Incentives for Optimal Management of Age-structured Fish Populations

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Abstract: We study optimal fishery management in an age-structured, bio-economic model where two age classes can be harvested independently. We show that the optimal amount of catch differs with age class, and we derive conditions under which it is optimal to harvest only one age class. The main policy implication is that optimal age-structured harvesting can be implemented by a single total allowable catch (TAC) and tradable harvesting quotas, where the latter are specified in terms of the number of fish harvested rather than in terms of biomass. In this case, gear restrictions (such as mesh-size prescriptions) turn out to be obsolete. We then apply our model to the Eastern Baltic cod fishery.

JEL-Classification: Q22, Q57, Q28

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1. Introduction

In all economic problems where market failure arises, economists typically ask two questions: first, what do efficient allocations look like, and secondly, how can these allocations be implemented through decentralized decision-making. This procedure also applies to problems in the optimal management of fish stocks. The workhorse model for answering these questions is the biomass model (also known as lumped-parameter or surplus-production model), which describes the dynamics of a fish stock in terms of its biomass [10, 23, 5]. This model has often been criticized for oversimplifying biological structures and thus for generating inadequate management recommendations.[1] The crucial weakness of the biomass model is that it is incapable of distinguishing between two aspects of overfishing: recruitment overfishing and growth overfishing. Recruitment overfishing refers to the problem of low reproduction, because the spawning stock has been fished down. Growth overfishing, by contrast, means that fish are caught in an inefficiently low age and weight class. In order to distinguish between these two forms of overfishing and hence formulate better management rules, it is necessary to look at the cohort, or age structure, of a given fish population.

In this paper we study how both problems, recruitment overfishing and growth overfishing, can be solved by means of incentive-based policy instruments. We first study optimal harvesting by asking which age classes should be harvested at all and what the optimal amounts of catch for each age class are. For this purpose we set up a simple dynamic cohort model with four age classes: eggs and larvae, juveniles, young fish at edible size but non-

[1] Tahvonen [25, 26] provides an overview of the criticism leveled at applying the biomass model in the economics of fisheries.

spawning age, and mature fish at spawning age. Only the young and mature fish are subject to a potential harvest. We consider selective fishing technology, which means that fishermen target the young and mature age groups independently. We use the concept of fishing technology in a broad sense, in that fishermen may choose different types of fishing gear as well as the time and location of harvest. Thus, fishermen's selective harvesting options are richer than the regulator's options for imposing selective harvesting by means of command-and-control.

Based on the results on optimal harvesting we ask how optimal management can be implemented by means of incentive-based policy instruments such as fees or tradable quotas.[2] We show that fixing the total allowable catch (TAC) and issuing tradable quotas , measured in terms of *biomass*, an instrument that is currently used in many fisheries[3], is bound to fail as a solution for the simultaneous problems of growth and recruitment overfishing. This may be one reason why, in most fisheries, tradable quotas are complemented by gear restrictions (such as minimum mesh-size) or minimum landing-size. In this paper we focus on the design of economic instruments that can implement first-best, age-structured harvesting more easily. We show that a single TAC and tradable quotas measured in terms of the *number* of fish rather than in terms of biomass, implements the first-best harvesting rule if natural survival rates of the different age classes subject to harvesting are identical. If natural survival rates differ with age, the instrument has to

[2] In accordance with our focus, we study how a system of tradable quotas could prevent growth overfishing. Since we do not study the effects of long-term use rights in fisheries, we refrain from using the term "individual transferable quotas (ITQs)" in this context.

[3] Individual quota systems in terms of biomass are used, for example in Iceland, New Zealand, Greenland and several member states of the European Union.

be modified slightly: the quotas for different age classes have to be traded at a fixed "exchange rate" that depends on the ratio of survival rates, and the TAC has to be adjusted accordingly. Furthermore, we show how a related price-based instrument in terms of harvesting fees can also implement the first-best harvesting rule. With the economic instruments we propose, additional regulations in terms of gear restrictions (such as mesh-size prescriptions) or minimum landing-sizes are obsolete. Finally, to quantify both the total allowable catch and the quota price for a real fish population, we apply our model and analysis to the case of the Eastern Baltic cod fishery.

The number of previous bio-economic studies of age-structured fisheries is still rather small, although such models have been developed and analyzed since the 1970's [11, 20, 9, 5]. Recently, Tahvonen [24, 25, 26, 27] has significantly advanced the analysis of optimally harvesting age-structured fish stocks. In particular, he studies the effects of different types of gear selectivity. One type is "knife-edge" selectivity [1]. This means that all age classes above a certain age are subject to fishing mortality, while all younger and smaller fish completely escape. The other type is non-selective fishing gear, which implies that all age classes are harvested in fixed (but not necessarily equal) proportions. With non-selective gear, the optimal harvesting strategy may be "pulse-fishing", where all fish are harvested at certain points in time with no fishing in between [11, 25]. The present paper differs from the previous studies by considering a perfectly selective fishing technology (in the broad sense discussed above) and by also suggesting new incentive-based policy instruments to decentralize optimal harvesting rules of age structured fish populations.

The paper is organized as follows: In the next section we present the bio-economic model comprising four age classes and an age-selective fishing technology. In Section 3 we derive general results on the structure of optimal age-dependent harvesting rules and in Section 4 we show how these can be implemented by means of economic instruments. We apply the model to the case of the Eastern Baltic cod fishery in Section 5. The final section concludes.

2. Analytical bio-economic model of an age-structured population

2.1. Population model

In this section we set up a simple model of an age-structured fishery that is sufficiently rich to analyze harvesting of different age classes. The fish population at time (year) t is divided into four stages: eggs and larvae $X_{E,t}$ (age < 1), juveniles $X_{J,t}$ ($1 \leq$ age < 2), young immature fish $X_{I,t}$ ($2 \leq$ age < 3), and mature fish $X_{M,t}$ (age ≥ 3). All stocks, $X_{j,t}$, $j \in \{E, J, I, M\}$, are measured in numbers of fish. Both eggs and larvae (age class E) and juveniles (age class J) are assumed to be too small to be harvested. In principle, these two age classes could be lumped together in one class, but we keep them separate to avoid time lags of different lengths. Age class I consists of immature, non-spawning fish that are sufficiently large to be of commercial value. Age class M consists of all mature fish that are three years and older. This age class is the spawning stock.

In a single time period (a year) four events occur in the following order: In the first step mature fish spawn, and in the second step fishermen harvest. In the third step, natural mortality further reduces the stocks of all classes, and finally somatic growth of individual fish takes place.[4]

[4]Generally, these events may occur differently throughout the year, or simultaneously during parts of the year [1]. However, the assumption of a particular order substantially simplifies the analysis. None of our central findings depend on this assumption.

To describe the population dynamics, we start with recruitment (the first event during a year). The stock of age-class 0 (eggs and larvae) in year $t+1$ depends on the size of year t's spawning stock and is governed by a non-linear recruitment function $r(X_{M,t})$, with $r(0) = 0$ and $r'(X_{M,t}) > 0$ for at least some range of $X_{M,t}$. An example of such a non-linear recruitment function is the Ricker function that we use in the case study (Section 5).

In the second step, only age classes I and M are subject to fishing mortality. Denoting harvest quantities of age classes I and M by $H_{I,t}$ and $H_{M,t}$, we obtain the escapement stocks $S_{I,t} = X_{I,t} - H_{I,t}$ and $S_{M,t} = X_{M,t} - H_{M,t}$, i.e. the numbers of fish that escape from harvesting. In the third step, all age classes are subject to natural mortality, which is assumed to be fixed and independent of the density of the fish population. The coefficients $b_{i,j}$ are the survival rates from age class i to age class j.

