Malthus0905

GROWTH AND LAND-USE IN A SIMPLE AGRARIAN ECONOMY WITH ENDOGENEOUS POPULATION

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Abstract: We analyze the relation between demographic transformation and agricultural development in a simple agrarian economy, where population growth is treated both as a cause and consequence of economic change. Labor is allocated to food production and wood harvesting (land clearing) and, consistent with stylized facts about the demographic transition, population growth is a non-linear function of food consumption. The economy can get 'stuck' in a poverty trap, which is the typical Malthusian situation, but may also develop towards an outcome with high per capita incomes. Scale economies in food production, induced through scarcity or otherwise, play a crucial role in the analysis. We interpret these findings by revisiting the Malthus - Boserup debate.

Key words: Agricultural development, poverty trap, Malthus, Boserup, demographic transition

Acknowledgements: Thanks to Amit Bhaduri for comments on an earlier draft. The paper has also benefited from discussions at seminars in Santiago, York, Oslo, Trondheim and Stocholm. The work has been partly funded by The Norwegian Research Council through the Biologial Diversity Programme.

1. Introduction

Economists have for a long time studied the relationships between agricultural growth, demographic transformation and land-use. In spite of its standing as a classical issue, this continues to be a relevant topic for the well-being and development prospects of hundreds of millions of people in developing countries today. In recent times it has even regained momentum because of its linkages with the contemporary problem of habitat and wilderness conversion, natural resource harvesting and associated loss of ecosystem services. Studies of population growth in agricultural economies have to a large extent been inspired by the theory of Thomas Malthus (1798) and its counter thesis developed by Esther Boserup (1965). Our approach is no exception to this rule, and one aim of this effort is to show how the disparate predictions implied by these seminal contributions may be reconciled.

Malthus' line of argument is that living conditions of people are the main determinant of population growth. Emphasizing the failure of humans to adapt their reproduction behavior to a limited resource base, he argued that, in good times, high per capita incomes would generate rapid population growth through a positive effect on fertility and negative effect on mortality. However, increased population size, combined with decreasing marginal productivity of labor in production, will reduce per capita consumption. As living conditions deteriorate, the fertility rate declines and the mortality rate increases. Eventually the population settles down at a subsistence level of consumption – the economy is ‘trapped’ in an undesirable equilibrium.[1]

Boserup (1965) has effectively reversed the main argument of Malthus. Rather than treating the capacity to produce food as given and as the main determinant of population growth, she argued that population growth is independent of the living conditions of the people. In contrast, population density is seen as the main factor determining labor productivity in agriculture. This argument hinges on the assumption that higher population pressure induces a shift to more labor-intensive production techniques and, possibly, triggers innovations in landuse. For a more full discussion and interpretation of the population theories of Maltus and Boserup, and extensions in various directions, see, Birdsall (1988), Cuffaro (1997), Kelley (1988), Nerlove and Raut (1997) and Robinson and Srinivasan (1997).

In light of Western world experiences during the nineteenth and twentieth centuries it appears as if Malthus' predictions were too pessimistic. In particular Malthus underestimated man's ability to deal with diminishing returns in food production and adjust his reproductive behavior (Robinson and Srinivasan 1997, Birdsall 1988, Kelley 1988). However, this optimism is challenged when we consider Third World population problems today (e.g., Platteau 1996 and Cuffaro 1997 for studies of sub-Saharan Africa)[2]. Population growth rates in the Third World countries today are considerably higher than ever experienced by the Western world[3], which, when combined with falling economic growth rates in the 1980s and 1990s across Africa, suggests an outcome akin to Malthusian stagnation.

In this paper we formulate a simple model of an agrarian economy that is 'Malthusian' in spirit. We may think of this as a small village in sub-Saharan Africa consisting of a number of farmers, or rural households. The Malthusian spirit is evident from our key assumption that reproduction is governed by consumption (affecting both birth and mortality rates). We focus on the simplest case where property rights to resources are not secured (or not enforced), so that people have few, if any, incentives to base their natural resource utilization on long-term considerations. We study a model without trade or aid – production is balanced by consumption – but consider the impact of various forms of intervention by governments or NGOs. In addition to modelling the interlinkages between population growth and food production, we also introduce a mechanism that leads to land-use changes – agricultural land develops endogenously. The model therefore pulls together population growth, standards of living and utilization of the natural resource base which in the present context is wilderness land used for firewood. As a byproduct, land is cleared and used for agricultural production

To some extent the model is a mirror image of work by Brander and Taylor (1998),

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[1] Leibenstein (1954) and Nelson (1956) are examples of early modelling efforts of the Malthusian trap.

[2] Whether Boserup's theory gives rise to an optimistic view on the population-food chain is a matter of controversy. Some interpret Boserup as linking population-induced technological and institutional innovations with long-term increases in labor productivity, while others confine her theory to explain the positive relation between population growth and land yields, leaving labor productivity basically unaffected by population growth in the long run.

[3] While the yearly growth rate was 1.1 % at its highest in the Western world under the industrial revolution, the typical rates of today's developing countries are well above 2 % (e.g., 2.7 % for the Africa as a whole during the period 1960-1990). Many developing countries, in spite of their poor living conditions, experience lower mortality rates than historically occurred in western agrarian societies, which is partly due to new and better

who also develop an open access renewable resource model with endogenous population. A crucial difference compared to their model is that we assume a non-linear relation between consumption and fertility (the demographical transition effect) and a linear relation to describe replenishment of the resource stock (return of land to wilderness). Instead, Brander and Taylor assume a resource base that grows in a logistic manner, while population growth is linearly related to consumption per capita of the natural resource[4]. It will become evident that this extension affects steady states and dynamics in a non-trivial manner.

The main results of our analysis are as follows. We identify the Malthusian poverty trap as one possible outcome of the model, but also demonstrate that this trap may be avoided. The model suggests two different growth regimes: '*The Poverty Trap Regime*' and '*The High Income Regime.*' While the conditions leading to the former regime are clearly Malthusian, the conditions leading to latter regime may be associated with the reasoning of Boserup. More specific, the High Income Regime is related to non-decreasing returns to scale in food production, which, in the spirit of Boserup, can be interpreted as being due to population-induced innovations. The present model therefore aims to represent a synthesis of the theories of Malthus and Boserup. We proceed by demonstrating that the effect of common policy interventions may yield ambiguous effects on rural welfare and resource conservation, depending on the nature of the equilibrium that emerges.

