

ANDERS SKONHOFT ET AL.

RH: Cost and income in red deer management

Balancing cost and income in red deer management – a case study from Norway

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1 **Abstract**

2 This paper presents a bioeconomic analysis of a red deer population within a Norwegian
3 institutional context. This population is managed by a well-defined manager, typically
4 consisting of many landowners operating in a cooperative manner, with the goal of
5 maximizing the present-value hunting related income while taking browsing and grazing
6 damages into account. The red deer population is structured in five categories of animals
7 (calves, female and male yearlings, adult females and adult males). It is shown that
8 differences in the per animal values and survival rates ('biological discounted' values) are
9 instrumental in determining the optimal harvest composition. Fertility plays no direct role.
10 The numerical analysis, represented by a case study from an area in the Western part of
11 Norway with high animal density, shows that the optimal harvest pattern stays quite stable
12 under various parameter changes, but it is also revealed which parameters and harvest
13 restrictions that is most important.

14

15

16 *Key word:*

17 Red deer, management, bioeconomic analysis, landowner, cost-benefit, hunting, meat value,
18 crop damage.

19 **Introduction**

20 The management of deer have become in focus with recent large increases in population sizes
21 in many areas of both Europe and North America (Gordon et al., 2004). Increased abundances
22 increases potential yields. However, the increasing number of deer-vehicle collisions and their
23 related costs (Langbein et al., 2010), deer damages to forestry and agriculture (Reimoser and
24 Putman, 2010) and their general ecosystem impact (Mysterud, 2006; Reimoser and Putman,
25 2010) are also important concerns accompanying the population increase. Large herbivores
26 are termed ecological keystone species because of their ability to affect vegetation community
27 patterns and ecosystem functioning (Côté et al., 2004; Hobbs, 1996). Deer management
28 decisions concerning population development are therefore important issues that generally
29 influence both ecological and economic aspects.

30
31 A good example of this development is the Norwegian red deer (*Cervus elaphus*) population,
32 which has increased and spread substantially during the last few decades (Milner et al., 2006).
33 The red deer has become the most numerous wild ungulate in Norway both measured as the
34 number of harvested individuals (Statistics Norway, 2010) as well as number of live animals
35 in the population (Andersen et al., 2010). In 1970 the total number of harvested red deer was
36 about 2,800, and animals were mainly found in scattered and low density populations along
37 the south-western part of the country (Figure 1). In 2009 red deer are found in nearly all
38 municipalities of southern Norway, and the total number of shot animals exceeded 39,000
39 animals. However, the main populations are still situated in the traditional core areas and
40 more than 90 % of the red deer harvested are shot within five counties in the south-western
41 part of Norway. The national population increase is therefore mainly due to increased
42 densities in these traditional core areas (Milner et al., 2006).

43
44 Red deer are extensively utilized as a game species, and the management strategy in Norway
45 has traditionally been oriented towards meat hunting rather than trophy hunting (Milner et al.,
46 2006). Large deer populations both represent a potential source of income for the landowners
47 possessing the hunting rights, as well as a potential cost related to damages from grazing and
48 browsing. The harvest related incomes are typically associated with sale of meat and hunting
49 rights, whereas additional services or products (cabin rental, guiding, etc.) generally represent
50 a modest source of income (Olaussen and Mysterud, Unpublished). The main cost component
51 is normally reduced production from agricultural crops (mostly grass production), both in
52 terms of quality and quantity. In addition, browsing damage to economically important tree
53 species may also be extensive. Costs related to deer-vehicle collisions are also significant
54 (Mysterud, 2004). Such costs are both complex to quantify (Bissonette et al., 2008) and paid
55 by the general public.

56
57 The present paper aims to analyze the cost and benefit of a red deer population from a
58 landowner perspective. We use a five stage population model (calves, female and male
59 yearlings, adult females and adult males) to simulate the population development and
60 response to various harvesting scenarios. The main goal is to analyze the basic driving forces
61 behind the optimal harvesting strategies given various constraints related to male-female
62 composition, and current code of conduct among hunters regarding to avoid shooting the
63 mother from calves. The meat value is assumed to be given by a fixed meat price, and the unit
64 costs related to grazing damage are assumed to be constant, but stage specific. Since costs
65 related to deer-vehicle collisions are external to landowners, they will not be taken into
66 account here. The landowner, typically consisting of several landowners acting as a single
67 agent through a landowner association (management unit), is assumed to be rational and well-
68 informed aiming to maximize the profit from hunting when accounting for grazing costs. We

69 also compare the results from our five stage model with the outcome of equivalent analyses
70 using a uniform harvesting scheme. Finally we compare our results with the present
71 harvesting pattern, which has been established through cohort analyses (Veiberg et al., 2010).

74 **Methods and material**

75 *Management organization and red deer hunting practice*

76 Decisions related to the management of red deer populations in Norway involve various
77 levels of the management hierarchy. General hunting periods and regulations are decided by
78 the Directorate for Nature Management. Within this framework, municipalities (local
79 governments) are given authority to limit the hunting period and to regulate the 'minimum
80 area required' (the basis for quotas) to local management aims, population situation and
81 current challenges. Each municipality is responsible for developing a management plan which
82 acts as a framework for private management. The municipality also has the responsibility for
83 coordinating local management plans, deciding hunting quotas and license fees, and reporting
84 harvest result to Statistics Norway (Andersen et al., 2010).

85
86 The hunting right belongs to the landowners. This means that landowners can benefit from
87 hunting related activities and products that relates to their property. Larger management units
88 are requested to present multi-annual management plans (3-5 years duration) containing
89 concrete management aims and a detailed harvest plan. These plans need to be approved by
90 the municipality, and harmonized with overall municipal management aims. This system
91 represents a more flexible and locally adapted management alternative compared to the
92 traditional alternative where the municipality enforce a more detailed and rigid practice
93 (Andersen et al., 2010). However, there are still some landowners receiving a quota from the
94 municipality. These quotas are related to the total area of suitable red deer habitat within an
95 area. The quota is typically found by dividing the total area of suitable red deer habitat within
96 an area by the minimum area required. The minimum area required does not differ between
97 red deer sex and age classes, but it typically differs between, or within, municipalities due to
98 differences in red deer population density and levels of deer related conflicts. Quotas are thus
99 regulated indirectly by adjusting the minimum area required per license. Since most
100 properties are rather small, association of two or more landowners into cooperative
101 management units is very common. This triggers allocation of more licenses, simplifies the
102 practical hunt and stimulates a more uniform management. For simplicity, we also refer to
103 such cooperative management units as landowners.

104
105 The red deer hunting starts September 10th and ends November 15th (in some areas December
106 23rd). Hunting on agricultural pastures from stands or ground blinds, and drive hunts for a
107 team of hunters, either with the aid from a dog on leach or a barking dog, are the most
108 frequently used hunting practices. Some stalking also occurs in forested areas, especially
109 during the rutting season when males are roaring (Meisingset, 2008).

111 *Study area*

112 Our study area is from the county of Sogn and Fjordane located on the western coast of
113 Norway. The county consists of 26 municipalities, and has the overall highest red deer
114 population in Norway. Flora and Gloppen (approx. 61.5°N and 5.5°E) are two neighboring
115 municipalities with a common total area of 1721 km². Flora is a coastal municipality, whereas
116 Gloppen is more of a fiord municipality located east of Flora (Figure 2). The topography
117 along the coast is characterized by islands and fiords with rolling country that change over to
118 narrow valleys with steep slopes leading to higher mountains and mountain range. Small scale

119 agriculture is widespread in the flatter lowlands, and the arable land is mainly cultivated to
 120 pastures and meadows for grass production. The forest is dominated by either Scots pine
 121 (*Pinus sylvestris*), alder (*Alnus incana*) or birch (*Betula* sp.), while planted Norway spruce
 122 (*Picea abies*) dominate locally. Common undergrowth species are juniper (*Juniperus*
 123 *communis*), bilberry (*Vaccinium myrtillus*), heather (*Calluna vulgaris*) and grasses and herbs.