The equations of motion describing the dynamics of the age-structured fish population subject to harvesting are[5]

$$
\begin{aligned}
X_{E,t+1} &= r(X_{M,t}) \\
X_{J,t+1} &= b_{E,J}\, X_{E,t} \\
X_{I,t+1} &= b_{JI}\, X_{J,t} \\
X_{M,t+1} &= b_{IM}\, S_{I,t} + b_{MM}\, S_{M,t}
\end{aligned}
\tag{1}
$$

The equilibrium properties (for fixed harvesting rules) of models with a similar structure (density-dependent recruitment and density-independent natural mortality) and the conditions for maximum sustainable yield harvesting have been analyzed by [20]. Here we focus on economically optimal harvesting and dynamics, and we investigate how first-best harvesting rules can be

[5] Alternatively, the population dynamics can be described by a Leslie Matrix with non-constant entries [9].

implemented by economic management instruments. For this purpose, we now turn to the economic part of the bio-economic model.

2.2. Harvesting and profit

As mentioned above, we assume that fishermen can perfectly select the age class they are targeting. Fishermen have several options to obtain a high degree of selectivity. For some species, selecting age classes is possible by choosing fishing grounds, as different cohorts can be found in different regions. Also, fishermen can choose fishing gear. Some passive gear types, such as traps, allow the targeting of specific size classes with a comparatively high degree of precision.

Annual profits are determined by the revenues from harvesting both age classes and by the harvesting cost of either age class. Using p_j to denote the price per kilogram and w_j to denote the weight of an individual fish of age $j = I, M$, the revenues are given by $p_I\, w_I\, [X_{I,t} - S_{I,t}] + p_M\, w_M\, [X_{M,t} - S_{M,t}]$. In Appendix A.1 we show that, with an instantaneous harvesting function of a generalized Gordon-Schaefer type, annual harvesting cost of age class j is given by $\frac{c_j}{1-\beta}\left[X_{j,t}^{1-\beta} - S_{j,t}^{1-\beta}\right]$. Here, the parameter $\beta \in [0, 1]$ is the stock elasticity of harvest (sometimes also called the "schooling parameter").[6] The unit cost parameters $c_j = \nu_j/\eta_j$ are increasing with the unit effort costs ν_j and decreasing with the catchability coefficients η_j.

In order to determine optimal harvesting of the age-structured fish stock, we consider a central planner who determines the escapements of both age classes optimally. The planner's objective is to maximize the present value of annual profits discounted at a constant factor $\rho \in (0, 1)$. The intertemporal

[6]The lower limit $\beta = 0$ describes a fish stock with strong schooling behavior, while the upper limit $\beta = 1$ describes a highly dispersed fish stock [12].

objective function is thus given by[7]

$$V = \sum_{t=0}^{\infty} \rho^t \Bigg[p_I \, w_I \, [X_{I,t} - S_{I,t}] + p_M \, w_M \, [X_{M,t} - S_{M,t}] - \frac{c_I}{1-\beta} \left[X_{I,t}^{1-\beta} - S_{I,t}^{1-\beta} \right] - \frac{c_M}{1-\beta} \left[X_{M,t}^{1-\beta} - S_{M,t}^{1-\beta} \right] \Bigg] \quad (2)$$

The central planner maximizes (2) subject to the population dynamics (1), together with the given initial number of fish in all four age classes $X_{j,0}$, $j \in \{E, J, I, M\}$, and the constraints that escapement must be positive (to exclude depletion of the stock) but no larger than the current stock $0 < S_{j,t} \leq X_{j,t}$, $j \in \{I, M\}$.

2.3. Conditions for optimal economic management

The necessary conditions for the optimal harvesting of the age-structured fish stock are obtained by applying the Lagrangian method together with the appropriate Kuhn-Tucker conditions. With $\lambda_{j,t} > 0$ ($j \in \{E, J, I, M\}$ and $t = 0, \ldots, \infty$) as the Kuhn-Tucker multipliers of the population growth equations (1) and $\mu_{j,t} \geq 0$, $j = I, M$ as the Kuhn-Tucker multipliers of the escapement constraints, $0 < S_{j,t} \leq X_{j,t}$, the Lagrangian function is given by

$$\begin{aligned} L = \sum_{t=0}^{\infty} \rho^t \Bigg\{ & p_I \, w_I \, [X_{I,t} - S_{I,t}] + p_M \, w_M \, [X_{M,t} - S_{M,t}] \\ & - \frac{c_I}{1-\beta} \left[X_{I,t}^{1-\beta} - S_{I,t}^{1-\beta} \right] - \frac{c_M}{1-\beta} \left[X_{M,t}^{1-\beta} - S_{M,t}^{1-\beta} \right] \\ & + \lambda_{E,t} \, [r(X_{M,t}) - X_{E,t+1}] + \lambda_{J,t} \, [b_{EJ} \, X_{E,t} - X_{J,t+1}] \\ & + \lambda_{I,t} \, [b_{JI} \, X_{J,t} - X_{I,t+1}] + \lambda_{M,t} \, [b_{IM} \, S_{I,t} + b_{MM} \, S_{M,t} - X_{M,t+1}] \\ & + \mu_{I,t} \, [X_{I,t} - S_{I,t}] + \mu_{M,t} \, [X_{M,t} - S_{M,t}] \Bigg\} \end{aligned} \quad (3)$$

[7]It can be shown that our general results on fishery management also hold for an objective function that is nonlinear in profits.

The first-order necessary conditions for optimal harvesting are

$$\rho^t \frac{\partial L}{\partial S_{I,t}} = 0 \qquad p_I\, w_I - c_I\, S_{I,t}^{-\beta} + \mu_{I,t} = b_{IM}\, \lambda_{M,t} \tag{4}$$

$$\mu_{I,t}\, [X_{I,t} - S_{I,t}] = 0$$

$$\rho^t \frac{\partial L}{\partial S_{M,t}} = 0 \qquad p_M\, w_M - c_M\, S_{M,t}^{-\beta} + \mu_{M,t} = b_{MM}\, \lambda_{M,t} \tag{5}$$

$$\mu_{M,t}\, [X_{M,t} - S_{M,t}] = 0$$

$$\rho^t \frac{\partial L}{\partial X_{E,t}} = 0 \qquad \rho\, b_{EJ}\, \lambda_{J,t} = \lambda_{E,t-1} \tag{6}$$

$$\rho^t \frac{\partial L}{\partial X_{J,t}} = 0 \qquad \rho\, b_{JI}\, \lambda_{I,t} = \lambda_{J,t-1} \tag{7}$$

$$\rho^t \frac{\partial L}{\partial X_{I,t}} = 0 \qquad \rho\, \left[p_I\, w_I - c_I\, X_{I,t}^{-\beta} + \mu_{I,t}\right] = \lambda_{I,t-1} \tag{8}$$

$$\rho^t \frac{\partial L}{\partial X_{M,t}} = 0 \quad \rho\, \left[p_M\, w_M - c_M\, X_{M,t}^{-\beta} + \mu_{M,t} + \lambda_{E,t}\, r'(X_{M,t})\right] = \lambda_{M,t-1} \tag{9}$$

Analysis of these conditions leads to a number of clear-cut results about optimal harvesting and management of the age-structured fish stock, to be discussed in the following two sections. A first set of results (presented in Section 3) characterizes optimal harvesting of the age-structured fish population under different conditions for biological parameters, costs and prices. The second set of results (presented in Section 4) shows how optimal management of the age-structured fish population can be decentralized by using suitable economic instruments.

3. Optimal harvesting structure

In our first result on optimal harvesting we show that under reasonable assumptions about market prices, harvesting costs, and survival rates, optimal escapement of immature (age class I) is larger than optimal escapement of mature fish (age class M). For both age classes, optimal escapement is governed by the trade-off between current benefit of immediate harvesting and future benefits in terms of next period's harvest and increased recruitment. The trade-off is different for the two age classes mainly because current benefits differ. Mature fish are usually much larger than young fish, and they often have a higher market price per kilogram. Thus, revenues are greater for mature than for immature fish. If unit harvesting costs are similar for both age classes, this implies that the current benefit of immediate harvesting is higher for mature than for immature fish. Future benefits are the same for both age classes, as immature fish become mature and thus contribute to the next period's spawning stock, just as the surviving mature fish do. The weight of future benefits in the trade-off may be different for both age classes as natural survival rates may differ with age. Since natural survival rates of immature and mature fish are typically of similar magnitude, it is reasonable to conclude that there is a higher benefit from escapement of immature fish than of mature fish. Formally, we can state our first result as follows.