The paper is organized as follows. In section two we outline the model, and in section 3 we show that the model's solution generally involves two steady states outcomes. In section 4 we analyze the Poverty Trap Regime in greater detail, and explore the adverse impacts of policy interventions aimed at improving living conditions and conserving the resource base. In section 5 we turn to the High Income Regime. Section 6 concludes.

2. The model

The model we develop consists of three interdependent modules: the allocation of labor between food production and natural resource harvesting, the change in agricultural land and resource harvesting over time, and growth of the human population. We assume

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medicines (Bairoch 1993).

[4] See also Pezzey and Anderies (2003) who extend the Brander-Taylor model by adding a resource subsistence requirement to the people's preferences. This generally destabilizes this model.

that households are myopic and solve a series of static optimization problems, but their choices, through their impact on land use and food consumption, drive the dynamic aspects of the model. Myopic behavior may be justified by insecure property rights to land and the natural resource base (e.g. Bulte and Horan 2003) and a potential slow institutional response to increasing scarcity (e.g. Brander and Taylor 1998, Copeland and Taylor 2003, but also see Pezzey and Anderies 2003 who analyze 'institutional departures' of open access extraction by exploring the consequences of implementing taxes and quotas).

Land and people are used in agricultural production to determine the current flow of food production C at the village level. Assume that this production is described as:

(1) $$C = qL^{\alpha} A^{\beta} ,$$

where L is the labour force allocation to food production, A is agricultural land currently under cultivation, and q is a productivity parameter. The size of the scale parameters α and β, with $0<\alpha<1$ and $0<\beta<1$, plays an important role in what follows.

With N as the number of people (households) in the village, the population constraint implies that $(N - L)$ units of labour are available for resource harvesting. Resource harvesting is supposed to be a flow of fuel wood, or construction wood. Through wood harvesting the forest vegetation is cleared and made available for crop production. Normalizing the total land base to one, non-cultivated, or wilderness, land $(1-A) \geq 0$ is used for fuel wood production. This amount is included as an input in the harvesting of fuel wood, and we specify such harvesting in a manner that is consistent with the well-known Schaefer production function in resource economics (see for example Clark 1990, Brander and Taylor 1998, Pezzey and Anderies 2003):

(2) $$F = \gamma(N - L)(1 - A) ,$$

where γ is a productivity parameter. This reflects that returns to labour are increasing in the resource stock, i.e., the amount of wilderness land, for example because distance to accessible wood increases as the wilderness area shrinks.

Next, consider the static optimization problem of households. We assume the representative individual, or household, derives utility from consumption of food, c, and resource harvest, f. Per capita consumption of food is simply defined as $c = C / N = qL^{\alpha} A^{\beta} / N$ while per capita consumption of fuel wood is defined as

$f = F / N = \gamma(N - L)(1 - A) / N$. Assuming Cobb-Douglas preferences for these two commodities, or:

(3) $\quad u = c^{v} f^{1-v}$

with $0 < v < 1$, utility maximization when land (and population) is kept fixed yields the following allocation of labour to agricultural production:

(4) $$L^* = \left(\frac{v\alpha}{v\alpha + (1-v)} \right) N = (1-s)N \,.$$

Wood production (and consumption) is readily solved for by substituting *N-L* = *sN* in (3):

(5) $$F^* = \gamma(N - L^*)(1-A) = \gamma sN(1-A).$$

Thus, static utility maximization of the Cobb-Douglas type implies that labour is allocated between food production and fuel wood harvesting (land clearing) according to the *fixed* share *s*[5]. Not surprisingly, the share of labour in food production is increasing in the preference parameter for food *v* as well as the production parameter of labour in agriculture α.

Next we turn to the dynamic implications of these choices. In our Malthusian model human population growth is governed by 'living conditions' as given by per capita food consumption. This relationship is specified in a logistic manner:

(6) $$\frac{dN}{dt} = N\{rc^*[1-(c^*/k)] - m\} = N\{rq(1-s)^{\alpha} N^{\alpha-1} A^{\beta} \left(1 - \frac{q(1-s)^{\alpha} N^{\alpha-1} A^{\beta}}{k} \right) - m\}$$

where *c** is the optimal per capita consumption that eventuates, for a given *A*, when substituting (4) in (1). The parameter *r* represents a (gross) intrinsic growth rate, *k* is the saturation level of growth and *m* > 0 is an autonomous mortality term.

The highly stylized and reduced form growth function in (6) is inspired by the remarkable work of Haavelmo (1954) who formulates various variants of Maltusian inspired growth models. It is specified to be in accordance with empirical evidence provided by Birdsall (1988) and Dasgupta (1995), and to represent the various phases of the demographic

[5] At the cost of considerable notational clutter one can derive qualitatively similar results when working with a more general CES utility function, where the elasticity of substitution σ between food and wood can differ from unity. When food and wood are net complements *(σ<1)* or substitutes *(σ>1)*, the allocation of labor over cropping and resource harvesting will be affected by relative scarcities. Specifically, when *σ<1 (σ>1)* holds, households will spend more (less) time harvesting fuel wood when that commodity becomes more scarce, relative to agricultural output. This implies that some of the effects discussed below will be accentuated or

transition. Accordingly, for 'low' levels of consumption per capita, population growth increases with increasing consumption per capita (the mortality rate falls while fertility is more or less constant). For 'high' levels of consumption population growth decreases with increasing consumption (the fertility rate falls while mortality is more or less constant). Our non-linear, humped-shaped function captures the negative relationship between per capita consumption and mortality for low consumption levels, and the negative relationship between fertility and consumption for high consumption levels.