124
 125 The climate in this region is characterized by the coastal location with annual precipitation
 126 between 1100-3500 mm, mean winter (December-February) temperatures between 0-3°C and
 127 mean summer (June-August) temperatures between 11-14°C (www.eklima.no). During winter
 128 number of days with snow cover normally increase and temperature decrease with altitude
 129 and distance from the coast (Langvatn et al., 1996; Mysterud et al., 2000).

131 **Population model**

132 In Norway mean dressed body weight (i.e., live weight minus skin, head, viscera, metapodials
 133 and bleedable blood) for males and females 2 years and older is usually between 60-125 kg
 134 and 50-70 kg, respectively (Solberg et al., 2010, see also Table 1). Due to lack of predators
 135 natural mortality rates are generally assumed to be low for all age and sex categories. The
 136 highest natural mortality rate is found for calves, followed by yearlings (Langvatn and
 137 Loison, 1999). As mortality throughout summer and fall is regarded to be small, all natural
 138 mortality is assumed to take place during the winter and after the hunting season. There has
 139 not been found evidence of density-dependent mortality for any age or sex categories in
 140 Norwegian red deer (Loison and Langvatn, 1998). On the other hand, fecundity is affected by
 141 female density (Langvatn et al., 2004), while the number of males seems to be of negligible
 142 importance for a small and moderately skewed sex composition (Mysterud et al., 2002).

143 However, it may play a role when the male population becomes small accompanied by a
 144 highly skewed sex composition (Milner-Gulland et al., 2003). From the hunters' point of
 145 view, a reasonable proportion of adult males is clearly both attractive and wanted (see also
 146 Result section below). To comply with this wish and to avoid speculations regarding potential
 147 long-term consequences of strongly biased sex ratios, we specify a male harvesting constraint.

148
 149 In the present analysis the population at time (year) t is structured in five stages; calves $X_{c,t}$
 150 ($yr < 1$), female yearlings $X_{yf,t}$ ($1 \leq yr < 2$), male yearlings $X_{ym,t}$, adult females $X_{f,t}$ ($yr \geq 2$
 151) and adult males $X_{m,t}$. The total population reads $X_t = X_{c,t} + X_{yf,t} + X_{ym,t} + X_{f,t} + X_{m,t}$. The
 152 population is measured in spring before calving. All stages may be harvested. Neglecting any
 153 stochastic variations in biology and environment, and any net dispersal in or out of the
 154 considered area, the number of calves (recruitment) is first governed by

155 $X_{c,t+1} = r'_{yf,t} X_{yf,t} + r'_{f,t} X_{f,t}$, so that $r'_{yf,t}$ is the fertility rate of yearlings and $r'_{f,t}$ the fertility rate
 156 of adults. Unless the number of males becomes extremely low, the male abundance and sex
 157 composition has probably negligible influence on female fertility rates. As already indicated,
 158 we therefore assume that only the total number of females, yearlings and adults, affect female
 159 fertility rates. Moreover, as female yearlings and adult females compete over the same
 160 resources, we assume that the fertility rates for both categories are affected only by the total
 161 number of females present. We then have $r'_{yf,t} = r'_{yf}(X_{yf,t} + X_{f,t})$, with $r'_{yf}(0) > 0$ and

162 $\partial r'_{yf} / \partial (X_{yf,t} + X_{f,t}) = r'_{yf} < 0$ for yearlings, and $r'_{f,t} = r'_f(X_{yf,t} + X_{f,t})$, with $r'_f(0) > 0$ and
 163 $r'_f < 0$, for adults. The density dependent recruitment function then yields:

$$164 \quad X_{c,t+1} = r'_{yf}(X_{yf,t} + X_{f,t})X_{yf,t} + r'_f(X_{f,t} + X_{yf,t})X_{f,t}. \quad (1)$$

165

166 The number of female and male yearlings follows next as:

$$167 \quad X_{yf,t+1} = \psi s_c (1 - h_{c,t}) X_{c,t} \quad (2)$$

168 and

$$169 \quad X_{ym,t+1} = (1 - \psi) s_c (1 - h_{c,t}) X_{c,t} \quad (3)$$

170 where s_c is the fixed calf natural survival rate (fraction) and ψ is the proportion of female
171 calves. The proportion of female and male calves born is assumed to be equal, i.e., $\psi \approx 0.5$
172 (Table 1, numerical section). $0 \leq h_{c,t} \leq 1$ is the calf harvesting rate. Finally, the abundance of
173 adult females and males become:

$$174 \quad X_{f,t+1} = s_{yf} (1 - h_{yf,t}) X_{yf,t} + s_f (1 - h_{f,t}) X_{f,t} \quad (4)$$

175 and

$$176 \quad X_{m,t+1} = s_{ym} (1 - h_{ym,t}) X_{ym,t} + s_m (1 - h_{m,t}) X_{m,t}, \quad (5)$$

177 respectively. s_f and s_m are the fixed natural survival rate of adult females and males, and
178 where $s_f \geq s_m \cdot s_{yf}$ and s_{ym} are the yearling survival rates, also with $s_{yf} \geq s_{ym}$ (Table 1).
179 $0 \leq h_{yf,t} \leq 1$ and $0 \leq h_{ym,t} \leq 1$ are the harvesting rates of female and male yearlings,
180 respectively, while $0 \leq h_{f,t} \leq 1$ and $0 \leq h_{m,t} \leq 1$ are the harvesting rates of the two adult stages.
181 Note that it is possible to hunt all adult females in a given year and still get production of
182 calves in the next year. The reason for this is that the yearlings also reproduce.

183 Equation (2) and (3) together yield:

$$184 \quad X_{ym,t} = [(1 - \psi) / \psi] X_{yf,t}. \quad (6)$$

185 Using this expression to eliminate $X_{ym,t}$ from equation (5), we find the abundance of adult
186 males as:

$$187 \quad X_{m,t+1} = s_{ym} (1 - h_{ym,t}) [(1 - \psi) / \psi] X_{yf,t} + s_m (1 - h_{m,t}) X_{m,t} \quad (7)$$

188 Equations (1), (2), (4) and (7) now constitute a reduced form model in four stages and five
189 harvesting rates, and where all equations are first order difference equations. We employ this
190 system of equations when studying exploitation below. The optimal number of animals and
191 harvest ratios will result from the optimization procedure, except for the number of male
192 yearlings which can be identified uniquely from the number of female yearlings using
193 equation (6). With $\psi = 0.5$, the number of female and male yearlings will be identical at
194 every point in time.