Proposition 1. *If $p_M\, w_M/b_{MM} > p_I\, w_I/b_{IM}$ and $c_M/b_{MM} \leq c_I/b_{IM}$, then optimal escapement of immature fish is larger than optimal escapement of mature fish if the stock of immature fish is large enough to allow harvesting, $S_{I,t} < X_{I,t}$.*

Proof. If we divide condition (4) by b_{IM} and condition (5) by b_{MM}, the right-hand sides of both conditions are identical. If $p_M\, w_M/b_{MM} > p_I\, w_I/b_{IM}$, the first term on the left-hand side (LHS) of (4) is smaller than the first term on the LHS of (5). For $\mu_{I,t} > 0$, and with $\mu_{M.t} \geq 0$, we conclude from (4) and (5) that $(c_I/b_{IM})\, S_{I,t}^{-\beta} < (c_M/b_{MM})\, S_{M,t}^{-\beta}$. Hence $S_{I,t} > S_{M,t}$, given that $c_M/b_{MM} \leq c_I/b_{IM}$. □

The condition $p_M\, w_M/b_{MM} > p_I\, w_I/b_{IM}$ has a straightforward economic interpretation: $p_j\, w_j/(b_{jM}\, \lambda_{M,t})$, $j = I, M$, is the ratio of benefit from immediate harvest to opportunity cost in terms of future harvests at time t. The condition states that this ratio is always larger for the older age class than for the younger one. The condition $c_M/b_{MM} \leq c_I/b_{IM}$ implies a corresponding relationship for the harvesting costs. As discussed above, the conditions of the proposition are likely to hold for many fisheries.

Since all fish of three years and older are grouped in the stock of mature fish, the number of mature fish may well be larger than the number of immature fish in the same year. If this is the case, Proposition 1 implies that the optimal number of immature fish harvested must be smaller than that of mature fish.

Our next result on optimal harvesting states that optimal harvest (and therefore the optimal TAC) of immature fish is *zero* if harvesting costs are independent of stock sizes, i.e. if $\beta = 0$, and if the conditions for prices, cost parameter, and survival rates apply as discussed before. This is because for stock-independent harvesting cost, the trade-off between current and future benefits is independent of the level of escapement. It is then always better to let all of the immature fish grow, become mature and spawn, and to harvest mature fish only. This is formally stated in the following proposition.

Proposition 2. *If harvesting costs are independent of stock sizes, i.e.* $\beta = 0$*, and if* $p_M\, w_M/b_{MM} > p_I\, w_I/b_{IM}$ *and* $c_M/b_{MM} \leq c_I/b_{IM}$*, then the optimal harvest of immature fish is zero; that is,* $S_{i,t} \equiv X_{I,t}$ *for all* t*.*

Proof. For $\beta = 0$, the result $\mu_{I,t} > 0$ and zero harvest of immature fish follow from conditions (4) and (5) under the assumptions $p_M\, w_M/b_{MM} > p_I\, w_I/b_{IM}$ and $c_M/b_{MM} \leq c_I/b_{IM}$, together with non-negative Kuhn-Tucker multipliers of the escapement constraints. □

For schooling fish such as the North Sea herring, the harvesting cost to be approximately independent of stock size [2, 12]. For such fisheries, complete escapement, i.e. zero harvest, of immature fish is optimal under the conditions discussed above. For many fisheries, however, it is more plausible that harvesting costs decrease with stock sizes. Therefore, our next result on optimal harvesting provides a condition, under which zero harvest of immature fish is optimal when harvesting cost decrease with the stock size. For analytical reasons, we concentrate on a steady state. A sufficient condition for zero harvest of immature fish is that the revenue of harvesting one immature fish in the current period is smaller than the present value of harvesting one mature fish one period later, and that unit harvesting costs of mature fish do not exceed those of immature fish, i.e. if $c_M \leq c_I$. Note that in a steady state all stocks and all current-value shadow prices are constant, i.e. $X_{j,t+1} = X_{j,t}$, $S_{j,t+1} = S_{j,t}$, and $\lambda_{j,t+1} = \lambda_{j,t}$ for $j \in \{E, J, I, M\}$. Using these conditions in the population dynamics (1) and the optimal control conditions (4–9), we obtain the following sufficient (but not necessary) condition for complete escapement of young immature fish in the optimal steady state.

Proposition 3. *If* $c_M \leq c_I$ *and*

$$\rho \, b_{IM} \, p_M \, w_M \geq p_I \, w_I \tag{10}$$

hold, then the optimal harvest of immature fish is zero in the steady state; that is, $S_I \equiv X_I$ *in the steady state.*

Proof. See Appendix A.2. □

As discussed above, the value of a mature fish harvested is usually larger than the value of an immature fish, that is, $p_M \, w_M > p_I \, w_I$. Inequality (10), however, provides a condition on the *present values* of mature and immature fish, which is obtained by discounting the value of mature fish by both the money-value discount factor ρ and the rate of survival b_{IM} from age class I to M, which may be interpreted as a biological discount factor. If the money-value discount factor is not too low, condition (10) is met for many fisheries. For the case of Eastern Baltic cod, for example, this holds for discount factors $\rho > 0.57$ (or discount *rates* lower than 76%, see Section 5).

One general conclusion from these results is that optimal harvest quantities of immature and mature fish differ. Thus to implement the optimal harvesting policy through quota setting (TAC), different TACs would have to be used for the two age classes. A second general conclusion is that corner solutions with a zero TAC for immature fish may well be optimal, depending on biological parameters (survival rates and body-weight growth rates), but also on prices and cost parameters. In theory, a zero TAC for immature fish could be implemented by a gear regulation that excludes the harvesting of two-year-old fish. Practically, however, such a policy is not easy to implement because it would require the regulator to prescribe a certain knife-edge selectivity of fishing gear. However, most specific types of fishing gear are imperfect at selecting for age.

4. Decentralization through incentive-based policy instruments

In this section we study how optimal harvesting structures as characterized above can be decentralized by implementing incentive-based policy instruments, such as harvesting fees and tradable harvesting quotas.[8] Start-

[8] In the literature on fisheries, price-based instruments are frequently referred to as "landing fees". We prefer to use the term "harvesting fees", as the source of market failure is not the landing but the harvesting. Since harvested fish may be discarded rather than landed, a landing fee may not be the appropriate instrument for implementing optimal

ing with harvesting fees, it is intuitive to guess that two harvesting fees are necessary to achieve the first best harvesting structure: one for the number of immature and another one for the number of mature fish harvested. These fees capture the marginal opportunity costs of harvesting in terms of foregone future benefits of the stock of mature fish, i.e. next period's harvest and increased recruitment. In general, two different harvesting fees are needed, as the age-specific survival rates may differ, and thus the rates at which escapement of immature and mature fish contribute to the next period's spawning stock. Harvesting fees exhibit two features rendering them the more attractive instrument compared with a set of different age-specific TACs. First, since the fees differ only by the constant survival rates b_{IM} and b_{MM}, the ratio of the fees is always constant even on the transitional path into a steady state. Second, no explicit distinction between an "interior" solution with positive harvesting quantities and a "corner" solution with zero harvest (complete escapement) of immature fish is necessary. The reason is that in the latter case the optimal harvesting fee for immature fish exceeds the marginal profit of harvesting the first fish. In the formal proposition we use $\phi_{j,t}$ to denote the harvesting fee for age class j in year t.

Proposition 4. *Optimal harvest of both age classes can be decentralized by setting two harvesting fees on the number of immature and mature fish, given by* $\phi_{I,t} = b_{IM}\,\lambda_{M,t}$ *and* $\phi_{M,t} = b_{MM}\,\lambda_{M,t}$.