Equation (6) implies there may be two equilibria with zero population growth, both taking place when per capita growth equals the autonomous mortality term *m*. One equilibrium occurs at a per capita consumption level below *k*/2, and the other occurs above *k*/2 (see Figure 1). We associate the former with the first phase of the demographic transition (i.e. high mortality and fertility rates), and the latter with the third phase of the demographic transition (low mortality and fertility rates). The interval where population growth is positive then corresponds with phase 2 of the transition[6].

<<Insert Figure 1 about here>>

Finally, the stock of agricultural land over time develops by the condition:

$$(7) \qquad \frac{dA}{dt} = \omega F^* - gA = \omega\gamma(sN)(1-A) - gA\,,$$

where multiplying (optimal) resource harvesting F^* and the parameter ω, reflecting a fixed fuel wood output per united land cleared, yields the gross increase in agricultural land. However, cultivated land regrows and returns to wilderness land, and *g* is a parameter that measures how fast this happens (akin to depreciation of man-made capital). We hence assume conversion of wilderness into arable land as a reversible process, and d*A/dt* >0 holds only as long as harvesting and its accompanying land clearing effort exceeds natural "decay". This equation also indicates that land clearing essentially is treated as a by-product of harvesting (for a discussion; see Arnold et al. 2003).

attenuated, depending on σ.

[6] This representation assumes an exogenous change in the parameters α and β over time – from $(\alpha + \beta) < 1$ initially to $(\alpha + \beta) > 1$ in later stages (see below). In this paper we don't model the cause of such technical innovations; we focus on analyzing the implications.

3. Dynamics and equilibria

Human population is constant when dN/dt=0. From (6) the following holds for $N > 0$:

(8) $$rc^*[1-(c^*/k)] = m, \text{ or:}$$

(8′) $$rq(1-s)^{\alpha} N^{\alpha-1} A^{\beta} \left(1 - \frac{q(1-s)^{\alpha} N^{\alpha-1} A^{\beta}}{k}\right) = m.$$

Typically, two configurations of agricultural land and population yield zero population growth, characterized by a low and a high per capita consumption. We denote $c^{*I}=(C/N)^{*I}$ and $c^{*II}=(C/N)^{*II}$, respectively, as the low and high consumption equilibria (see Figure 1). Equation (8′) may be solved for two N-isoclines, along which per capita consumption is constant. Both isoclines are positively sloped and run through the origin in the A-N plane. They are linear in the presence of constant returns to scale ($\alpha+\beta=1$), and concave (convex) functions of A when there are decreasing (increasing) returns to scale. Moreover, the slope in the origin is infinite (zero) in the case of decreasing (increasing) returns to scale (see Appendix).

In the Appendix we also demonstrate that the human population will shrink when above the isocline with the lowest land-labor ratio (or lowest per capita consumption), or when below the isocline with the highest land-labor ratio (highest per capita consumption). The human population will grow when between these two isoclines. Figure 2 illustrates the two isoclines together with the out-of-equilibrium dynamics for the case of decreasing return to scale ($\alpha+\beta<1$**)** in food production In contrast, Figure 3 illustrates the increasing returns to scale case (more details below).

Next we turn to the dynamics of agricultural land. Land-use is fixed when dA/dt = 0 so that, from (7), the following holds:

(9) $$N = \frac{gA}{s\omega\gamma(1-A)}.$$

This isocline is a convex function that starts in the origin. The slope is equal to $g/s\omega\gamma$ in the origin, and becomes infinitely steep as all land is made available for crop production and A approaches one. This effectively ensures an interior solution for the dynamic problem provided that the N-iscolines are concave, or sufficiently convex in the increasing return to scale case. Above the *A*-isocline, clearing effort exceeds decay and $dA/dt > 0$, while the reverse holds below the isocline. Firewood production is given by (5), and when combining with (9) we find wood production along the A-isocline as $F^* = (g/\omega)A$. As the A-isocline is a convex function, *per capita* wood production therefore declines when more land is made up for agriculture along $dA/dt = 0$.

Since both the *N*-isoclines and the *A*-isocline slope upward the system is symbiotic – more agricultural land implies a larger equilibrium human population, and more people imply more agricultural land in the long-term. The two *N*-isoclines imply there are two interior equilibria. A^{*I} and N^{*I} (defining $c^{*I}=(C/N)^{*I}$) occur along the *N*-isocline with the low land-labor ratio. Similarly, N^{*II} and A^{*II} (defining $c^{*II}=(C/N)^{*II}$) occur along the *N*-isocline with the high land-labor ratio.

The stability of the two equilibria is determined by the slopes of the isoclines. The crucial point is whether the *A*-isocline intersects the *N*-isoclines from above or below. Since the outcome hinges on the curvature of the isoclines, the economic significance of the equilibria is related to the specification of the production technology in food production. If $(\alpha+\beta) < 1$ holds and the *N*-isoclines are concave they will necessarily cut the *A* isocline from above. Conversely, for $(\alpha+\beta) > 1$ the *N*-isoclines will necessarily cut the *A*-isocline from below, assuming they are sufficiently convex[7].

The reasoning above about relative slopes of isoclines has important consequences for equilibrium stability. It follows that $(C/N)^{*I}$ will be approached in case of non-increasing returns to scale in food production; that is, if $(\alpha + \beta) \leq 1$ holds, the $(C/N)^{*I}$ is stable. We refer to this outcome as the Poverty Trap Regime. In contrast, A^{*II}, N^{*II} and $(C/N)^{*II}$ can only be approached when $(\alpha+\beta)>1$ holds. In what follows we refer to this outcome as the High Income Regime (see Appendix).

This implies it is not a matter of 'history or luck' – initial conditions – that drives the growth pattern and long-run outcome of this economy. The main determinant is technology. Decreasing returns to scale, as envisioned by Malthus, condemn the agrarian society to the poverty trap. However, if due to innovations (population-induced as argued by Boserup, or

[7] When the *N*-isoclines are not sufficiently convex to generate an interior solution (i.e. the *N* isoclines are below the *A*-isocline when *A* is equal to 1), it is readily verified that the system ends up in the corner where $A = N = 0$.

otherwise) there are increasing returns to scale, the economy will settle in a more prosperous steady state. Note that the economy cannot grow without bounds. Even with increasing returns to scale in both food production and the land-clearing activity (and in the absence of a binding land constraint) the economy will settle down to zero growth in the long term. This result is caused by the demographic transition – eventually population growth will fall which will choke economic growth, even if there is abundant land to expand on.

