.1. Data. To shed some further light on the above analysis, the model is illustrated numerically, applying the above-specified vegetation natural growth function and animal consumption curve. Only simulations in which the farm capacity is not binding are reported. We consider a rather large farm (cf. introductory section) located in an area with relatively high vegetation productivity. The baseline parameter values, in which sheep biological data and economic values are related to Norwegian conditions, are shown in Table A1 (Appendix). The sheep biological data are based on a large set of observations, whereas we lack reliable data for the vegetation parameter values. However, these last values are calibrated such that the ewe weight in our model, as in reality, is always higher than that of lambs, and the animal slaughter value in the model, as in reality, is always higher for lambs (see Appendix).

The size of the farm is scaled by the vegetation carrying capacity Q. With $Q = 500$ (tons of vegetation biomass), we find $V^{msy} = Q/2 = 250$. Accordingly, for the baseline parameter values (Table A1), the highest possible stocking rate (and winter population size) is $X^* = r(Q + 2c)/4k(1 + b) = 108$ (ewes). We contrast this farm with a larger

TABLE 1. X^* stocking rate (number of ewes), V^* vegetation (tons), H^* slaughter (number of lambs), w_Y^* slaughter weight (kg/animal), and π^* profit (in NOK). Farm capacity not binding.

	X^*	V^*	H^*	w_Y^*	π^*
Baseline	102	278	135	23.1	106,400
50% price increase ($p_Y = 75$)	104	267	140	22.5	185,200
50% cost increase ($\alpha = 750$)	96	296	129	23.8	81,700
33% reduction vegetation productivity ($r = 0.33$)	67	278	90	22.9	70,200
30% increase carrying capacity ($Q = 650$)	118	352	158	25.9	146,400

one, with a higher fixed carrying capacity, as well as with one located in an area with lower vegetation productivity, captured by a different intrinsic vegetation growth rate. We also study the effects of changes in prices, costs, and sheep biological factors.

7.2. Results. In the results (Table 1), the lamb-harvesting rate (fraction) is always $h_Y^* = 0.93$, determined by (sheep) biological parameters alone (Result 1). In the baseline scenario (row one), the optimal ewe flock size is 102 animals (ewes). The lamb slaughter weight is slightly above 23 (kg) and the number of animals slaughtered (lambs) is 135. The vegetation quantity is 278 (tons), somewhat above that of the maximum sustainable yield value $V^{msy} = 250$. The lamb value (NOK per lamb) becomes $p_Y w_Y^* = p_Y qg(V^*) = 1,153$ compared with the ewe fixed value of $p_X w_X = 3530 = 1,050$ (Table A1, Appendix).

When the slaughter price rises while all other parameters remain fixed (Table 1, row 2), the stocking rate increases while the vegetation quantity falls (Result 4). Increased indoor feeding cost (line 3) works in the opposite manner. The numerical simulations also confirm (not shown in the table) that the hypothetical case of small and negligible costs yields a stocking rate consistent with the highest possible vegetation natural growth (Result 3). The effects of changing sheep biological factors are studied as well, and higher fertility and reduced mortality increase the lamb harvest rate and number of animals slaughtered while the size of the optimal flock decreases (not reported).

Table 1 (row 4) also illustrates the effects of a shift in the vegetation productivity (intrinsic growth rate) parameter. The stocking rate X^* becomes lower when the vegetation productivity falls, while the vegetation quantity stays unchanged compared with the baseline scenario (Result 5). For the given specific functional forms, we also find that the stocking rate increases linearly with increasing vegetation productivity. This is recognized by writing the vegetation equilibrium condition as $X^* = [(V^* + c)(1 - V^*/Q)/k(1 + b)]r$ (see Section 5), which for the baseline parameter values (except r) yields $X^* = [(278 + 300)(1 - 278/500)/0.50(1 + 1.53)]r = 202.9r$ (animals). Accordingly, we find $X^* = 67$ when $r = 0.33$ while the baseline value $r = 0.50$ yields $X^* = 102$ (first row of Table 1). The effects of a larger carrying capacity, indicating a larger farm size, are shown in the last row of the table. As expected, the optimal vegetation utilization increases significantly. Therefore, we also find a substantially higher lamb slaughter weight. Because a higher number of animals are slaughtered (lambs), profit also increases significantly.