Proof. For $\mu_{I,t} = \mu_{M,t} = 0$ and harvest in both stages, this follows immediately from conditions (4) and (5). (The spawning stock's shadowprice $\lambda_{M,t}$ is determined by the conditions (6)–(9) for optimal harvesting.) For $\mu_{j,t} > 0$, $j = I, M$, we have $p_j\,w_j - c_j\,X_{j,t}^{-\beta} > \phi_{j,t}$. Hence, $S_{j,t} = X_{j,t}$ and no fishing is

management.

optimal for individual fishermen, as is the socially optimal solution. □

Decentralizing the optimal harvest structure by fees is even simpler if the survival rates of immature and mature fish are identical. In this case, a single fee for all age classes is sufficient to decentralize the social optimum. The assumption of equal survival rates for the different age classes is appropriate for several fisheries, including the Eastern Baltic cod fishery studied in Section 5. Formally, this result is a corollary to Proposition 4.

Corollary 1. *If* $b_{IM} = b_{MM}$*, a single fee* $\phi_t = b_{IM}\,\lambda_{M,t} = b_{MM}\,\lambda_{M,t}$ *for the number of harvested fish decentralizes the optimal harvest of both age classes.*

It is important to note that the harvesting fee is related to the *number* of fish harvested, not to the weight, or the biomass, of catch. Let us consider the case of identical survival rates to illustrate the important difference between the instrument proposed here and a traditional harvesting fee based on weight resulting from the biomass model. In particular, we want to demonstrate that an unmodified "biomass" fee may generate inadequate incentives for fishermen. To illustrate this, we use $\tilde{\phi}_t^{\text{b-m}}$ to denote a fee per kilogram harvested. Converting this into a fee per fish of age class j we obtain $w_j\,\tilde{\phi}_t^{\text{b-m}}$, as a fish of age j has a weight of w_j kilograms. Thus, the fee per individual immature fish derived from a uniform "biomass" fee is much smaller than the corresponding fee per individual mature fish, i.e. $w_I\,\tilde{\phi}_t^{\text{b-m}} \ll w_M\tilde{\phi}_t^{\text{b-m}}$, as normally $w_I \ll w_M$. This implies that a uniform biomass fee induces a considerable distortion towards over-harvesting of immature fish. Put differently, the optimal fee per kilogram of immature fish would have to be higher than the optimal fee per kilogram of mature fish. It is easy to see that for the case of different survival rates, a "biomass" harvesting fee will also induce a distortion towards over-harvesting of immature fish.

Next, we study tradable harvesting quotas as the corresponding quantity-based economic instrument. Similar to management by fees, the important difference to traditional management systems is that both the total allowable catch and the quotas are measured in numbers of fish rather than in units of biomass. If the survival rates of immature and mature fish are identical, optimal harvesting can be implemented by means of a single TAC and a system of individual tradable harvesting quotas that can be traded on a one-to-one basis. Note that the regulator does not need to prescribe the allocation of quotas among the different age classes, as this is done by the quota market. This result is formally stated in the following proposition.

Proposition 5. *If $b_{IM} = b_{MM}$, then the optimal harvest of both age classes is decentralized by setting a total allowable catch of size $X_{I,t} - S_{I,t} + X_{M,t} - S_{M,t}$ on the overall number of fish harvested and implementing it by means of tradable harvesting quotas in numbers.*

Proof. See Appendix A.3. □

If, by contrast, the survival rates of immature and mature fish differ, the instrument has to be slightly modified. Harvesting quotas must then not be traded on a one-to-one basis among age classes, but on the basis of a constant "exchange rate" determined by the ratio of survival rates, as the following corollary to Proposition 5 states.

Corollary 2. *If $b_{IM} \neq b_{MM}$, then the optimal harvest of both age classes can be decentralized by i) setting a TAC of size $(b_{IM}/b_{MM})\,[X_{I,t} - S_{I,t}] + X_{M,t} - S_{M,t}$ on the overall number of fish harvested, ii) issuing tradable harvesting quotas in numbers and iii) fixing an exchange rate of b_{IM}/b_{MM} units of immature fish for one mature fish.*

Proof. See Appendix A.3. □

In this setting, the harvesting quota may be thought of as a license to reduce the stock of fish of a particular age by no more than a specified amount.[9] For harvesting a number h_I of immature fish (or a number h_M of mature fish), fishermen would need a license that allows them to reduce the spawning stock at the beginning of the next period by $b_{IM}\, h_I$ (or $b_{MM}\, h_M$) fish.

With a similar line of reasoning, the traditional TAC/quota system in terms of biomass could be modified to decentralize optimal harvesting of the different age classes. A harvesting quota of one ton of mature fish may be thought of as the license to reduce the current spawning stock by one ton, or, equivalently, as the license to reduce the spawning stock at the beginning of the next period by b_{MM} tons. In the same vein, a harvesting quota of one ton of immature fish may be thought of as the license to reduce the current stock of immature fish by one ton, or, equivalently, as the license to reduce the spawning stock at the beginning of the next period by $b_{IM}\, w_M/w_I$ tons. This is because a fraction b_{IM} of currently immature fish that escape fishing would become mature, accompanied by an increase in weight by a factor of w_M/w_I. In other words, the "biomass" quota system could be modified in such a way that one ton of quota is needed to catch one ton of mature fish, while $(b_{IM}\, w_M/w_I)/b_{MM}$ tons of quota are needed to catch one ton of immature fish. The "exchange rate" $(b_{IM}\, w_M/w_I)/b_{MM}$ for biomass quotas will typically be much larger than one. For Eastern Baltic cod, for example, this exchange rate is 2.1 (see Section 5).

[9] This interpretation of harvesting quotas is reminiscent of the Montgomerys [19] concept of pollution licenses with exchange rates to account for the spatial dimension.

5. Application: Eastern Baltic Cod fishery

The Eastern Baltic cod stock is historically the third largest stock in the North Atlantic [6] with a long-term mean spawning stock biomass (SSB) of 400,000 to 500,000 tons. The cod is of considerable commercial importance for the region's fisheries. All countries bordering the Baltic Sea are involved in the cod fishery, and all of them, except Russia, are member states of the European Union (EU). Management decisions are settled in bilateral agreements between the EU and Russia. Between 1983 and 1992 a combination of high fishing pressure and low recruitment resulted in a decrease of the spawning stock biomass from over 600,000 to less than 100,000 tons, reaching a record low level in 2005 (66,000 tons; [15]). Landings from this fishery reached a peak of almost 400,000 tons in 1984 and then started to decline significantly, reaching a minimum of 45,000 tons in 1993 and remaining at low levels ever since. Although the present estimates of stock biomass are uncertain due to misreporting of landings, discarding, and age-reading problems, the available information indicates that the SSB has recently increased. This is mainly due to the unusual strength of the 2005 and 2006 year classes [16].

Current management measures are based on a formal recovery and management plan implemented since January 2008 with an overall target fishing-mortality level of 0.3.[10] Besides setting the annual total allowable catch (TAC), the fishery is further managed through mesh size regulations (130 mm), minimum landing sizes (38 cm), seasonal fishery restrictions, and area closures mainly designed to protect spawning fish in the three main deep

[10]This figure corresponds to an instantaneous fishing mortality of 0.3 throughout the year, which implies an escapement of $\exp(-0.3) = 74\%$ of the stock.

basins of the Baltic Sea, i.e., the Bornholm Basin, the Gotland Basin, and the Gdansk Deep [16]. The latter two management instruments are not the subject of this study and may well be part of an overall optimal fishery management. The two instruments that currently aim at preventing growth overfishing, mesh size regulations and minimum landing sizes, would be superfluous under a management by TAC and tradable harvesting quotas in terms of the number of fish, as proposed here. This would imply a significant reduction of transaction costs connected to monitoring and enforcement of these regulations.