4. The Poverty Trap Regime

We first analyze the case with decreasing returns to scale in food production; $(\alpha+\beta) < 1$. This case is depicted in Figure 2. The low land-labor equilibrium ratio with A^{*I} and N^{*I} is locally stable, while the equilibrium A^{*II} and N^{*II} is unstable (again, see the Appendix where we demonstrate that point II is a saddle-point equilibrium)[8]. Under the present technology and scale conditions, the low-income trap A^{*I} and N^{*I}, and associated consumption level $c^{*I} = (C/N)^{*I}$, therefore represents the only interior long-term equilibrium[9].

<< *Insert Figure 2 about here* >>

Figure 2 provides a possible development trajectory when starting from a population level N_0 equipped with a small amount of agricultural land A_0. Initially, the economy displays high rates of mortality and fertility, but mortality dominates. Consequently, the population declines and Malthus' notion of 'positive checks' is in effect. Even though the population declines, however, it will clear additional land in its search for fuel and construction wood. The combination of a falling population and an increasing area of agricultural land implies that living conditions improve. Mortality decreases, and the population decline is reversed. The increasing population will now, because of decreasing returns to scale, counterbalance the gain from new investments in land and living conditions will eventually start to deteriorate. The economy expands with continued land conversion, a growing population and decreasing consumption per capita until it settles at the long-term

[8] In the absence of a planner it is virtually impossible to place the economy on the separatrix leading to the saddlepoint, arriving in $(C/N)^{II}$ when $(\alpha+\beta)<1$.

[9] When the economy starts below the high-income N-isocline, the human population falls until it goes extinct (a corner solution).

equilibrium (A^{*I},N^{*I}).

During the entire course of transition, the land-labor ratio in food production $A/(1\text{-}s)N$ increases. Because of the fixed labor allocation fraction s, the labor productivity $C/(1\text{-}s)N$ develops identically to consumption per capita. Recalling that the N-isoclines represent constant food per capita paths, it is therefore clear that labor productivity improves in the beginning. It will increase during the first part when the population grows, but eventually productivity growth ceases and starts to decline before it stabilizes at the long-term equilibrium. It can be shown that land productivity C/A decreases continually. This occurs because the constant land productivity locus defines N as convex functions of A when we have decreasing returns to scale. By inspection of the labor productivity identity $C/(1\text{-}s)N \equiv (C/A)[A/(1\text{-}s)N]$ (Hayami and Ruttan 1985), it is also clear that declining land productivity more than outweighs the positive productivity effect from more land per agricultural worker during the last part of the growth path.

4.1 Policy intervention and the poverty trap

Assume an economy is 'trapped' in the Malthusian steady state (A^{*I},N^{*I}). How does outside intervention by the government or an international NGO affect the economy's performance? We argue that such 'outsiders', depending on their type, may have two reasons to intervene: to promote rural welfare (reduce poverty) and/or to promote wilderness conservation. We therefore evaluate the consequences of intervention along the two following dimensions: (i) per capita utility u, and (ii) extant wilderness area (1-A). Motivated by the key role played by technology transfer in agricultural development programs, we highlight the effect of interventionist policies aimed at promoting productivity improvements in crop production. This may take place through Hicks neutral technical improvements (i.e. raising parameter q in equation 1), or through manipulation of scale parameters (i.e. raising α and/or β). Welfare and the natural resource base may also be changed by polices aiming to influence the demography.

The consequences of Hicks neutral technical change are straightforward. Raising parameter q shifts the N-isocline upwards and leaves the A-isocline unchanged. For given population and agricultural land, food production will increase. Thus, in the short term, per capita food consumption is increased, making people better off. The Malthusian dynamics as captured by our demographic transformation imply that the human population will grow and, thus, clear more land. However, due to decreasing returns to scale in food production, the expansion of food supply does not exactly match the increment in factors. This means that the economy expands while per capita food consumption gradually declines. In the long run the economy settles down at a new equilibrium where the initial (pre-intervention) consumption per capita level is reached. Per capita food consumption is unaffected by intervention in the long run.

In addition, since the expansion of agricultural land comes at the cost of wilderness, it is evident that Hicks neutral change does not represent a prospective scenario. The fall in wilderness area is not only bad because the intervening agency might care about wilderness conservation; it also implies per capita fuel wood production will fall – recall that consumption decreases wile moving upwards along the *A*-isocline. Since utility of rural households is a function of both food and wood, utility must decline (unchanged food consumption and lower wood consumption). Hicks' neutral technical change is therefore unambiguously bad when societies are caught in a poverty trap – it lowers utility and is detrimental for the conservation of wilderness.

If intervening agencies aim to affect long term per capita food consumption (as opposed to utility), they might instead attempt to change demographic parameters. The workings of such policies depend on whether the equilibrium human population grows larger or smaller as a result. For example, vaccination programmes aiming at reducing mortality rate m will increase steady-state population and agricultural acreage. The long run outcome will be a fall in per capita consumption of food and wood and, hence, reduced utility. This fits intuitive reasoning. Due to decreasing returns to scale, increases in factors will not be matched by an equal expansion of output. Fertility control will obviously work in the opposite direction, making people better off.

Finally, more can be said about technical change. As discussed by Hayami and Ruttan (1985) and Bairoch (1975), crop production technological change can be biased. Technical change may be occur through an increase in α ('mechanisation'), β ('biology'), or both parameters. A change in β shifts the *N*-isoclines, while a change in α shifts both the *N*-isoclines and the *A*-isocline (through the effect on the fixed labor allocation share s). A sufficiently large increment in these parameters so that $(\alpha+\beta)>1$ holds after intervention, implies a qualitative change in the equilibrium. The change in the N-isoclines, from concave to convex, implies that the poverty trap outcome is no longer stable. Instead, the system will now "jump" to the high-income equilibrium (A^{*II}, N^{*II}). This represents a trade-off between development and wilderness conservation – per capita income increases while agricultural area expands. Depending on the priorities of intervening agencies, such an outcome may be beneficial or not. We now discuss the high-income regime in more detail.