195
196 In biological equilibrium it is possible to say something about the adult sex ratio in our
197 population model. Combing (4) and (7) when $X_{i,t+1} = X_{i,t} = X_i$ ($i = c, yf, ym, f, m$) for fixed
198 harvesting rates, gives this sex ratio as

$$199 \quad X_m / X_f = [(1 - \psi) / \psi] [(1 - s_f (1 - h_f)) s_{ym} (1 - h_{ym})] / [(1 - s_m (1 - h_m)) s_{yf} (1 - h_{yf})]$$

200 after some
201 straightforward manipulations. Therefore, harvest of both the yearling and adult categories
202 influence the adult equilibrium sex ratio. However, the calf harvest plays no role because
203 there is no distinction between female and male calves in the model. With unprofitable
204 yearling harvest so that $h_{ym} = h_{yf} = 0$, the adult sex ratio becomes

$$205 \quad X_m / X_f = [(1 - \psi) / \psi] (1 - s_f (1 - h_f)) s_{ym} / (1 - s_m (1 - h_m)) s_{yf}.$$

206 For identical female and male
207 yearling survival rates together with similar proportion of female and male calves entering the
208 yearling stages, $\psi = 0.5$, the ratio reduces further to

$$209 \quad X_m / X_f = [(1 - s_f (1 - h_f))] / [(1 - s_m (1 - h_m))].$$

209 harvesting rates directly translate into sex-skewed adult abundance. This last relationship may
 210 also be written as $h_f = (1/s)(1-s)(X_m/X_f - 1) + (X_m/X_f)h_m$ when $s_m = s_f = s$ (see Table 1).
 211 Therefore, in biological equilibrium, or steady state, we find that a skewed sex ratio like
 212 $X_m/X_f = 0.5$, can be met with hunting rates in the ranges $(1-s)/s \leq h_m < 1$ together with
 213 $0 \leq h_f < (s-0.5)/s$.
 214

215 **Landowner cost and benefits**

216 The traditional landowner management goal has been directed towards maximizing the
 217 population productivity in order to produce meat, but also with a strong recreational
 218 component (Andersen et al., 2010; Milner et al., 2006). Typically, hunters have been local
 219 people, the landowners and their families and friends, and the hunting has had some flavor of
 220 subsistence hunting (Andersen et al., 2011; Olaussen and Mysterud, Unpublished). It has been
 221 a gradual shift towards more commercialization of the hunting during the last two decades or
 222 so. This is due both to higher red deer populations and to increased awareness of the
 223 economic potentials of red deer hunting and related products and activities (e.g., cabin renting
 224 and guiding; see e.g. Natur og Fritid, 2005), but also due to easier access to the various
 225 hunting areas and increased income among people. This has caused increased introduction of
 226 non-local hunters and a higher willingness to pay among all hunters regardless of local
 227 belonging. Still, the most important motivation for hunting among the local people is the
 228 prospect for meat and recreation (Olaussen and Mysterud, Unpublished), and only these kinds
 229 of hunting related income are considered here (but see Naevdal et al., Submitted, who
 230 analyzes moose trophy hunting). Therefore, we are thinking of the hunter as the local people,
 231 the landowners and their friends, enjoying the meat value and the recreational value of the
 232 hunt.
 233

234 The number of animals removed through hunting in year t is defined as $H_{i,t} = h_{i,t}X_{i,t}$
 235 ($i = c, yf, ym, f, m$). The value (price) per kg harvested meat \hat{p} is assumed to be independent
 236 of number of animals hunted, population density and animal category, as well as assumed to
 237 be fixed over time. The current gross meat value is accordingly
 238 $\hat{p}(w_c h_{c,t} X_{c,t} + w_{yf} h_{yf,t} X_{yf,t} + w_{ym} h_{ym,t} X_{ym,t} + w_f h_{f,t} X_{f,t} + w_m h_{m,t} X_{m,t})$ and where
 239 $w_c < w_{yf} \leq w_{ym} < w_f < w_m$ are the (average) dressed weights (kg per animal) of the different
 240 categories of animals (Table 1). For every animal shot the landowner pays a fixed fee q_i (NOK
 241 per animal) to the municipality. This fee is typically differentiated between calves and older
 242 animals (Table 2). For landowners the yearly net benefit from hunting measured as meat
 243 value is then defined as:

$$244 \quad V_t = p_c h_{c,t} X_{c,t} + p_{yf} h_{yf,t} X_{yf,t} + p_{ym} h_{ym,t} X_{ym,t} + p_f h_{f,t} X_{f,t} + p_m h_{m,t} X_{m,t}, \quad (8)$$

245 and where $p_i = (\hat{p}w_i - q_i)$ ($i = c, yf, ym, f, m$) is the net per animal meat value (NOK per
 246 animal).
 247

248 In addition to the meat value, the hunters also obtain a recreational value related to the
 249 hunting. Studies from elk hunting in US (Aiken and Pullis La Rouche, 2001) report net
 250 consumer surplus for resident elk hunters (total willingness to pay minus actual fees and
 251 costs) of about \$60 per day in Wyoming. Empirical studies from Sweden (Boman et al., 2011)
 252 also indicates that time spent on hunting is valuable. In what follows, only the hunting time is
 253 considered as an indicator for recreational value and hence all other valuable aspects of the
 254 hunting, except of the meat value, are lumped into this indicator. When using a production

255 function approach and where the production (harvesting) function is assumed to be of the
 256 standard Schaefer type with perfect selectivity (e.g., Clark, 1990), the number of hunted
 257 animals of category i becomes $H_{i,t} = \theta_i N_{i,t} X_{i,t}$ ($i = c, yf, ym, f, m$). $N_{i,t}$ indicates the hunting
 258 effort, measured in number of hunting days, and θ_i is the catchability (productivity)
 259 coefficient (1/hunting days). The catchability coefficient yields the hunting efficiency, and a
 260 higher value hence indicates a more efficient hunt. The value will typically differ somewhat
 261 between the various categories of animals, but probably not too much. In what follows, it is
 262 assumed to be identical for all categories. Therefore, with z as the recreational value (NOK
 263 per hunting day), the recreation value writes $B_t = z(N_{c,t} + N_{yf,t} + N_{ym,t} + N_{f,t} + N_{m,t})$, or

$$264 \quad B_t = (z / \theta)(h_{c,t} + h_{yf,t} + h_{ym,t} + h_{f,t} + h_{m,t}) \quad (9)$$

265 when substituting for the harvesting functions¹. The total yearly hunting benefit is hence
 266 described as $(V_t + B_t)$.

267
 268 The underlying causes explaining the occurrence and extent of deer damages are complex and
 269 not fully understood, but crop type and population density are assumed to be the most relevant
 270 factors (Reimoser and Putman, 2010). In a descriptive study from a neighboring municipality
 271 of our study area, Thorvaldsen et al. (2010) found that crop damages on agricultural land was
 272 more important than damages to commercial tree species. Based on knowledge about the
 273 primary production in our study area, we therefore make the assumption that only agricultural
 274 damages is of significant importance to the general landowner.

275
 276
 277 As crop damage on agricultural land takes place during the spring, summer and early fall, it is
 278 the pre-harvest population size that is relevant. Separate damage function for each animal
 279 category is specified. These cost functions may be written as $D_i(X_{i,t})$ ($i = c, yf, ym, f, m$),
 280 and where no animals means zero damage and more animals means more damage; that is,
 281 $D_i(0) = 0$ and $D_i' > 0$. Except for these two properties, we do not know very much about
 282 these damage cost functions (but see Wam and Hofstad, 2007 for an assessment of forest
 283 damage costs). They may be concave or convex, or partly concave and convex. For simplicity
 284 we use linear functions, i.e., $D_i(X_{i,t}) = d_i X_{i,t}$, and where d_i is the fixed per animal damage
 285 cost. The per animal damage varies for different categories of animals, and we assume
 286 $d_c < d_{yf} \leq d_{ym} < d_f < d_m$ (Table 2). The agricultural damage costs are thus:

$$287 \quad D_t = d_c X_{c,t} + d_{yf} X_{yf,t} + d_{ym} X_{ym,t} + d_f X_{f,t} + d_m X_{m,t}. \quad (10)$$

288 Therefore, the landowner yearly net benefit is described by:

$$289 \quad R_t = V_t + B_t - D_t \quad (11)$$

290

291 **Optimal management**

292 *Outcome based on hunting meat value benefit only*

293 We first study the situation where the landowner maximizes the hunting benefit meat value
 294 only. This means that the recreation aspects of the hunting, as well as the damage costs, are
 295 not taken into account when optimizing the harvesting strategy. The goal is thus to find
 296 harvest rates, and hence hunting quotas, that maximizes the present-value

¹ The recreation value z is here assumed fixed. In reality it is likely to decrease with effort use, at least above a certain level, making B_t a concave function of the number of hunting days. However, to say more about this, the number of hunters has to be included in the analysis which is beyond the scope of our study.