5.1. Data and calibration of the model

The parameterization of the population model for the eastern Baltic cod case is based on the best available biological data. Age-specific abundance data, the proportion of mature fish per age-class, and natural mortality rates are based on assessment data using a stochastic multispecies model (SMS; [14]). The weight of young immature fish ($w_I = 0.44$ kg/individual) is directly taken from the ICES [15] assessment report. The weight of mature fish ($w_M = 0.96$ kg/individual) was estimated as the mean weight of cod aged three years and older, weighted by relative age-class-specific relative abundance from 1974–2007 (data from ICES [15]).

We are not interested in calculating optimal stock numbers of eggs and juveniles. Accordingly, we estimate $r(x_{M,t})$ as the stock-recruitment relationship between the number of mature and the number of immature cod (first quarter, lagged for two years) and set $b_{EJ} = b_{JI} = 1$. The two other survival rates, $b_{IM} = 0.81$ and $b_{MM} = 0.82$, are taken from the ICES [15] assessment report. For short-term forecasting, ICES standard stock assessment does not currently use any stock-recruitment function but rather uses a geometric mean of years 1987–2005 [15]. For our longer-term simulations, however, a stock-recruitment function is needed. We use the Ricker specification [22]

$$r(X_{M,t}) = \gamma_1 \, X_{M,t} \, \exp(-\gamma_2 \, X_{M,t}) \tag{11}$$

which has a maximum at $X_M^{\text{peak}} = 1/\gamma_2$. This type of stock-recruitment relationship is an appropriate description of recruitment biology of Baltic cod, as there are clear indications of increased cannibalism at high stock numbers, mainly affecting juvenile fish. This phenomenon is due to a higher spatial overlap between juvenile nursery grounds and an outspreading adult population when stock numbers are high. In order to find estimates for the two parameters, γ_1 and γ_2, we use ICES [14] data for the number of mature Eastern Baltic cod, $X_{M,t}$, and for the young immature recruits two years later, $X_{I,t+2}$, for the period 1974–2007. Taking logs of (11) and applying a simple OLS regression to $\ln(X_{I,t+2}/X_{M,t}) = \ln(\gamma_1) - \gamma_2 \, X_{M,t}$, we obtain $\gamma = 1.54$ (standard error 0.159) and $\gamma_2 = 1.5 \cdot 10^{-3}$/million fish (standard error $0.6 \cdot 10^{-3}$/million). The peak value of $X_M^{\text{peak}} = 667$ million individuals is about 10% greater than the spawning stock observed in the early 1980s (approx. 600 million individuals).

With weights below one kilogram, both immature and mature cod fall in the same size category. We therefore use $p_I = p_M$ in the simulation.[11] We normalize the price to unity, i.e. $p_I = p_M = 1$, and calculate unit effort costs in terms of the average cost/price ratio. For effort and cost, the data do not allow differentiating between age classes. We therefore assume that harvest-

[11] According to European regulation (Council Regulation No 2406/96), this is the category of 0.3–1 kg. In 2007, the ex-vessel price for cod in this size category was 12.63 Danish Crowns (DKK) per kilogram [7]. Overall, the price is increasing with weight. For the next higher size category of 1-2 kg the price was 19.48 DKK/kg in 2007. As in practice, since some of the mature cod will fall in this or an even higher size category, our assumption $p_I = p_M$ tends to overestimate the value of immature cod harvested.

ing functions and cost parameters are the same for immature and mature cod. To estimate the parameters of the harvesting function (see Appendix A.1), we use the stock numbers from the ICES [14] report (years 1974–2007). We estimate escapement from stock numbers and the fishing mortalities reported in [14]. Historical data on spawning stock numbers (SSN) and escapement from 1974 to 2007 are shown in Figure 1. Effort data, measured in days at sea, are available for the Danish fleet for the years 1987–2007 from [14]. Dividing the effort of the Danish fleet by its harvesting share (also from [14]), we obtain an estimate for total effort. Assuming stock elasticity of $\beta = 1$, an OLS regression of the harvesting function (see Appendix A.1) yields a catchability coefficient $\eta = 2.08 \cdot^{-6}$ (standard error $0.83 \cdot 10^{-6}$).[12] Using the method of [18], and using data on Danish fishery accounts from 1995–2007, we obtain an average unit effort cost parameter of $\nu = 0.554$ (see Appendix A.4). With this, we obtain the cost parameter $c_I = c_M = \nu/\eta = 2.66 \cdot 10^5$.

Table 1 summarizes the parameters used in the following simulation. In the table we also report the 90% confidence intervals for the parameters estimated, i.e. γ_1, γ_2, η and ν, as well as an according interval of discount factors. We use these intervals for the sensitivity analysis of the numerical optimization.

Table 1 about here

[12] We also performed a non-linear least-squares regression of the harvesting function allowing for $\beta < 1$ (using the Levenberg-Marquardt least squares algorithm). We could not reject the null hypothesis $\beta = 1$ at the 10% level. The assumption $\beta = 1$ is also supported by previous findings [18, 13].

5.2. Optimization results

From the parameters given in Table 1, we have $w_M/b_{MM} = 1.17$ and $w_I/b_{IM} = 0.54$. Hence the assumption $p_M\, w_M/b_{MM} > p_I\, w_I/b_{IM}$ is satisfied for Baltic cod with $p_M = p_I$. Condition (10) holds whenever $\rho > w_I/(b_{IM}\, w_M) = 0.57$, i.e. for annual discount rates lower than 76%. Assuming that this is the case, and as, by assumption, the cost of harvesting immature cod is not lower than that of harvesting mature cod ($c_M \leq c_I$), it is optimal to exclusively harvest mature cod in a steady state (Proposition 3). In the optimal steady-state, the spawning stock consists of $X_M = 625$ million individuals, and the escapement of $S_M = 390$ million individuals (the equations that determine the steady state are given in Appendix A.2). The optimal steady-state spawning stock is slightly below the value representing the peak of the recruitment function (625 as compared to 667 million individuals).

In order to assess the uncertainty involved in calculating the optimal steady state we perform the following sensitivity analysis: we calculate the optimal steady state for each combination of both the reference parameter set and the upper and lower boundaries of their 90% confidence intervals (see Table 1). The resulting minimum and maximum steady-state values for the optimal steady state spawning stock are $\underline{X}_M = 327$ and $\overline{X}_M = 2824$ million individuals. For the optimal steady-state escapement, the corresponding minimum and maximum values are $\underline{S}_M = 322$ and $\overline{X}_M = 1942$ million individuals. These figures show that the optimal steady-state values are subject to considerable uncertainty, especially with regard to the upper bounds on the steady-state spawning stock and escapement. This reflects the general uncertainty associated with biological stock assessment, which is amplified by the uncertainties in the economic parameters used to calculate the optimal steady state. Nevertheless, the results of the sensitivity analysis clearly indicate that the Baltic is overfished despite the recent increase in spawning stock numbers: the lower bound of the calculated optimal steady-state levels of escapement (233 million individuals) is about 60 percent above the value of 2007 when escapement was 137 million individuals.

Figure 1 about here

We perform the dynamic optimization using the reference parameter set reported in Table 1.[13] The resulting optimal developments of the spawning stock and escapement of mature Eastern Baltic cod are shown in Figure 1.[14] According to the results, it is optimal to stop harvesting for three years, as in this period optimal escapement equals the spawning stock. After this period, harvesting is gradually increased to the steady-state values.[15]

Figure 2 shows the optimal harvesting fee and the instantaneous profits per unit of harvest at the beginning and end of the harvesting seasons. For the transition period of three years, the optimal harvesting fee already exceeds the profit per unit of harvest at the beginning of the season, where harvesting costs are at a minimum, so that no fisherman would have an incentive to start fishing. After the transition period, the harvesting fee and the current profit

[13]For the numerical calculation we employed the interior-point algorithm of the Knitro (version 6.0) optimization software with Matlab [3, 4].

[14]The developments of the stock and escapement of immature cod (age class I) are not shown, as over the whole time horizon complete escapement (i.e. zero harvesting) of the young immature cod is optimal.