5. The High Income Regime

The equilibrium represented by A^{*II}, N^{*II} (defining $c^{*II}=(C/N)^{*II}$) can be approached when the 'Boserupian' condition of increasing returns to scale in food production holds; $(\alpha+\beta)>1$. If an additional condition related to the magnitude of the decay parameter g is satisfied, equilibrium point II will be locally stable and may be reached either by cyclical or monotonic convergence (see the Appendix). The other equilibrium A^{*I}, N^{*I} with $c^{*I} =(C/N)^{*I}$ is now a saddle point, and reaching it is a zero probability event.

<< *Insert Figure 3 about here* >>

In Figure 3 we show one possible development trajectory for an economy initially equipped with a small amount of agricultural land and a low land-labor ratio: A_0 and N_0. Initially the situation is just as under the poverty trap regime above; 'positive checks' are in effect and the population shrinks. However, the low land-labor ratio implies expansion of agricultural land (little depreciation), which, combined with fewer mouths to feed, implies that living conditions improve. Mortality decreases, and the population decline slows down and eventually is reversed. From this stage onwards, being in phase two of the demographic transition, the economy expands. Increasing returns to scale imply that continued land conversion, population growth and improvements in living conditions go hand in hand. When the High Income equilibrium (A^{*II}, N^{*II}) is reached the economy has gone through all the phases of the demographic transition.

As above, labor productivity in food production, $C/(1–s)N$, develops in accordance with food per capita and, consequently, increases steadily during the growth trajectory. From Figure 3 it is also evident that the land-labor ratio $A/(1–s)N$ increases as well. The fixed land productivity loci *C/A* are now concave functions running through the origin. Land productivity will therefore first decrease during the growth trajectory, but after the population has started to grow land productivity will increase as well. When inspecting the labor productivity identity, $C/(1-s)N \equiv (C/A)[A/(1-s)N]$, we now find that the increased land labor ratio more than outweighs reduced land productivity during the first part of the growth path.

5.1 Policy intervention and the high-income equilibrium

We again explore the consequences of policy intervention, focussing on the impact of promoting Hicks neutral technical change in food production (or increasing q). Raising q still shifts the N-isocline upwards and leaves the A-isocline unaffected, such that the intersection of N and A isoclines now occurs closer to the origin – both $A^{*\mathrm{II}}$ and $N^{*\mathrm{II}}$ decrease.

The reason is as follows: The short-term boost in consumption that follows from raising q triggers reduced fertility. As a result the population declines. This leads to a decline in harvesting effort, and a return of some agricultural land back to wilderness. The economy contracts until it settles down at a long-term equilibrium where the initial (pre-intervention) level of per capita food consumption is again reached. However, since the wilderness area has grown (an input in wood collection), it follows that per capita wood production and consumption are higher than before. Therefore welfare in the High Income Regime has increased as a result of intervention, even if long-term per capita food consumption is unaffected.

Similarly, vaccination programmes or other types of intervention that reduce the mortality rate m or affect other demography parameters, increasing the steady state human population, have a different effect than the above case where $(\alpha + \beta) \leq 1$. Increasing the steady state levels of N and A now means that per capita food consumption unambiguously increases. However, the expansion of agricultural land implies less wood production per capita. A trade-off between food and wood consumption is therefore present, so utility is ambiguously impacted by the intervention (depending on parameters).

6. Discussion and concluding remarks

We have developed a model where population growth depends on living conditions and where living conditions depend on population size. While Malthusian in spirit, the analysis extends the standard Malthusian theory along two dimensions. First, we allow land clearing as a by-product of resource harvesting. This means that labor productivity may increase as the human population grows, even if the typical Malthusian assumption of diminishing returns to labor in production is retained. Second, consistent with demographic evidence, we assume population growth is described by a humped function of per capita consumption. While Malthus was concerned with 'preventive checks' to keep the population from outgrowing food production, our model captures preventive checks by allowing net fertility to decrease with improved living conditions.

Our generalized Malthusian theory demonstrates that the poverty trap is but one of two possible equilibria. Society will typically become trapped when its production technologies display decreasing returns to scale. Increasing returns to scale in food production imply the poverty trap may be escaped. As population growth goes hand in hand with increased labor productivity, we associate the high-income regime with the reasoning of Boserup.

In the context of our model with myopic crop producers harvesting a renewable resource, we find that Hicks neutral technical change will *never* affect long-term food consumption. However, several effects are noteworthy. First, resource consumption will be affected in the long run and, hence, so is utility. Specifically, rural communities caught in a poverty trap will see their utility fall due to technical progress, while high-income economies will experience a gain. Second, there are short-term effects. Immediately following a Hicks neutral innovation living conditions will *always* improve. Third, the conservation effects are ambiguous, depending on the nature of the equilibrium prior to intervention. In the low-income regime we will have environmental degradation, while the environment will improve in the high-income regime[10].

Models only approximate reality and are as useful as the assumptions on which they are based. In the present model we disregard trade and assume myopic behavior. Even for

[10] Our short-term results bear some resemblance with the so-called Environmental Kuznets Curve (EKC). Temporarily, a Hicks neutral technical improvement will improve living conditions both for low and high-income communities. Thus, low-income communities will experience higher living standards combined with environmental degradation, and high-income communities will experience higher living standards combined with an improving environment. This fits well with the well-known features of the EKC. However, the analogy breaks down in the long run.

poor rural societies these assumptions may be questioned. The analysis is also carried out for specific functional forms of utility and production. For example, resource harvesting is specified as a Schaefer function. By assuming different technologies, as for instance the case where the extant wilderness area is not an input in wood collecting, indicating that returns to labor are independent of the resource stock, some other features of the model emerge. Two results are noteworthy. First, assuming interior solutions, Hicks neutral technical change no longer has an impact on utility because, unlike the 'general' case explored in the main text, we now find that neither food nor wood consumption per capita in equilibrium is affected by technical parameters. Second, omitting the resource stock as an argument in the production function implies that the costs of land clearing do not increase as the wilderness area is drawn down. Therefore it is possible that the system ends up in a corner where all land is converted to agricultural use. In this case, access to the High Income steady state may be blocked due to a binding land constraint. even when having increasing returns to scale in food production. It can be shown that the economy now will settle down in an outcome akin to the poverty trap, in spite of its superior technologies. One conclusion, therefore, is that land scarcity may condemn a society to poverty and low utility levels as much as inferior technologies.