A striking point of these calculations is the modest changes in stocking rate, number of animals slaughtered, and vegetation utilization because of shifting economic conditions. This picture is confirmed when other economic parameter values are applied. Therefore, price and cost variations more or less spill over to profitability changes only (last column of Table 1). This indicates that the profit function $\pi^* = p_Y q[bs_Y - (1 - s_X)]g(V^*)X^* - \alpha X^*$ (Section 6, above) is only weakly nonlinear (at least in the actual range of parameter values) in p_Y and α. Figure 4 demonstrates this aspect of our model in another way; profit is depicted for different stocking values under the baseline parameter scenario. The optimal ewe stocking value is $X^* = 102$ (see also Table 1), but the figure demonstrates a rather flat profitability curve in the neighborhood of this optimum. For example, a figure of 90 animals instead of the optimal 102 reduces the profit from (NOK) 106,400 (Table 1) to just 103,200 (left-hand scale). At the same time, the number of animals slaughtered changes from 135 to 121. This animal output reduction is thus counterbalanced by a higher, albeit

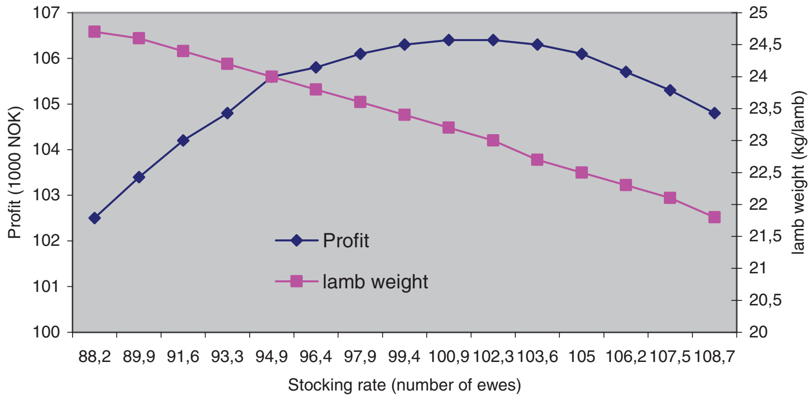

FIGURE 4. Variations in stocking rate. Baseline parameter values. Profitability (left-hand scale) and per-lamb productivity (right-hand scale).

quite modest, per-animal (lamb) slaughter weight (right-hand scale of Figure 4).

Figure 4 illustrates at the same time the basic sheep–vegetation trade-off taking place in our ecological–economic system without a binding farm capacity and when the slaughtering policy is fixed according to the difference in per-animal value. For an initial low stocking rate, expansion and more animals added is beneficial for the farmer as increased production in number of animals slaughtered (lambs) more than outweighs reduced productivity in weight per slaughtered animal, together with additional winter fodder costs. However, when farm size is expanded further above the optimal stocking rate, reduced vegetation quantity translates into a lower weight per slaughtered animal and higher winter fodder costs, which dominates the additional income gain from the larger number of animals slaughtered.

8. Concluding remarks. This paper has analyzed the economics of sheep farming in a two-stage model of lambs and adult females (ewes). The analysis is at the farm level in a Nordic context with a crucial distinction between the outdoor grazing season and the winter indoor feeding season, and where a Noy-Meir [1975] type model describes the animal–vegetation interaction. The farmer is assumed to be "rational" and well-informed, and aims to find the level of animal slaughter that maximizes profit, the accompanying number of summer grazing animals, and the number of animals to be kept indoors during the winter. This problem is analyzed as an equilibrium-harvesting problem with zero vegetation and animal growth.