[15] The most rapid approach to constant escapement, which would be optimal in the corresponding biomass model with a linear objective function (both in continuous and discrete time, [5, 21]), is not optimal in the age-structured setting considered here. This result is similar to a model with multi-species interactions, where again the most rapid approach is generally not the optimal solution [5, chapter 10].

at the end of the fishing season coincide (cf. condition 5).[16] Overall, the harvesting fee is substantial, with values of almost 60 percent of the ex-vessel price of landed fish.

Figures 2 and 3 about here.

In order to study the sensitivity of optimal dynamics to changes in the parameter values, we perform the dynamic optimization with the two boundary values of the 90% confidence interval of the catchability parameter η (Table 1), while the other parameter values are set as in the reference case. The optimization results are shown in Figure 2. The lower curves, for the maximum value of catchability parameter, i.e. the case of low harvesting cost, are qualitatively similar to the curves for the reference parameter set shown in Figure 1. However, we observe two quantitative differences: the period of zero harvest lasts only two years, and both the optimal steady-state stock and escapement are considerably lower. Also, the steady-state harvest is lower than in the case of the reference parameter set.

In quantitative terms, the upper curves in Figure 3, which display the optimal values for stock and escapement for the minimum value of the catchability parameter (i.e. high harvesting cost), differ from the reference case in that the period of zero harvest is considerably longer (eight years) and their steady-state values are much higher. Steady-state harvest, on the other hand, is lower than in the reference case. In addition, the optimal paths of stock and escapement differ in qualitative terms from the reference case. After the period of zero harvesting, the steady state is approached in damped oscillations with periods of "over-shooting", when stock and escapement exceed

[16] In this period with positive optimal TAC, the price for tradable quotas would be equal to the optimal harvesting fee.

the steady-state values.

6. Conclusion and discussion

In this paper we set up an age-structured fishery model that allows us to distinguish spawning stock and non-spawning stock and to disentangle the problems of recruitment overfishing and growth overfishing. We have shown that, in general, harvesting from the spawning stock and from the non-spawning stock has to be targeted in different ways, meaning that, except in special cases, immature fish and mature fish should be harvested in different quantities. Specifically, we have identified conditions under which it is optimal to harvest mature (spawning) fish only.

Our study provides important policy implications. First, the type of quota management currently implemented in most fisheries fails to solve the problem of growth overfishing, as quotas are expressed in terms of biomass. We have secondly shown that, although optimal harvesting quantities differ from one age class to another, optimal age-structured management can be implemented by means of incentive-based policy instruments, provided they are specified in terms of the *number* of fish harvested. We have shown that for a single harvesting fee will be sufficient to solve the problems of growth and recruitment overfishing simultaneously if natural survival rates of the different age classes subject to harvesting are identical. Alternatively, optimal harvesting can be decentralized by setting a single TAC and by issuing or auctioning off tradable harvesting quotas. The quota market will then efficiently allocate the TAC among the different age classes. One notable aspect of this result is that the quota market will thereby bring about an optimal age structure in the fish stock. This is an important difference to the conclusion from the "biomass" model, where the ecological effectiveness of management is guaranteed by setting the appropriate TAC, irrespective of whether quotas are tradable or not.

A practical implementation of fishery management requires quantifying the TAC or, with the price-based approach, quantifying the harvesting fee. As an illustration, we have applied our age-structured model to the Eastern Baltic cod fishery. Here it turns out that the conditions for a zero-harvest policy for immature cod are satisfied. For the older cod, we have computed a time path for total allowable catch that maximizes the present value of resource rents. It involves zero harvesting for a period of three years (due to the linear objective function), then a period of gradually increasing TACs, and ultimately a yearly harvest substantially higher than current harvests. A comprehensive sensitivity analysis shows that current management is clearly not optimal. However, the sensitivity analysis also shows that the quantification of optimal TACs is subject to considerable uncertainty. One conceivable source of uncertainty is that our model is a single species model. Many species, including the Baltic cod, interact with other commercially valuable species. For example, Baltic cod feeds on sprat and herring, while the sprat feeds on cod eggs and larvae. Further research should therefore try to integrate species interaction into age-structured models to reduce the uncertainties and to allow for the development of an integrated policy regulating several commercial species simultaneously.

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Appendix

A.1 Derivation of harvesting and profit functions: We assume that instantaneous harvest flows $h_{I,t}(\tau)$ and $h_{M,t}(\tau)$ are determined by current fishing effort $e_{j,t}(\tau)$ targeting age class $j = I, M$ (at time τ) within the fishing season in year t:[17]

$$h_{I,t}(\tau) = \eta_I \, e_{I,t}(\tau) \, x_{I,t}(\tau)^{\beta}$$
$$h_{M,t}(\tau) = \eta_M \, e_{M,t}(\tau) \, x_{M,t}(\tau)^{\beta} \tag{A.12}$$

Here, $x_{j,t}(\tau)$ is the stock of age $j = I, M$ at time τ, such that $x_{j,t}(\tau) = X_{j,t}$ at the beginning of year t's fishing season and $x_{j,t}(\tau) = S_{j,t}$ at the end. The parameters η_j, $j = I, M$, are the catchability coefficients, and $\beta \in (0, 1]$ is the stock elasticity of harvest. Harvesting costs are proportional to effort, with ν_j as the unit effort cost. Introducing the cost parameter $c_j = \nu_j/\eta_j$, the instantaneous profit flows from harvesting these two age classes at time τ within fishing period t are:

$$\pi_{I,t}(\tau) = p_I \, w_I \, h_{I,t}(\tau) - c_I \, x_{I,t}(\tau)^{-\beta} \, h_{I,t}(\tau)$$
$$\pi_{M,t}(\tau) = p_M \, w_M \, h_{M,t}(\tau) - c_M \, x_{M,t}(\tau)^{-\beta} \, h_{M,t}(\tau) \tag{A.13}$$

During the harvesting season, each fish caught diminishes the stock by one unit, i.e. $\dot{x}_{j,t}(\tau) = -h_{j,t}(\tau)$, $j = I, M$, as, by assumption, the natural processes of mortality and growth do not happen during the harvesting season. The aggregate annual profit (Π_t) from fishing both age classes in period t is obtained by integrating the flow of profits over the whole fishing season [5].

$$\begin{aligned} \Pi_t = & \int_{S_{I,t}}^{X_{I,t}} \left[p_I \, w_I - c_I \, x_{I,t}^{-\beta} \right] \, dx_{I,t} + \int_{S_{M,t}}^{X_{M,t}} \left[p_M \, w_M - c_M \, x_{M,t}^{-\beta} \right] \, dx_{M,t} \\ = & \, p_I \, w_I \, [X_{I,t} - S_{I,t}] - \frac{c_I}{1-\beta} \left[X_{I,t}^{1-\beta} - S_{I,t}^{1-\beta} \right] \\ & + p_M \, w_M \, [X_{M,t} - S_{M,t}] - \frac{c_M}{1-\beta} \left[X_{M,t}^{1-\beta} - S_{M,t}^{1-\beta} \right] \end{aligned} \tag{A.14}$$

The corresponding harvesting function is $H_{j,t} = X_{j,t} - \left[X_{j,t}^{1-\beta} - (1-\beta) \, \eta_j \, E_{j,t} \right]^{\frac{1}{1-\beta}}$, where $E_{j,t}$ is total effort directed at harvesting age class $j = I, M$ in year t.

[17] Alternatively, τ may be interpreted as an index that orders the different fishermen harvesting sequentially.

For $\beta = 1$, this equation may be written as $\ln(S_{j,t}) = \ln(X_{j,t}) - \eta_j \, E_{j,t}$.