How can rural society escape the poverty trap? The model suggests there are two options for policy makers to promote such an escape. First, as demonstrated, fertility control will increase per capita consumption of food and wood, and improve standards of living. Second, a "sufficiently large" transfer of technology (to be precise: from a condition where decreasing returns to scale in food production is replaced by increasing returns) will push the economy from the poverty trap regime to the high income regime. This may be achieved by raising scale parameters α ('mechanisation') and/or β ('biology').

Finally, the various trajectories discussed in the model may be viewed in light of empirical overviews given by Timmer (1988) and Hayami and Ruttan (1985)[11]. Timmer suggests that there have been various stylized 'transformation paths' for the agricultural

[11] The present model exercise can also be related to the conditions for agricultural and economic 'take off' as discussed by Bairoch (1975). Bairoch sees the agricultural sector as the key sector for economic and social development in most 'backward' Third World economies, and pinpoints that an essential condition for increased labor productivity in the agricultural sector is that the area cultivated per agricultural worker shifts up. As demonstrated above, however, the land-labor ratio increases along the development paths both in The High Income Regime and The Poverty Trap Regime. Thus, the model suggests that more land per labor is not a *sufficient* condition for improved labor productivity and better living conditions. The beneficial effects can be

sector. One characterizes newly opened countries in the last part of the 19th century with abundant surplus land (e.g. USA, Canada and Australia). These countries moved almost uniformly in the direction of higher labor productivity and lower land productivity. The growth in land productivity was quite slow until recently. Another path is sketched by developing countries in Asia, where high population growth and scarce land resources are common characteristics leading to rapidly increasing land productivity and slowly increasing (or falling – think of Bangladesh) labor productivity. Timmer also sees a particular African path over the last decades. Between 1965 and 1973, Africa's productivity performance in the agricultural sector was much like that of the newly opened countries in the last century; slow growth in land productivity and more rapid growth in labor productivity. However, after the early 1970s the productivity in both land and labor declined for sub-Saharan Africa, as did the land-labor ratio. Taken together this suggests that the development path under the high-income regime can be related to experiences of new countries opened up during the last century, and that the paths within the poverty trap regime share some similarities with developments in Africa and to some extent, Asia.

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Appendix

The N-isoclines

Solving equation (8'), we obtain the *N*-isoclines explicitly as

$$\text{(A1)}\qquad N = \left(\frac{k \pm \sqrt{k(k-4m/r)}}{2q(1-s)^{\alpha}}\right)^{\frac{1}{\alpha-1}} A^{\frac{\beta}{1-\alpha}} = \phi A^{\frac{\beta}{1-\alpha}}, \text{ where } \phi > 0 .$$

The isocline with the highest population for a given amount of agricultural land corresponds to the low consumption per capita equilibrium $c^{*I}=(C/N)^{*I}$ in Figure 1, while the other isocline corresponds to the high per capita consumption equilibrium $c^{*II}=(C/N)^{*II}$. The slope reads

$$\frac{dN}{dA} = \phi\frac{\beta}{1-\alpha}A^{\frac{\alpha+\beta-1}{1-\alpha}} = \frac{\beta}{1-\alpha}\frac{N}{A} > 0 .$$

The dynamics outside the isoclines are found by differentiating dN/dt. After some rearrangements this yields:

$$\text{(A2)}\qquad \left.\frac{d\dot{N}}{dN}\right|_{\dot{N}=0} = (\alpha-1)rq(1-s)^{\alpha}N^{(\alpha-1)}A^{\beta}\left(1-\frac{2q(1-s)^{\alpha}N^{(\alpha-1)}A^{\beta}}{k}\right).$$

Hence, we have:

$$\left.\frac{d\dot{N}}{dN}\right|_{\dot{N}=0} \begin{cases} <0 & \text{if} \quad N > \left(\dfrac{k}{2q(1-s)^{\alpha}}\right)^{\frac{1}{(\alpha-1)}} A^{\frac{\beta}{(1-\alpha)}} \\ =0 & \text{if} \quad N = \left(\dfrac{k}{2q(1-s)^{\alpha}}\right)^{\frac{1}{(\alpha-1)}} A^{\frac{\beta}{(1-\alpha)}} \\ >0 & \text{if} \quad N < \left(\dfrac{k}{2q(1-s)^{\alpha}}\right)^{\frac{1}{(\alpha-1)}} A^{\frac{\beta}{(1-\alpha)}} \end{cases}$$

$N = \left(\frac{k}{2q(1-s)^{\alpha}} \right)^{\frac{1}{(\alpha-1)}} A^{\frac{\beta}{(1-\alpha)}}$ is a line between the two isoclines. Above this line $\left.\frac{d\dot{N}}{dN}\right|_{\dot{N}=0}$ is negative, and N decreases (increases) when above (below) the upper isocline (with the lowest land-labor ratio). On the other hand, below this line $\left.\frac{d\dot{N}}{dN}\right|_{\dot{N}=0}$ is positive. N increases (decreases) when above (below) the isocline with the high land-labor ratio.