In this two-stage model of lambs and ewes, the harvesting decision is shaped by economic factors alone. For the given price and market conditions whereby the value of lambs is higher than that of ewes, lamb-only slaughtering at the highest possible level is the optimal strategy. On the other hand, the optimal lamb slaughter fraction is determined by sheep biological factors alone. The reason for this sharp distinction between the effects of economic and biological forces is the lack of any density-dependent factors regulating sheep population growth. In contrast, and in line with standard bioeconomic harvesting theory, biological and economic factors jointly determine optimal flock size and vegetation utilization. Our stocking problem is analyzed with binding and nonbinding farm capacity constraints. In the last case, as explained, the basic trade-off mechanism is that with an initial low stocking rate, additional expansion and animals are beneficial for increased production. Number of animals slaughtered more than outweighs reduced productivity in weight per slaughtered animal (lambs). At the optimum level, the marginal meat income should equal the marginal cost of keeping the stock plus the user cost of the pasture vegetation, evaluated by its shadow price.

The numerical illustrations indicate that shifting economic conditions for the farmer have small effects on the stocking rate and vegetation utilization. Such shifts, at least within the actual range of parameter values, spill over to changing farm profitability. On the other hand, we find vegetation productivity to have crucial allocation effects. For example, when comparing two equally sized farms located in areas with differing productivity, the farmer that benefits from high productivity will find it rewarding to keep a significantly higher stocking rate than the other one. The high productivity farmer will receive substantially higher economic benefits as well. The optimal sheep farming decision may hence be more sensitive to changes in pasture quality and productivity than changes in economic conditions.

Meat production alone is included in our study because this accounts for most of the income of the Nordic sheep farmer. The remainder comes from wool. When also adding the wool value, however, we find that the slaughtering decision is the same as that without wool. On the other hand, including the wool value generally influences the optimal stocking decision and the optimal number of animals to be kept during the winter. For this reason, pasture utilization is also affected. During the grazing period, sheep in Scandinavia may be vulnerable to predation from four large predators: bears, wolverines, wolves, and lynxes. Considering predation would not change the slaughtering decision, but the optimal flock size would be lower (Skonhoft [2008]). Adding more stages of the sheep population, with natural mortality and fertility generally differing among the stages (Mysterud et al. [2002]), may also increase the realism of the analysis. However, such an extension would not change the principal application of our model because differences in the per-animal economic value, not natural mortality and fertility, determine the optimal slaughtering decision of the farmer. Therefore, Result 1 and lamb-only slaughtering still hold when the per-animal slaughter value is highest for the lambs. The basic sheep–vegetation trade-off taking place in our system and depicted in Figure 4 would also be left more or less unchanged.

Only equilibrium harvesting is analyzed in this paper. If the optimal equilibrium stock size is not realized because of shifting economic, ecological, or other conditions, it raises the question of how the farmer should adjust slaughtering to reach equilibrium. If the farm is initially below the optimal stocking rate, slaughtering below the equilibrium lamb slaughter rate should occur temporarily. Because of the high animal growth potential, however, this adjustment period will typically be short, possibly only one year. For example, following the animal growth equation (1), we find that the ewe number increases by more than 60% within 1 year without slaughtering (cf. parameter values in Table A1). On the other hand, if the stocking level is initially too high relative to the vegetation resources, some ewe harvesting, in addition to slaughtering all the lambs, should possibly be included if the farmer aims to adjust to the optimal equilibrium as rapidly as possible. Although it is quite simple to find harvesting policies leading to an equilibrium when initially outside it, it is more difficult to find optimal transitional paths leading toward an equilibrium, or steady state. To find such paths, a complete dynamic analysis of our farm problem is required.

APPENDIX TABLE A1. Baseline ecological and economic parameter values.

Parameter	Parameter description	Value
s_Y	Natural survival fraction lambs	0.91
s_X	Natural survival fraction ewes	0.95
b	Fertility rate	1.53 (lamb/ewe)
ψ	proportion of female lambs	0.50
w_X	Adult (ewes) slaughter weight	30 (kg/animal)
q	Biomass translation parameter	96 (kg meat/ton vegetation biomass)
r	Intrinsic vegetation growth rate (pasture productivity)	0.50
Q	Vegetation carrying capacity	500 (tons of vegetation biomass)
k	Vegetation saturation parameter	0.50 (tons of vegetation biomass/animal)
c	Shape animal consumption parameter	300 (tons of vegetation biomass)
p_X	Adult (ewe) slaughter price	35 (NOK/kg)
p_Y	Lamb slaughter price	50 (NOK/kg)
α	Marginal cost, winter	500 (NOK/animal)

Note: Exchange rate: 1 Euro = 8.70 NOK (Sept. 2009).