A.2 Steady state conditions and proof of Proposition 3: In a steady state with complete escapement of age class I, i.e. $S_I = X_I$ and partial escapement of age class M, i.e. $0 < S_M < X_M$, the first-order necessary conditions (4)–(9) imply

$$\mu_I = b_{IM} \, \lambda_M - \left[p_I \, w_I - c_I \, X_I^{-\beta} \right] \tag{A.15}$$

$$p_M \, w_M = b_{MM} \, \lambda_M + c_M \, X_M^{-\beta} \tag{A.16}$$

$$\lambda_E = \rho \, b_{EJ} \, \lambda_J = \rho^2 \, b_{EJ} \, b_{JI} \, \lambda_I = \rho^3 \, b_{EJ} \, b_{JI} \, b_{IM} \, \lambda_M \tag{A.17}$$

$$\rho \, p_M \, w_M = \rho \, c_M \, X_M^{-\beta} - \rho \, \lambda_E \, r'(X_M) + \lambda_M \tag{A.18}$$

Combining the last two equations, we can calculate the shadow price λ_M as a function of the spawning stock $\rho \, p_M \, w_M = \rho \, c_M \, X_M^{-\beta} - \rho^4 \, r'(X_M) \, b_{EJ} \, b_{JI} \, b_{IM} \, \lambda_M + \lambda_M$, or:

$$\lambda_M = \rho \, \frac{p_M \, w_M - c_M \, X_M^{-\beta}}{1 - \rho^4 \, r'(X_M) \, b_{EJ} \, b_{JI} \, b_{IM}} \tag{A.19}$$

This is the first of two conditions determining the steady state. The other condition is obtained by substituting $X_I = b_{JI} \, X_J = b_{EJ} \, b_{JI} \, X_E = b_{EJ} \, b_{JI} \, r(X_M)$ and equation (A.16) into (1)

$$X_M = b_{EJ} \, b_{JI} \, b_{IM} \, r(X_M) + b_{MM} \, \left[\frac{c_M}{p_M \, w_M - b_{MM} \, \lambda_M} \right]^{\frac{1}{\beta}} \tag{A.20}$$

To prove the proposition, we now show that $\mu_I > 0$ provided that $c_I \geq c_M$ and condition (10) hold. For this purpose, we consider (A.15) in the form $\mu_I = b_{IM} \, \lambda_M - \left[p_I \, w_I - c_I \, S_I^{-\beta} \right]$ with $\mu_I = 0$ if $S_I < X_I$. Substituting (A.19) into , we obtain

$$\mu_I = \rho\, b_{IM}\, \frac{p_M\, w_M - c_M\, X_M^{-\beta}}{1 - \rho^4\, r'(X_M)\, b_{EJ}\, b_{JI}\, b_{IM}} - \left[p_I\, w_I - c_I\, S_I^{-\beta}\right] \tag{A.21}$$

$$> \rho\, b_{IM}\, \left[p_M\, w_M - c_M\, X_M^{-\beta}\right] - \left[p_I\, w_I - c_I\, S_I^{-\beta}\right] \tag{A.22}$$

$$\stackrel{(10)}{>} c_I\, S_I^{-\beta} - \rho\, b_{IM}\, c_M\, X_M^{-\beta} \geq c_M\, \left[S_I^{-\beta} - \rho\, b_{IM}\, X_M^{-\beta}\right] \tag{A.23}$$

$$= c_M\, \left[S_I^{-\beta} - \rho\, b_{IM}\, \left[b_{IM}\, S_I + b_{MM}\, S_M\right]^{-\beta}\right] \tag{A.24}$$

$$\geq c_M\, \left[S_I^{-\beta} - \rho\, b_{IM}^{1-\beta}\, S_I^{-\beta}\right] = c_M\, S_I^{-\beta}\, \left[1 - \rho\, b_{IM}^{1-\beta}\right] > 0 \tag{A.25}$$

Hence, we also conclude $S_I = X_I$.

A.3 Proof of Proposition 5: We consider one representative fisherman harvesting age class $j = I, M$ who chooses the quota $H_{j,t}$ at a quota price ψ_t (which is independent of j) such as to maximize

$$\max_{H_{j,t}} \left\{ p_j\, w_j\, H_{j,t} - \frac{c_j}{1-\beta}\, \left[X_{j,t}^{1-\beta} - \left[X_{j,t} - H_{j,t}\right]^{1-\beta}\right] - \psi_t\, H_{j,t} \right\} \tag{A.26}$$

The first-order conditions of profit maximization are

$$p_j\, w_j - c_j\, \left[X_{j,t} - H_{j,t}\right]^{-\beta} \leq \psi_t \qquad\qquad j = I, M \tag{A.27}$$

with equality for $H_{j,t} > 0$. Comparison with the first-order conditions for the social optimum, (4) and (5), yields that the quota price is $\psi_t = b_{IM}\, \lambda_{M,t} = b_{MM}\, \lambda_{M,t}$ if $H_{j,t} = X_{j,t} - S_{j,t}$, $j = I, M$, i.e. if the quota market clears. The proof for the case when $b_{IM} \neq b_{MM}$ (Corollary 2) is straightforward.

A.4 Estimation of cost parameters: Effort is measured in days at sea. The variable costs of fishing per day at sea are calculated according to [17]. Data is available for fishing vessels operating in the region of Bornholm, a major fishing area in the Eastern Baltic Sea, for the years 1996 to 2007 from the [8]. To obtain the variable cost per day at sea in Danish Crowns (DKK), the variable cost of harvesting cod is divided by the days at sea a firm is harvesting cod in the Bornholm region. The variable cost of harvesting cod is obtained as the product of variable cost (in 1000 DKK per firm) and the share of cod expressed through the quotient of gross output in the cod fishery and gross output in total. The variable costs are derived adding the labor cost of fishermen and total cost (in 1000 DKK per firm) and subtracting depreciation. Specifically, we use the following data and calculations, here illustrated for data from 2007.

A)	Gross Output for cod, 1000 DKK/firm	1201
B)	Gross Output in Total; 1000DKK/firm	1782
C)	Share of cod (equal to A/B)	60.67
D)	Total cost, 1000 DKK/firm	1240
E)	Total cost of hired labor, 1000DKK/firm	383
F)	Depreciation, 1000 DKK/firm	160
G)	Labor input of crew, Days at sea/firm	178
H)	Labor input of fisherman, Days at sea/firm	110
I)	Wage per day (equal to E/G)	2.15
J)	Labor cost of fisherman, 1000DKK/firm (equal to I*H)	236.68
K)	Variable cost, 1000DKK/firm (equal to J+D-F)	1316.69
L)	Variable cost of cod, 1000DKK/firm (equal to K*C)	882.18
M)	Days at sea harvesting cod per firm	120
N)	Variable cost per day, DKK. (equal to L/M*1000)	7351.5

With an average price in 2007 of 13.36 DKK/kg, we find $\nu^{2007} = 0.550$ tons/day. The overall figure of $\nu = 0.554$ is obtained as the average over these estimates for 1995 to 2007. For the years 1995–1999 we used the estimations of Kronbak [17], while for the years 2000–2007 we calculated the cost/price ratio with the method described above using data from [8]. The 90% confidence interval of cost/price ratios is [0.414,0.693].

Parameter	Symbol	Value	Confidence interval
Survival rate eggs/larvae–juveniles	b_{EJ}	1	
Survival rate juveniles–immature	b_{JI}	1	
Survival rate immature–mature	b_{IM}	0.81	
Survival rate mature–mature	b_{MM}	0.82	
Weight of immature	w_I	0.44 kg/fish	
Weight of mature	w_M	0.96 kg/fish	
Parameters of recruitment	γ_1	1.54	[1.17,2.02]
function	γ_2	$1.5 \cdot 10^{-9}$/fish	$[0.47 \cdot 10^{-9}, 2.53 \cdot 10^{-9}]$
Stock elasticity of harvest	β	1	
Catchability coefficient	η	$2.08 \cdot 10^{-6}$	$[0.64 \cdot 10^{-6}, 3.51 \cdot 10^{-6}]$
Ex-vessel price	$p_I = p_M$	1	
Cost/price ratio	$\nu_I = \nu_M$	0.554	[0.413,0.69]
Discount factor	ρ	0.95	[0.90,1.0]

Table 1: Estimated parameters and confidence intervals (90%, where applicable). Sources and methods are described in the text.