297 $\max_{h_{c,t}, h_{yf,t}, h_{ym,t}, h_{f,t}, h_{m,t}} \sum_{t=0}^{t=\infty} \rho^t V_t$ subject to the biological growth equations (1), (2), (4) and (7).

298 $\rho = 1/(1 + \delta)$ is the discount factor with $\delta \geq 0$ as the (yearly) fixed discount rate.

299
300 Due to the common code of conduct among hunters to avoid shooting the mother from calves,
301 a female – calf harvest restriction is included (Olaussen and Skonhøft, 2011). This restriction
302 is included simply as:

$$303 \quad h_{f,t} X_{f,t} \leq h_{c,t} X_{c,t}. \quad (12)$$

304 In addition, a restriction on the number of harvested adult males is incorporated to avoid a too
305 skewed sex ratio. One way to include this restriction is simply to impose an upper limit on the
306 adult male harvesting fraction. Such male harvesting restriction may be strengthened if we
307 additionally also impose a similar restriction on the male yearling hunting. This is what is
308 actually done and we hence include:

$$309 \quad h_{m,t} \leq \bar{h}_m \text{ and } h_{ym,t} \leq \bar{h}_m. \quad (13)$$

310
311 In the Supplement section the first order necessary control conditions are stated and it is
312 shown that it is possible to say something about the basic driving forces steering the harvest
313 composition. We now discuss these driving forces, first by disregarding the code of conduct
314 constraints (12) and (13).

315
316 For both sexes, we find that simultaneous harvest of both yearlings and adults does not satisfy
317 the optimality criteria. Next, it can be shown that positive female yearling harvest together
318 with zero adult female harvest implies $p_{yf} / s_{yf} > p_f / s_f$; that is, the ‘biological discounted’
319 female yearling hunting value (in NOK per animal) exceeds the ‘biological discounted’ adult
320 female hunting value. Because the adult category is more valuable than the yearling category,
321 $p_f > p_{yf}$, and the survival rates s_{yf} and s_y do not differ too much (Tables 1 and 2, numerical
322 section), this inequality represents a contradiction. In the same manner, we also find that
323 positive male yearling harvest together with zero adult male harvest implies $p_{ym} / s_{ym} > p_m / s_m$
324 which also represent a contradiction within the range of actual parameter values. Therefore,
325 positive female yearling harvest and zero female adult harvesting and positive male yearling
326 harvest and zero adult male harvesting are not possible options when the code of conduct
327 constraints (12) and (13) are not binding and when the harvest recreation value is small and
328 hence neglected.

329
330 These results show that the marginal hunting benefit (in NOK per animal) and the survival
331 rates are instrumental in determining the optimal harvest composition (see also Olaussen and
332 Skonhøft, 2011; Reed, 1980). Since, the ‘biological discounted’ value for the calf category is
333 lower than for the yearlings, $p_c / s_c < p_i / s_i$ ($i = yf, ym$) (see Table 1 and 2), we may also
334 suspect that calf harvest together with zero adult harvest will not represent an optimal
335 harvesting policy when the code of conduct constraint (12) is not binding. It is not possible to
336 prove this by only using the first order necessary control conditions, but numerical solutions
337 demonstrate that this will be the case for realistic parameter values.

338
339 On the other hand, when the female – calf constraint (12) and the male harvesting constraint
340 (13) still do not bind, we find that zero yearling harvest together with positive harvest of one,
341 or both, of the adult categories do not contradict the optimality conditions. The reason for this
342 outcome is simply that the per animal values are higher for the adults than for yearlings while

343 the survival rates do not differ too much. However, based on the optimality conditions it is
 344 difficult to say something definite about the harvest composition among the adults. There are
 345 two effects involved here. First, a direct economic effect working through the relative hunting
 346 value of females and males, and where males are more valuable due to a higher net per animal
 347 hunting value. Second, only the females contribute to reproduction (under our assumption of a
 348 not too skewed sex ratio). This last effect may also indirectly (because fertility is not included
 349 in the control conditions; see Supplement section) pull in the direction of a higher harvesting
 350 pressure on males than females.

351
 352 However, if both adult females and males are harvested in the unbinding case, this result may
 353 change when the restrictions (12) and (13) are included. First suppose that restriction (12) is
 354 included, while we still neglect restriction (13) and the harvest recreation value. If females are
 355 harvested, the code of conduct restriction (12) indicates that at least the same number of
 356 calves must be harvested. Since the restriction is unbinding if more calves than females are
 357 harvested, we already know from the unbinding case that no calves should be harvested.
 358 Therefore, harvesting more calves than females represents a contradiction both in the
 359 unbinding as well as the binding case. Hence, the restriction must bind with the same positive
 360 number of calves and females harvested, or zero harvest of both. When constraint (12) binds,
 361 it will certainly also influence the various possibilities of yearling harvest together with adult
 362 female and male harvest. We may hence find that yearling harvest together with male harvest
 363 occurs in the optimal solution. When condition (13) binds and $h_{m,t} = h_{ym,t} = \bar{h}_m$, this will
 364 generally also influence the various optimal harvest options. Both yearling female, calf and
 365 adult female harvest may be affected due to less yearling and adult male harvest. The intuition
 366 is straightforward as e.g., calf harvest is an indirect way of controlling the yearling category
 367 when the yearling male harvest is restricted. Hence, even if calf harvest is not an optimal
 368 option in the unrestricted case, it may very well be optimal in the restricted case.

369
 370 Summing up, we have demonstrated that differences in the ‘biological discounted’ value of
 371 various animal categories are the basic driving forces behind the optimal harvest composition.
 372 Fertility plays no direct role. However, certain modifying and complicating factors related to
 373 the common code of conduct constraint (12) and male harvest constraint (13) are present.
 374 These constraints also complicates our understanding of how the harvest composition and the
 375 populations may evolve over time; that is, the dynamic properties of the model. However,
 376 because of the strong degree of linearity in the model together with density dependent
 377 regulation through the recruitment function, we may suspect that oscillations will not be
 378 present and that the harvesting composition will approach a stable equilibrium.

380 *Crop damage taken into account*

381 We then look at the problem when also the damage costs are taken into account, but the
 382 recreational hunting value is still not included. This damage cost component is often
 383 inadequately compensated in real life management situations. One important reason for this is
 384 that animal migration pattern often generates mismatch between landowner hunting benefits
 385 and damage costs (Skonhøft and Olaussen, 2005). Therefore, the problem now is to find the

386 harvesting strategy that $\max_{h_{c,t}, h_{yf,t}, h_{ym,t}, h_{f,t}, h_{m,t}} \sum_{t=0}^{\infty} \rho^t (V_t - D_t)$, again subject to the biological growth
 387 equations (1), (2), (4) and (7) and the code of conduct constraints (12) and (13).

388
 389 Because the damage in year t takes place *before* the harvest (see Eq. 10), the first order
 390 necessary control conditions will be identical to the above problem of maximizing the hunting

391 benefit only (see Supplement section). Therefore, the equations determining the structure of
 392 the harvesting composition remain unchanged. On the other hand, we find that a new term
 393 reflecting the fixed marginal damage costs will be included in all the first order necessary
 394 portfolio conditions. The harvest of the different stages and the harvest composition will thus
 395 generally change. One reason for this is that the shadow prices of the various restrictions,
 396 including the female–calf harvest constraint, change. Just as in biomass models (e.g., Clark,
 397 1990), we may also suspect that the harvesting pressure will be higher and the number of
 398 animals lower when damage costs are included (see e.g., Skonhøft and Olausson, 2005). This
 399 is confirmed by the numerical results.