APPENDIX

Data and parameter values. Aunsmo [1998], Nersten et al. [2003], and Asheim [2007] provide economic data. Prices and costs are in 2003 values. The sheep biological baseline parameter values are based on Mysterud et al. [2002] and Aunsmo [1998]. As a background for the vegetation growth values, there are some studies indicating the amount of fodder production. However, alpine pastures are heterogeneous, and estimations of fodder production from two alpine ranges in

Norway (Setesdalsheiene and Hardangervidda) show large variations. Vegetation types with a limited biomass production dominate. There are also meadows that produce a large amount of fodder of very high quality. However, the meadows cover only a small proportion of these areas (Austrheim et al. [2008a]). The vegetation consumption values build on detailed animal food intake data but are also calibrated based on weight and price data. This is also true for the vegetation growth values. First, the ewe slaughter weight is assumed to dominate the lamb weight for all vegetation quantities, $w_X \geq w_Y = qg(V)$. With Q as the vegetation carrying capacity, we simply assume $w_X = qg(V = Q)$. For the specific functional form $g(V) = kV/(V + c)$, this yields

$$w_X = qkQ/(Q + c). \tag{a1}$$

Next, the lamb slaughter price is higher than the ewe slaughter price, $p_Y qg(V) > p_X w_X$. Following the equilibrium optimization model (main text, Section 5), the lowest possible vegetation quantity is $V^{msy} = Q/2$. Inserted into the vegetation consumption function $g(V)$, $p_Y qkQ/(Q + 2c)$ yields the lowest possible lamb price. Therefore, $p_Y qkQ/(Q + 2c) > p_X w_X$ should hold. Inserting (a1), we then find $c < Q(p_Y/p_X - 1)/(2 - p_Y/p_X)$ after a small rearrangement. A necessary and sufficient condition for a unique vegetation equilibrium is $f'(V^*) < g'(V^*)(1 + b)X$ (main text, Section 4). This condition is equivalent to $f'(0) > g'(0)(1 + b)X$, or $r > (k/c)(1 + b)X$. After a small rearrangement and also using the above slaughter price constraint, we have

$$(k/r)(1 + b)X < c < Q(p_Y/p_X - 1)/(2 - p_Y/p_X). \tag{a2}$$

Based on food intake data, the vegetation saturation parameter is fixed as $k = 0.50$ (tons of vegetation biomass/animal) while the intrinsic vegetation productivity parameter value (baseline) is assumed to be $r = 0.5$. When further scaling the farm size through the vegetation carrying capacity given as $Q = 500$ (tons), and inserting the slaughter price ratio and fertility (Table A1), condition (a2) reads $2.53X < c < 375$. As indicated above (Section 7), the highest possible stocking rate is $X = 108$ (ewes). Based on this inequality, the animal consumption shape baseline parameter value is simply scaled to

$c = 300$ (tons of vegetation biomass). Finally, using (a1) and inserting Q, k, c, and $w_X = 30$ (kg/animal) (Table A1), we find the value of the biomass translation parameter to be $q = 96$ (kg meat/ton vegetation biomass).

ENDNOTES

1. Vegetation consumption (or grazing pressure) is given by the number of animals in the *beginning* of the grazing season. An average over the season may better describe actual grazing pressure but comes at the cost of considerable notational clutter without altering the qualitative aspect of the model. Decisions on this and similar questions are an inherent problem of time-discrete models.

2. The problem of also allowing for physical capital accumulation and changing farm capacity is progressively more difficult to analyze because one has to account for irreversibility (see the pioneering work of Clark et al. [1979] in a fishery context).

3. This notion of overgrazing is obviously related to the specification of the natural growth function. For a skew-distributed logistic growth function with its peak value located to the left of $Q/2$, we thus find that overgrazing following this definition also may take place when $V > Q/2$.

4. Therefore, we find $\partial V^*/\partial p_Y < 0$, suggesting that $d^2\pi/dV^2 < 0$ holds when $d\pi/dV = 0$.

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