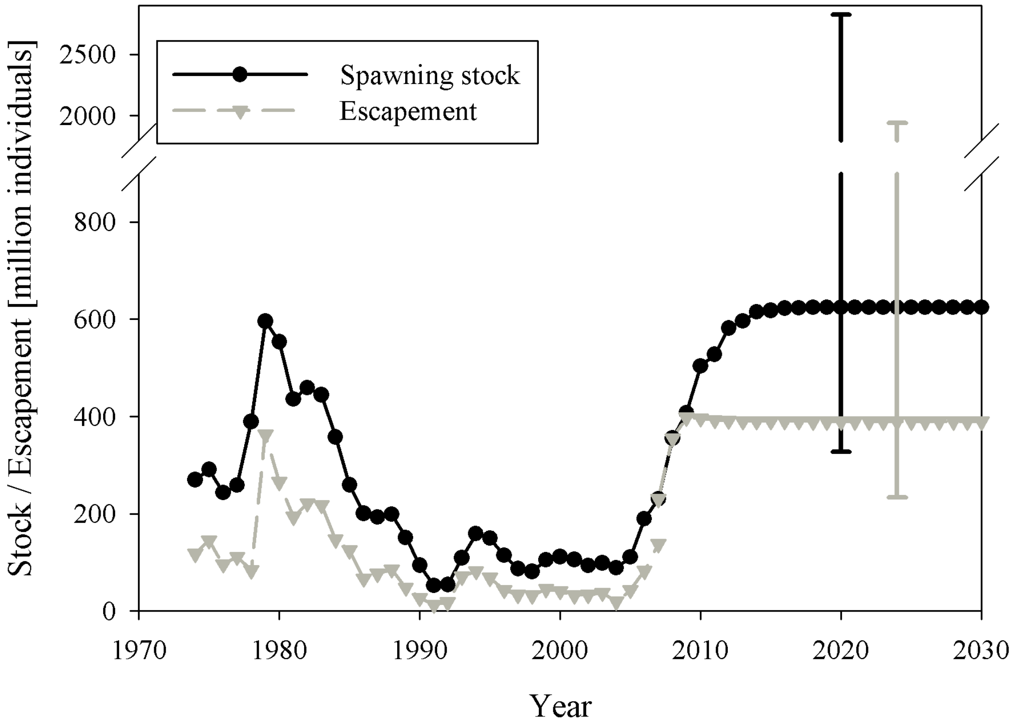

Figure 1: Spawning stock and escapement of Eastern Baltic cod. ICES data from 1974–2007, results of numerical optimization from 2007 to 2030. Parameter values are given in Table 1.

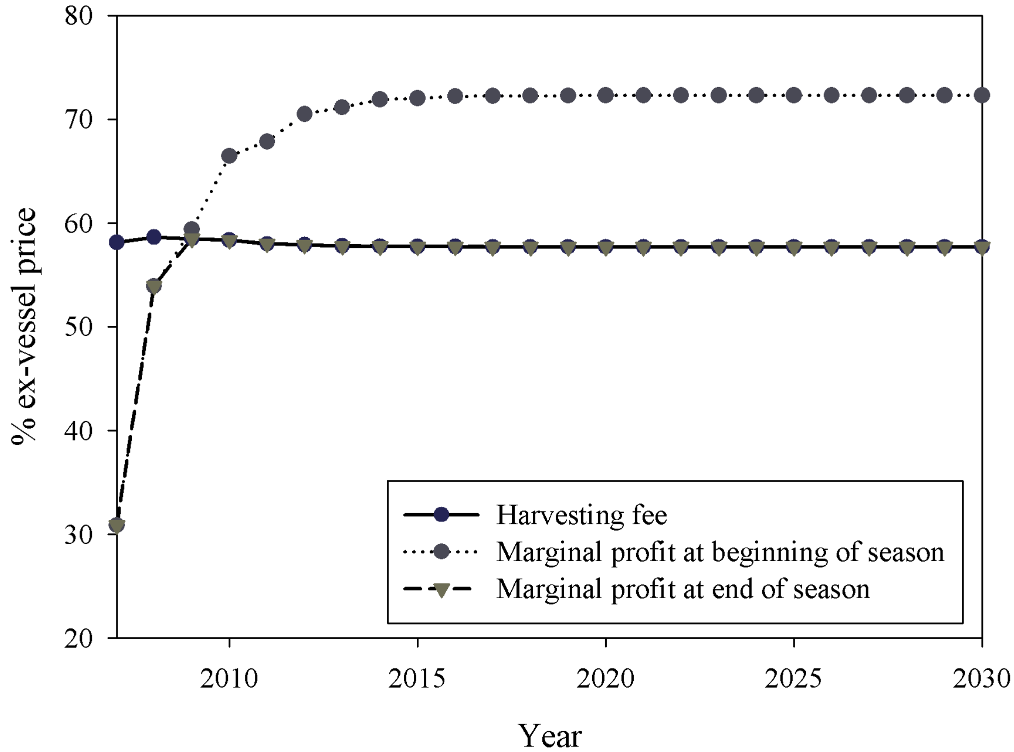

Figure 2: Harvesting fee and instantaneous profit at beginning and end of fishing seasons. Parameter values are given in Table 1.

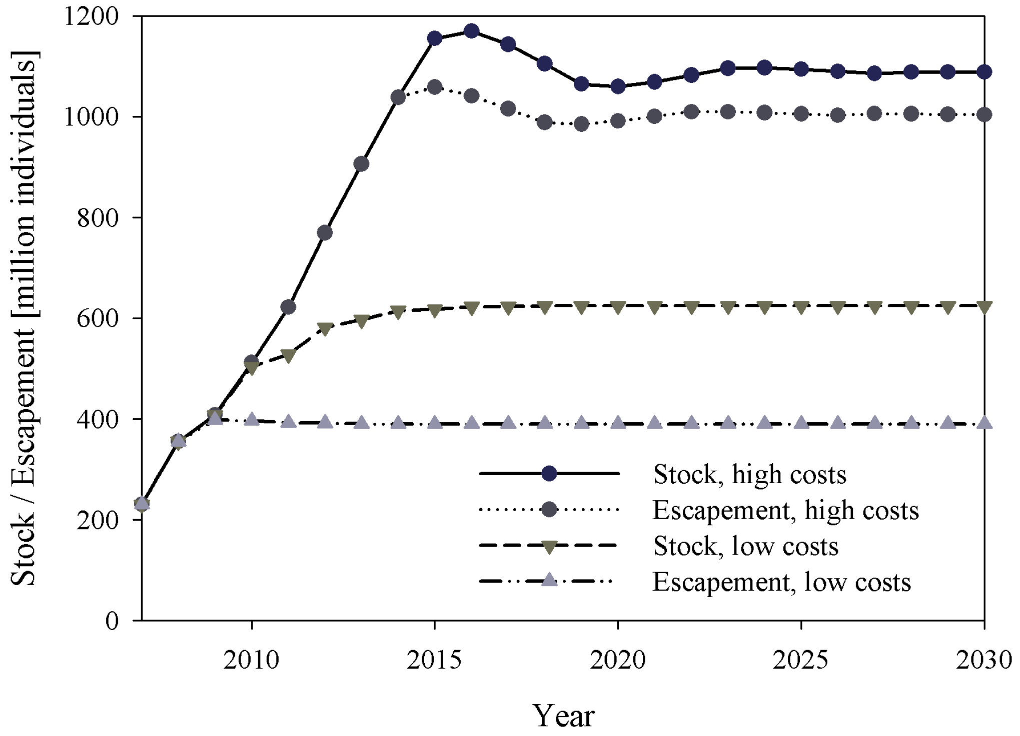

Figure 3: Sensitivity analysis of optimal dynamics. The upper curves show the result for the minimum, the lower curves for the maximum catchability coefficient out of the 90% confidence interval reported in Table 1. The other parameters are given in Table 1.