Stability conditions

The Jakobi-matrix of the reduced form dynamic system (6) and (7) is given as

$$J = \begin{bmatrix} \frac{\partial \dot{A}}{\partial A} & \frac{\partial \dot{A}}{\partial N} \\ \frac{\partial \dot{N}}{\partial A} & \frac{\partial \dot{N}}{\partial N} \end{bmatrix}.$$

Evaluated at the equilibrium we obtain after some few manipulations:

$$\frac{\partial \dot{A}}{\partial A} = -\omega\gamma sN - g = -\frac{g}{1-A},$$

$$\frac{\partial \dot{A}}{\partial N} = \omega\gamma s(1-A),$$

$$\frac{\partial \dot{N}}{\partial A} = \beta rq(1-s)^{\alpha} N^{\alpha} A^{\beta-1} \left(1 - \frac{2q(1-s)^{\alpha} N^{\alpha-1} A^{\beta}}{k} \right),$$

and

$$\frac{\partial \dot{N}}{\partial N} = (\alpha-1) rq(1-s)^{\alpha} N^{\alpha-1} A^{\beta} \left(1 - \frac{2q(1-s)^{\alpha} N^{\alpha-1} A^{\beta}}{k} \right).$$

The trace reads:

(A3) $$Trace(J) = (\alpha-1) rq(1-s)^{\alpha} N^{\alpha-1} A^{\beta} \left(1 - \frac{2q(1-s)^{\alpha} N^{\alpha-1} A^{\beta}}{k} \right) - \frac{g}{1-A}$$

$$= (\alpha-1) r \frac{C}{N} \left(1 - \frac{2C/N}{k} \right) - \frac{g}{1-A},$$

and the determinant is:

(A4) $$Det(J) = \frac{g}{(1-A)} rq(1-s)^{\alpha} N^{\alpha-1} A^{\beta} \left(1 - \frac{2q(1-s)^{\alpha} N^{\alpha-1} A^{\beta}}{k} \right) \left[1 - \alpha - (1-A)\beta \right]$$

$$= \frac{g}{(1-A)} r \frac{C}{N}\left(1-\frac{2C/N}{k}\right)\left(1-\alpha-(1-A)\beta\right).$$

At the equilibrium ($A^{*\mathrm{I}}$,$N^{*\mathrm{I}}$) we have $C/N < k/2$, i.e., $\left(1-\frac{2C/N}{k}\right) > 0$ while $\left(1-\frac{2C/N}{k}\right) < 0$ holds at ($A^{*\mathrm{II}}$,$N^{*\mathrm{II}}$). The stability properties are then as follows:

The low consumption per capita equilibrium (A^{*I}, N^{*I})

When $\left(1-\frac{2C/N}{k}\right) > 0$, $Det(J) > 0$ if $\left[1-\alpha-(1-A)\beta\right] > 0$, or $1 > (\alpha + (1-A)\beta)$, while $Trace(J) < 0$ if $\alpha < 1$. Both these conditions are met when $(\alpha + \beta) \leq 1$. The low consumption per capita equilibrium is therefore (locally) stable when having decreasing returns to scale (DRS) in food production. $Det(J)$ may also be positive with increasing return to scale (IRS) combined with a 'large' amount of land for crop production. However, due to the assumed Cobb-Douglas technology in food production, the A-isocline can never intersect with the low per capita N-isocline from below (the equivalent condition of $Det(J) > 0$) with IRS. With Cobb-Douglas technology, we have $Det(J) < 0$ and a 'small' amount of land made up for crop production, and the IRS equilibrium case will be a saddle point. The low consumption per capita equilibrium can hence only be approached with DRS in food production.

The high consumption per capita equilibrium (A^{*II}, N^{*II})

We now have $Det(J) > 0$ if $1 < (\alpha + (1-A)\beta)$, which never holds when there are DRS in food production. On the other hand, $Det(J) > 0$ holds with IRS combined with a 'small' fraction of land made up for food production. As the high per capita N-isocline always intersects with the A-isocline from below with IRS (now the equivalent condition of $Det(J) > 0$), this condition is always met. A sufficiently convex N-isocline that ensures an interior solution therefore implicitly yields $Det(J) > 0$ (see also the main text section three). We now find that $Trace(J) < 0$ if α is high combined with a large decay parameter. Under these conditions the high consumption per capita equilibrium is stable when having IRS in food production.

The locally stable equilibrium when non-DRS is approached by monotonic convergence if

$$Trace(J)^2 - 4Det(J) = \left((\alpha-1)r\frac{C}{N}\left(1-\frac{2C/N}{k}\right)-\frac{g}{1-A}\right)^2 - \frac{4g}{(1-A)}r\frac{C}{N}\left(1-\frac{2C/N}{k}\right)\left(1-\alpha-(1-A)\beta\right) > 0$$

and by cyclical convergence if

$$Trace(J)^2 - 4Det(J) = \left((\alpha-1)r\frac{C}{N}\left(1-\frac{2C/N}{k}\right)-\frac{g}{1-A}\right)^2 - \frac{4g}{(1-A)}r\frac{C}{N}\left(1-\frac{2C/N}{k}\right)\left(1-\alpha-(1-A)\beta\right) < 0$$

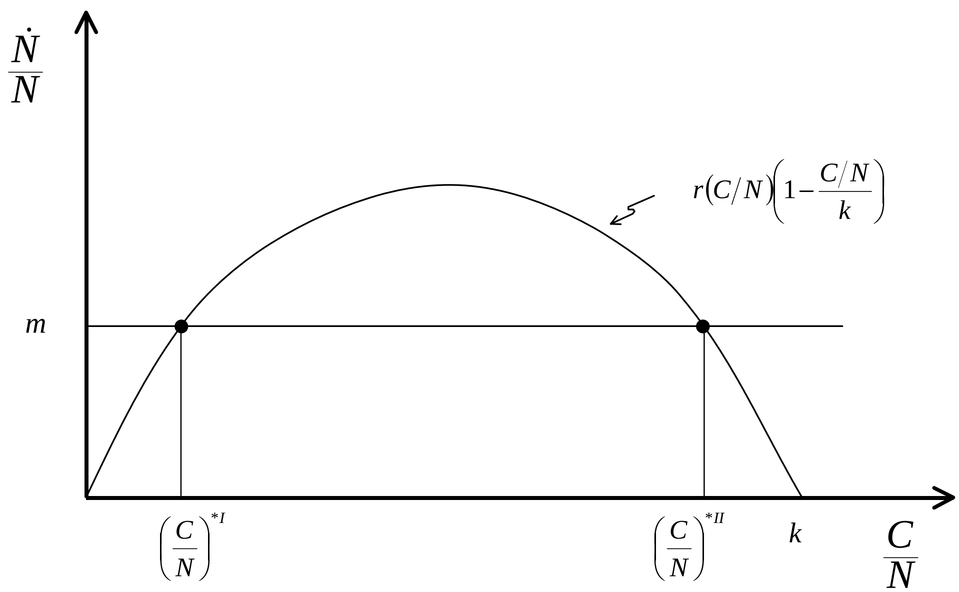

Figure 1: The population growth function. The low consumption per capita equilibrium $c^{*I} = \left(\frac{C}{N}\right)^{*I}$ and the high consumption per capita equilibrium $c^{*II} = \left(\frac{C}{N}\right)^{*II}$.