400
 401 *Recreational value included*

402 When taking the damage costs into account, the current net benefit is defined by equation

403 (11), and the problem is hence to $\max_{h_{c,t}, h_{yf,t}, h_{ym,t}, h_{f,t}, h_{m,t}} \sum_{t=0}^{t=\infty} \rho^t (V_t + B_t - D_t)$, subject to the same

404 constraints as above. The solution to this problem gives the ‘overall’ optimal solution for the
 405 landowners. In this case, however, we find it difficult to draw any analytical conclusions
 406 about the harvest composition because the population sizes $X_{i,t}$ ($i = c, yf, ym, f, m$) now are
 407 included in the control conditions such that the number of animals directly influence which of
 408 the different categories of animals it is beneficial to harvest (see Supplement section).

409
 410 Therefore, not only differences in per animal values and survival rates (‘biological
 411 discounted’ values) are instrumental in determining the harvest composition when the
 412 recreational aspect of the hunting is included, but also the demographic composition of the
 413 population. This effect may of course be of particular importance if the recreation value is
 414 significant; that is, the value per hunting day z is ‘high’. We may also suspect that the
 415 harvesting composition is more prone to changes over time because the strong degree of
 416 linearity in the model is no longer present. Generally, we find the dynamics now to be
 417 strikingly different. See Results section.

418
 419

420 **Results**

421 *Data and specific functional forms*

422 The fertility rate (see Methods and material section) is expressed as a sigmoid function
 423 predicting increased degree of density dependence with higher population densities (Langvatn
 424 et al., 2004). The function reads:

$$425 \quad r_{i,t} = r_i \left(X_{yf,t} + X_{f,t} \right) = \frac{\tilde{r}_i}{1 + \left[\left(X_{yf,t} + X_{f,t} \right) / K \right]^{b_i}}; \quad (i = yf, f) \quad (14)$$

426 with $\hat{r}_i > 0$ as the intrinsic growth rate (maximum number of calves per female) and $K > 0$ as
 427 the female population level for which density-dependent fertility is equal to density-
 428 independent fertility (female ‘carrying capacity’). Thus, for a population level above K
 429 density-dependent factors dominate. This parameter scales the population sizes, and its value
 430 is contingent upon factors such as the size of the area, the biological productivity of the
 431 grazing and browsing resources, and so forth. The compensation parameter $b_i > 0$ indicates to
 432 what extent density-independent effects compensate for changes in the population size.
 433 Equation (14) implies a recruitment function (1) as

434 $X_{c,t+1} = \tilde{r}_y X_{y,t} / \left\{ 1 + \left[(X_{y,t} + X_{f,t}) / K \right]^{b_y} \right\} + \tilde{r}_f X_{f,t} / \left\{ 1 + \left[(X_{y,t} + X_{f,t}) / K \right]^{b_f} \right\}$ and is of the
 435 so-called Shepherd type.
 436

437 Table 1 gives the baseline parameter values for this function as well as the other biological
 438 parameter values. The parameter K is scaled for our study area (Methods and material
 439 section) and set to 4,000. The survival rates are from a population in the northern margin of
 440 the red deer range (Langvatn and Loison, 1999), and might underestimate survival further
 441 south. The fertility rates are based on ovulation rates (Langvatn et al., 2004), and may thus
 442 overestimate recruitment. However, for our purpose, these potential biases are likely to be
 443 unimportant.
 444

445 The baseline economic parameter values are given in Table 2. The value of the catchability
 446 coefficient θ is calibrated based on the assumption that three hunting days are needed to
 447 shoot one animal when the population size is about 3,000 animals; that is catch per unit effort
 448 ($CPUE$) is 0.33 (animal/day). With $H_{i,t} = \theta N_{i,t} X_{i,t}$ (Methods and material section), or $CPUE$
 449 as $H_{i,t} / N_{i,t} = \theta X_{i,t} = 0.33$ and $X_{i,t} = 3,000$, we then have $\theta \approx 0.0001$ (1/hunting day). The per
 450 day recreational hunting value $z = 50$ (NOK/day) is based on studies from the actual area
 451 (Thorvaldsen et al., 2010). The crop damage cost parameters are taken from Olausen and
 452 Mysterud (unpublished), and range from 175 (NOK/calf) to 532 (NOK/adult male). The meat
 453 value \hat{p} is assumed similar among all categories of animals, and is set to 75 (NOK/kg) (see
 454 also Methods and material section). The per animal hunting fee is 210 (NOK/calf) and 350
 455 (NOK/yearling and adult). We assume a zero discount rate in the baseline scenario. As is well
 456 known (e.g., Clark, 1990), this means that the (possible) steady state of the dynamic
 457 optimization problem coincides with the solution to the problem of maximizing current net
 458 benefit in biological equilibrium. The numerical results are first presented when only meat
 459 value and damage costs are included. Then we present results when also the recreation value
 460 of the hunting is taken into account.
 461

462 *Results when recreation value is not included*

463 We first present the basic dynamic harvesting results². As already indicated, because of the
 464 strong degree of linearity in the model together with density dependent regulation through the
 465 recruitment function, the model solution approaches a stable equilibrium. As the profit
 466 function is linear in harvest, economic theory suggests that harvest should be adjusted such as
 467 to lead the populations to their optimal steady states as rapidly as possible – if no constraints
 468 are binding (for proof of the Most Rapid Approach Path theorem, see Spence and Starrett,
 469 1975). Hence, populations that are below their optimal steady state will not be harvested until
 470 they have reached their optimal size, and populations above their steady states will be
 471 harvested down to the optimal level within the first year.
 472

473 Figure 3 demonstrates the optimal harvest rate dynamics with the baseline parameter values,
 474 and when both meat value and grazing damage are included (Case 1 in Table 3 below). The
 475 results here seem to partly confirm the most rapid approach principle. Starting with
 476 populations equal to 300 for all categories, calves and adult females are not harvested until
 477 they have grown to their optimal population sizes. Adult males, on the other hand, are
 478 harvested at the maximum rate even though the population is below its optimal steady state.
 479 This occurs because constraint (13) binds for adult males in the steady state, so postponing

² The dynamic optimization was performed with the KNITRO for MATLAB solver from Ziena Optimization, with MATLAB release 2011b.

480 harvest until the optimal population size is reached would lead to overshooting. The reason
 481 why we find that calves are harvested in the steady state, despite the fact calves have a lower
 482 'biologically discounted' value than the other age classes, is that constraint (12) binds in the
 483 steady state, meaning that some calves must be harvested to allow any harvest of adult
 484 females.

485
 486 Figure 4 demonstrates the effect of changing the discount rate on the time path and steady
 487 state of the total population size (with all categories taken together), and where the baseline
 488 scenario (Case 1) is included with zero discount rate (upper curve). We find, not surprisingly,
 489 that increasing the discount rate results in progressively smaller populations, with
 490 correspondingly higher harvest rates, while the dynamics do not change qualitatively. The
 491 effects of other initial population values were examined as well without changing the
 492 qualitative structure of the dynamics; that is, the transitional dynamics were following the
 493 most rapid approach principle. Neither was other steady state values found ('ergodic'
 494 dynamics).

495
 496 Tables 3 and 4 report the detailed steady state results for different configurations of parameter
 497 values and constraints. Table 3 shows the optimal harvest ratios and population sizes for all
 498 stages, along with total harvest and number of animals. Table 4 indicates the economic
 499 results.

500
 501 Again, Case 1 describes the outcome under the baseline parameter values, and with all
 502 constraints included. From Table 3 we see that 20 % of the calves are harvested, because the
 503 female-calf harvesting constraint (12) is binding, and the harvest rate of adult females is 0.15.
 504 The adult male harvest constraint (13) also binds with $h_m = 0.55$. In addition, in Case 1 as in
 505 all cases in Table 3 except from Case 5, we find that yearlings of both sexes are not harvested.
 506 This is because the 'biologically discounted' values are lower for yearlings than for adults.
 507 From Table 4, we find the total yearly meat value to be 4,071 (1,000 NOK), with grazing
 508 damage costs being 1,843. Net average yearly income per hunted animal is 2.102 (1,000
 509 NOK), while net income per living animal is 403 (NOK).

510
 511 The recruitment function (1) assumes that males (adults as well as yearlings) represent no
 512 constraint in relation to female reproduction rates. Still, a harvest restriction on both yearlings
 513 and adult males is imposed to avoid a too skewed sex ratio. In the baseline Case 1, we find the
 514 adult male/female sex ratio to be $740 / 2,158 \approx 0.34$ (Table 3). In this case, as well as in the
 515 other cases, we find identical numbers of yearling males and yearling females in the pre-
 516 harvest population. This is simply a consequence of equal sex ratio in the recruitment, $\psi = 0.5$
 517 , equal harvesting rates for male and female calves, and identical survival rates for both sexes
 518 through their first winter (see Methods and material section and Table 1).

519
 520 Case 2 describes what happens when the meat value is increased with 50 % while all other
 521 parameter values are kept constant. This leads to a slight adjustment in the harvest ratios for
 522 calves and adult females, but probably more important, to higher standing populations (Table
 523 3). This is not generally obvious as standard biomass harvesting theory (e.g., Clark, 1990)
 524 predicts that populations should be kept at a level where the animal biomass growth rate
 525 equals the discount rate when meat income only matters and there are no population
 526 dependent costs; a criterion that is independent of the harvesting value. When a population
 527 effect, such as the grazing damage, is included in the benefit function, however, the meat
 528 price will influence the optimal population sizes. Obviously, the net economic value per
 529 animal both when hunted and when alive (as 'capital') increases when the meat value shifts

530 up. For a given production level, or harvest activity in our case, a price increase leads to an
 531 equal percentage increase in income. When costs are unchanged, the percentage increase in
 532 profits – income minus costs - will always be higher. In addition, the harvest activity will be
 533 adjusted to take further advantage of the price change. Following the 50 % price increase, we
 534 hence find a doubling of total net benefit (Table 4).
 535

536 In Case 3 the grazing damage D_t in equation (10) is ignored; that is, the landowners optimize
 537 without taking the grazing damage into account. Optimal population sizes are affected as the
 538 cost of keeping the population, as viewed by the landowner, reduces. This leads to a change in
 539 harvest pattern and population size that works in the same direction as the price increase, and
 540 in both cases the net value per living animal (as ‘capital’) goes up. The overall population size
 541 now increases with more than 20 % compared to Case 1. The population increase is
 542 particularly strong for adult females, and harvest ratios are correspondingly lower. The
 543 implied economic effect, on the other hand, goes in the opposite direction as the *actual*
 544 grazing damage will be larger, and amounts to a 10 % decrease in total net benefit (Table 4).
 545 In Cases 2 and 3, populations are increased through lowering the harvest ratios of calves and
 546 adult females, which implies a decrease in the adult male/female sex ratios to
 547 $778/2,449 \approx 0.32$ and $830/3,042 \approx 0.27$, respectively.
 548

549 The female-calf constraint (12) is excluded in Case 4. As expected, there is no harvesting of
 550 calves in this scenario. The male harvesting constraint still binds with $h_m = 0.55$, and the
 551 adult female harvest rate increases to $h_f = 0.21$. Population sizes increase moderately, except
 552 for adult females. Total net benefit increases with about 14 %, from 2,228 (1,000 NOK) to
 553 2,537, and there is a significant increase in the net benefit per hunted animal as the average
 554 weight of a hunted animal is higher. Therefore, the cost of the code of conduct constraint (12)
 555 is quite significant. As only the stock of adult females reduces, we also find an increase in the
 556 adult male/female sex ratio to $916/2,097 \approx 0.44$.
 557

558 The contours in Figure 5 indicate the steady state net economic benefit ($V - D$) for different
 559 combinations of h_f and h_m , and demonstrates the sensitivity of the net economic benefit to
 560 changes in the adult harvesting rates, when all the other harvesting rates are kept constant at
 561 their optimal levels. Panel a) is for the baseline Case 1, while panel b) is for Case 4 (without
 562 the cow-calf constraint). When the female-calf harvesting constraint is included as in Case 1,
 563 the maximum possible harvest ratio of adult females is around 0.15, as a higher harvest ratio
 564 implies a violation of constraint (12) when the harvest rate of calves is kept constant. In Case
 565 4 however, the maximum sustainable harvest rate of adult females is around 0.4, which is
 566 equal to the recruitment rate of adult females when no calves or female yearlings are
 567 harvested.
 568

569 From panel a) it is seen that combinations of the male harvesting rates within the range 0.40 –
 570 0.55 and female rates within the range 0.15 – 0.10 yields a net benefit of about 2,000 (1,000
 571 NOK) which implies an approximate 10 % reduction in net benefit compared to the optimal
 572 steady state (see also Table 4). This indicates that the profit loss is modest within a quite wide
 573 range of harvesting combinations. In panel b), where the code of conduct constraint (12) is
 574 omitted, net benefit is even less sensitive to adjustments in the adult harvest ratios. The
 575 inclusion of the zero profit contour line indicates when profit turns negative because of high
 576 grazing damage, and occurs when harvest rates become low and populations become large. In
 577 Case 4, net benefits decrease to around -7 mill. NOK for zero harvest of adults (while still

578 keeping other harvest rates constant at their optimal levels). This dramatic impact on profits is
 579 to a large extent due to grazing damage being linearly related to population size, an
 580 assumption that is possibly unrealistic for large populations.

581

582 *Uniform harvesting pattern*

583 One interesting aspect of stage structured harvesting is to what extent the complexity involved
 584 in this kind of management practice pays its way through a significantly higher economic
 585 benefit yield than a *uniform* harvest rate across all stages. Such uniform hunting pattern
 586 implies an exploitation scheme where the present value benefit is maximized under the same
 587 biological constraints as above (but not constraints 12 and 13), but with the additional
 588 constraint of equal harvest rates across *all* age classes imposed. Results from this uniform
 589 harvesting scheme are reported as Case 5 in Table 3 and 4. As indicated here, uniform harvest
 590 implies a highly inefficient exploitation pattern, where the net benefit is reduced with about
 591 50 % compared to the baseline Case 1. Population sizes are significantly reduced, except for
 592 adult males, due to excess harvesting of younger age classes. The positive effect on grazing
 593 damage of smaller populations is also somewhat counteracted by the fact that the younger
 594 stages are the ones that impose the least grazing damage. The implied inefficient composition
 595 of age categories results in substantially reduced net benefit both per hunted animal and per
 596 living animal. The male–female ratio also changes significantly, and is in this Case 5 equal to
 597 $1,102/1,165 = 0.95$. The reason why we do not find the exact same number of adult females
 598 and males is because of the survival rates among female and male yearlings are different
 599 (Table 1).