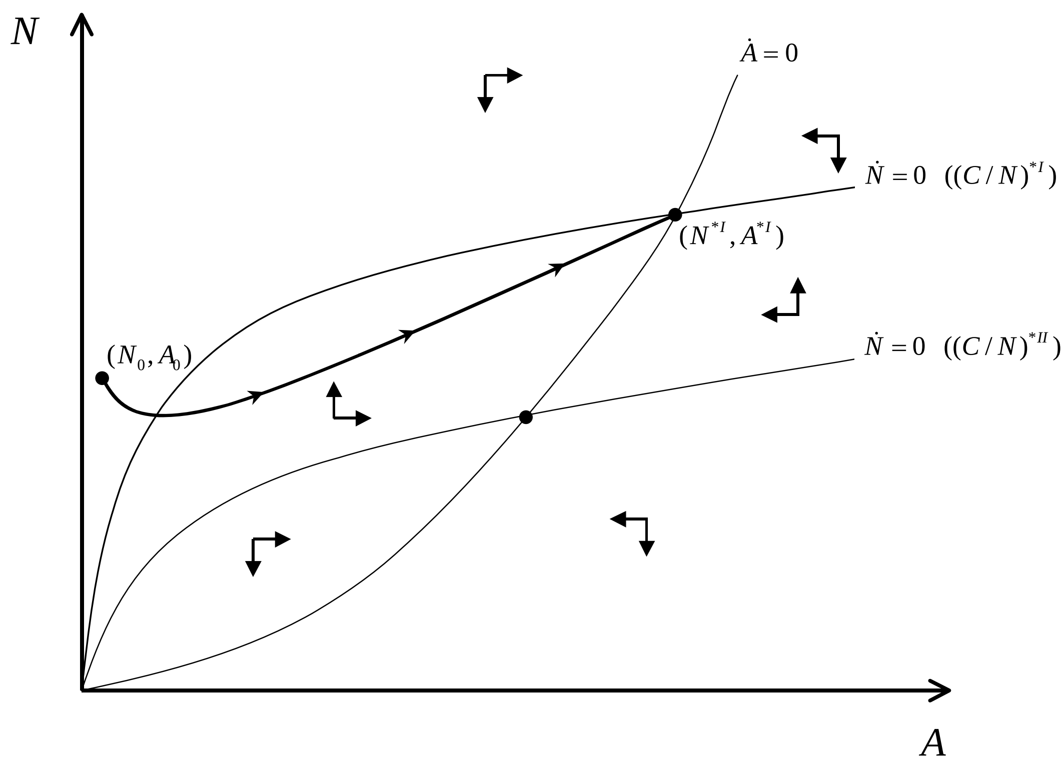

Figure 2: Dynamics and equilibrium under The Poverty Trap Regime

$(\alpha + \beta < 1)$.

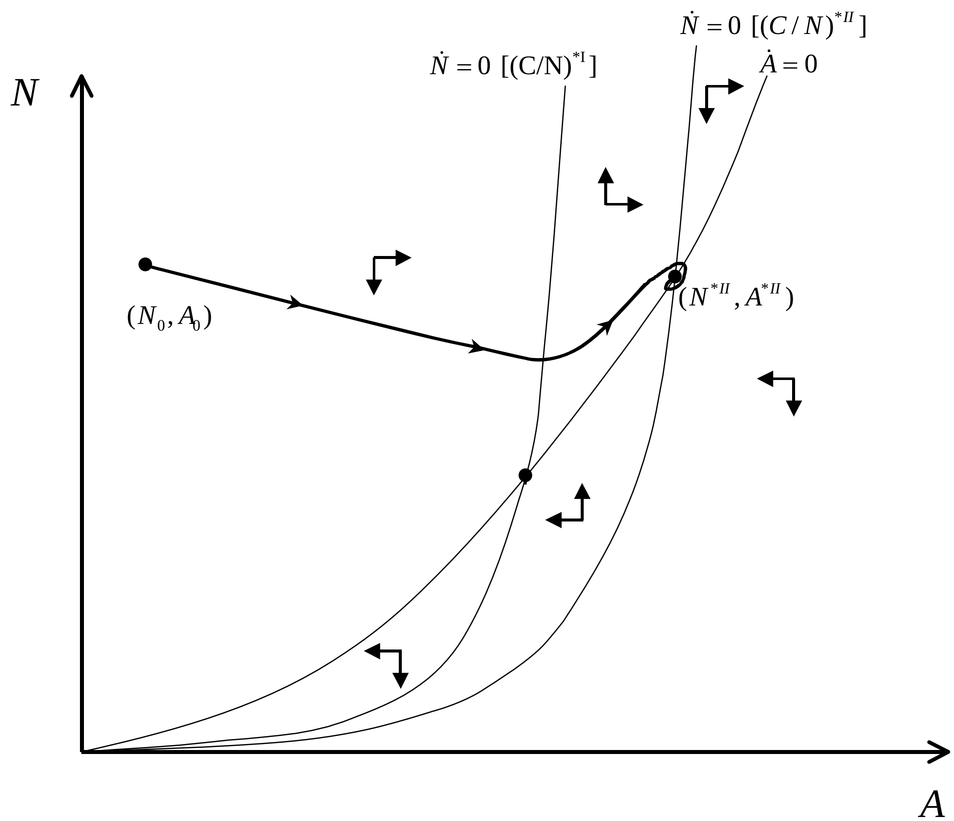

Figure 3: Dynamics and equilibrium under The High Income Regime

$(\alpha + \beta > 1)$.