600

601

602 *Also including the recreational aspect*

603 So far, the hunting recreational value has not been included in the numerical analysis. As
 604 mentioned above (Optimal management section) this adjustment complicates the solution
 605 considerably. The long term optimal solution may no longer be a steady state, but perpetual
 606 oscillations between more profitable, but unsustainable, outcomes. Figure 6 shows that, under
 607 the same assumptions as the Case 1 scenario and with a recreational value of $z = 50$
 608 (NOK/day) included, oscillating harvest rates for the calves and the female categories emerge.
 609 The harvest rates of both adult and yearling males become constant and equal to their upper
 610 constraint levels, whereas the harvest rates of calves, yearling and adult females oscillates
 611 between zero and 0.45, 0.3 and 1, respectively. As indicated above (Optimal management
 612 section), cycles may evolve when the strong degree of linearity, or concavity, in the model is
 613 no longer present. Once the non-concavity is introduced by the recreational value, the
 614 possibility for oscillations depends on combinations of all parameters in the model. Generally,
 615 it seems that this non-concavity effect is stronger for smaller populations, so that everything
 616 that leads to reduced populations will increase the possibility of oscillations, once the
 617 recreational value is incorporated into the analysis. This includes higher discount rates, higher
 618 grazing damage, lower meat price and also the magnitude of the recreational value itself. For
 619 baseline parameter values, pulse harvesting occurs only if the recreational parameter z value
 620 exceeds a certain critical value ($z = 25$).

621

622 **Concluding Remarks**

623 In this paper we have analyzed the cost and benefit of a red deer population within a five
 624 stage model with density-dependent fertility and density independent mortality, and where the
 625 cost and benefit functions are approximated by linear functions. This population is managed
 626 by a well-defined manager, typically consisting of many landowners operating in a
 627 cooperative manner, with the goal of maximizing the present-value hunting related income
 628 while taking grazing damages into account. The different ways to compose the harvest across

629 age and sex categories and the implied economic benefits are highlighted. Without any
630 restrictions on the harvest, and ignoring the recreation aspect of the hunting, we find the
631 optimal harvest composition to be determined basically by the same factors as in Reed (1980)
632 - such that differences in the 'biological discounted' values are instrumental in determining
633 the optimal harvesting composition. The meat value – survival ratio comprises the 'biological
634 discounted' values. Therefore, it may be seen as a marginal gain – cost ratio, where the
635 marginal gain is given by the by the meat value increase from a younger to an older animal
636 category while the marginal cost as reduced survival.
637

638 The Results section illustrates the predictions from the theoretical model. In the baseline case
639 (Case 1) where the meat value and the grazing damage, but not the recreational value, are
640 included, we find that all stages except yearlings are harvested. The females and calves are
641 harvested only moderately, and calves are harvested due to the imposed code of conduct
642 restriction (12). We also find that the same pattern persists when the meat price goes up (Case
643 2) and when grazing damage costs are ignored (Case 3), but the total population becomes
644 substantially higher in both cases. However, the implied economic effect differs between the
645 two cases, and yields a 100 % increase and 10 % decrease in total net benefit respectively.
646 When the code of conduct constraint (12) was excluded (Case 4), we found that no calves
647 were harvested, and the harvest of adult females increased to 21 %. Net benefit increased by
648 14 %.
649

650 The landowner net benefit was also calculated when the optimal selective harvesting patterns
651 were replaced by 'optimal' uniform harvest rates (Case 5) as considered in the traditional
652 bioeconomic models ('an animal is an animal'). We found that such a uniform harvesting
653 pattern typically reduced the economic benefit by about 50 % compared to the optimal
654 selective scheme. This clearly underlines the importance of using stage dependent models
655 when populations can be structured into sex and age categories that significantly differentiate
656 either in reproductive value or in economic return, and when perfect selectivity in harvest is
657 possible.
658

659 When the recreational hunting value was included in the objective function, we found that the
660 optimal harvesting scheme involved oscillating harvest rates. This result is not a new one
661 within the resource management literature; it has been shown to exist in various models (Wirl,
662 1995), and in particular when there is imperfect selectivity in the harvest (Tahvonen, 2009).
663 In our case however, there are reasons to be cautious when interpreting this result, as the
664 formulation of the assumption of constant recreational value per hunting day may be
665 questionable. In any case, it is doubtful whether a landowner would consider a pulse
666 harvesting regime to be 'optimal' in a context such as this, not the least because of the
667 difficulties involved with implementing such a management programme in practice.
668

669 Our results may be compared to the current red deer harvesting practices. The present
670 harvesting regimes of red deer in Norway varies a great deal as a consequence of variation in
671 local population density, demography and red deer related conflicts. On the other hand, a
672 general pattern is that calves and yearlings are rather extensively harvested, and this is also so
673 in our study area. As the traditional management practice in general has been oriented towards
674 stimulating population increase, males have also been subject to a more extensive harvest
675 compared to females. This male-biased harvest has resulted in female-biased populations with
676 a high growth potential. Based on extensive data series from culled animals, population size
677 and demographic composition have been reconstructed through cohort analyses for three
678 regions in Norway (Veiberg et al., 2010). The adult sex ratio varied between 3-3.7 females

679 per male. The harvest rates for adult females were rather moderate and stable through all of
 680 their lifespan (0.13-0.16). In contrast, the harvest rates of adult males were significantly
 681 higher (0.40-0.49) and varied considerably with age. The empiric harvest rates for adult
 682 females are quite in line with the steady state alternatives described for Case 1-3. These three
 683 cases also represent the most realistic harvest scenarios given the present code of conduct and
 684 the Norwegian hunting tradition. The experienced adult sex ratio was also quite equal to the
 685 steady state sex ratios in Case 1-3. A striking difference between the current hunting practice
 686 and all the steady state alternatives, apart from Case 5, is that yearling harvest represents a
 687 significant proportion of the total red deer harvest. On the national level yearlings in general
 688 represent 30 % of the total red deer harvest (Statistics Norway, 2010), and the sex specific
 689 harvest rates were found to vary between 0.16-0.26 for yearling females and 0.40-0.49 for
 690 yearling males (Veiberg et al., 2010). This indicates that there is scope for improving the
 691 economics of the present hunting practice by lowering the yearling harvest.

692 Defined management aims and basic knowledge about the productivity and the demographic
 693 structure of the local population is important to accomplish a given management practice.
 694 Still, considerations regarding practical feasibility and necessary harmonization with common
 695 codes of conduct represent important limitations. In addition, landowners and hunters share a
 696 common interest of securing predictable future population development. Management
 697 practices that involve regular alterations of the harvest pattern (see Figure 6) are therefore as
 698 indicated less likely to be implemented than more static practices. Apart from this, all the
 699 presented case studies represent highly feasible management practices.

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801 **Table 1.** Baseline biological parameter values.

Parameters	Description	Value	Reference/source
\tilde{r}_{yf}	maximum specific growth rate yearlings	0.6	Langvatn et al. (2004)
\tilde{r}_f	maximum specific growth rate adults	0.96	Langvatn et al. (2004)
K	carrying capacity females	4000 (animals)	Calibrated
b_{yf}	density compensation parameter yearlings	2	Nilsen et al. (2005)
b_f	density compensation parameter adults	2	Nilsen et al. (2005)
w_c	mean weight calves	25.5 (kg)	Solberg et al. (2010)
w_{yf}	mean weight female yearlings	43.1 (kg)	Solberg et al. (2010)
w_{ym}	mean weight male yearlings	49.4 (kg)	Solberg et al. (2010)
w_f	mean weight females	57.3 (kg)	Solberg et al. (2010)
w_m	mean weight male	77.6 (kg)	Solberg et al. (2010)
s_c	survival rate calves	0.75	Langvatn and Loison (1999)
s_{yf}	survival rate female yearlings	0.92	Langvatn and Loison (1999)
s_{ym}	survival rate male yearlings	0.87	Langvatn and Loison (1999)
s_f	survival rate adult females	0.93	Langvatn and Loison (1999)
s_m	survival rate adult males	0.93	Langvatn and Loison (1999)
ψ	calf sex ratio	0.5	

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803 **Table 2.** Baseline economic parameter values.
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Parameters	Description	Value	Reference/source
\hat{p}	meat price	75 (NOK/kg)	Olaussen and Mysterud (Unpublished)
q_c	hunting fee calf	210 (NOK/animal)	Directorate for Nature Management
q_{yf}	hunting fee yearling female	350 (NOK/animal)	Directorate for Nature Management
q_{ym}	hunting fee yearling male	350 (NOK/animal)	Directorate for Nature Management
q_f	hunting fee female	350 (NOK/animal)	Directorate for Nature Management
q_m	hunting fee male	350 (NOK/animal)	Directorate for Nature Management
θ	catchability coefficient	0.0001 (1/hunting day)	Calibrated
d_c	crop damage cost calves	175 (NOK/animal)	Olaussen and Mysterud (Unpublished)
d_{yf}	crop damage cost yearling female	295 (NOK/animal)	Olaussen and Mysterud (Unpublished)
d_{ym}	crop damage cost yearling male	338 (NOK/animal)	Olaussen and Mysterud (Unpublished)
d_f	crop damage cost adult female	393 (NOK/animal)	Olaussen and Mysterud (Unpublished)
d_m	crop damage cost adult male	532 (NOK/animal)	Olaussen and Mysterud (Unpublished)
z	recreational value	50 (NOK/day)	Thorvaldsen et al. (2010)
δ	discount rate	0	

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807 **Table 3.** Steady state harvest rates and populations. Recreational value not included. h_c , h_{yf} ,
 808 h_{ym} , h_f , and h_m are harvest rates for calves, female yearlings, male yearlings, adult females
 809 and adult males, respectively. H is total harvest (in number of animals) and X_c , X_{yf} , X_{ym} ,
 810 X_f , and X_m are population sizes (in number of animals) of calves, female yearlings, male
 811 yearlings, adult females and adult males, respectively. X is total population size.

	Case 1	Case 2	Case 3	Case 4	Case 5
h_c	0.20	0.19	0.18	0	0.18
h_{yf}	0	0	0	0	0.18
h_{ym}	0	0	0	0	0.18
h_f	0.15	0.13	0.11	0.21	0.18
h_m	0.55	0.55	0.55	0.55	0.18
H	1,060	1,088	1,096	951	732
X_c	1,651	1,717	1,799	1,632	1,165
X_{yf}	494	520	555	612	360
X_{ym}	494	520	555	612	360
X_f	2,158	2,449	3,042	2,097	1,165
X_m	740	778	830	916	1,102
X	5,532	5,984	6,780	5,869	4,151

812 Case 1: Baseline parameter values. Maximizing meat value minus grazing damage. Both constraints
 813 (12) and (13) included, and (13) with $\bar{h}_{ym} = \bar{h}_m = 0.55$.

814 Case 2: Meat price increased by 50 %

815 Case 3: Maximizing meat value only

816 Case 4: Excluding constraint (12)

817 Case 5: Uniform harvest pattern

818

819 **Table 4.** *Steady state landowner yearly cost and benefit (in 1,000 NOK).*
 820

	Case 1	Case 2	Case 3	Case 4	Case 5
Meat value (V)	4,071	6,477	4,303	4,521	2,620
Crop damage (D)	1,843	2,006	2,303	1,984	1,476
Net benefit ($R=V-D$)	2,228	4,472	2,000	2,537	1,144
Net benefit per hunted animal (R/H)	2.102	4.108	1.825	2.667	1.563
Net benefit per living animal (R/X)	0.403	0.747	0.295	0.432	0.276

821 Case 1: Baseline parameter values. Maximizing meat value minus grazing damage. Both constraints
 822 (12) and (13) included, and (13) with $\bar{h}_{ym} = \bar{h}_m = 0.55$.
 823 Case 2: Meat price increased by 50 %
 824 Case 3: Maximizing meat value only
 825 Case 4: Excluding constraint (12)
 826 Case 5: Uniform harvest pattern
 827

828

829 **Figure 1.** Number of harvested red deer per 10 km² in Norwegian municipalities in 1970
830 (left) and 2009 (right). The figure illustrates both the change in population density and
831 geographical distribution. Source: Statistics Norway.

832

833 **Figure 2.** The study area consisted of two municipalities, Flora (A) and Gloppen (B), situated
834 on the west coast of Norway.

835

836 **Figure 3.** Dynamic harvest pattern Case 1. Harvesting rates h and time (years). Harvest rates
837 of both male and female yearlings are zero all the time.

838 **Figure 4.** Dynamic harvest pattern and discount rate. Case 1 is for zero discount rate. Total
839 population size X and time (years).

840

841 **Figure 5.** Contour plots showing how different levels of landowner net benefit, meat hunting
842 value minus grazing damage ($V - D$) (1,000NOK) relates to variation in harvest rates of adult
843 males (h_m) and adult females (h_f). The left and right panel illustrates the conditions described
844 in Case 1 and Case 4 respectively. The optimal steady state is identified with dots in both
845 panels. The numbers along the level curves indicate net economic benefit (in 1,000 NOK),
846 and the distance between two adjacent curves is 1 mill. NOK. Note the different scaling of the
847 two panels.

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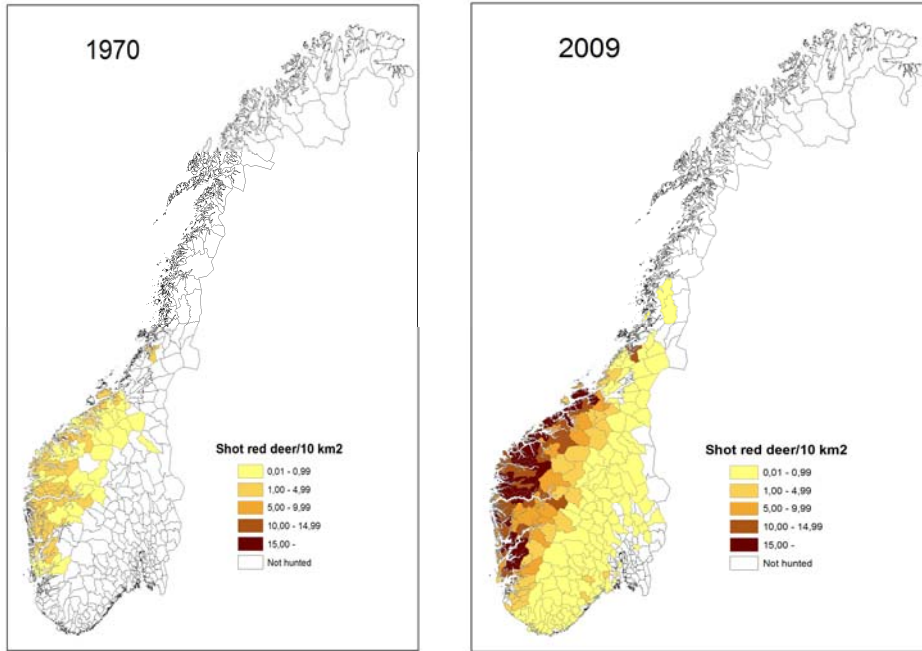
849 **Figure 6.** Dynamic harvest pattern when recreational value is included. $z = 50$ (NOK/hunting
850 day). Other parameter values and constraints as in Case 1. Harvesting rates and time (years).
851 Harvest rates of both adult males and yearlings are kept constant at 0.55.

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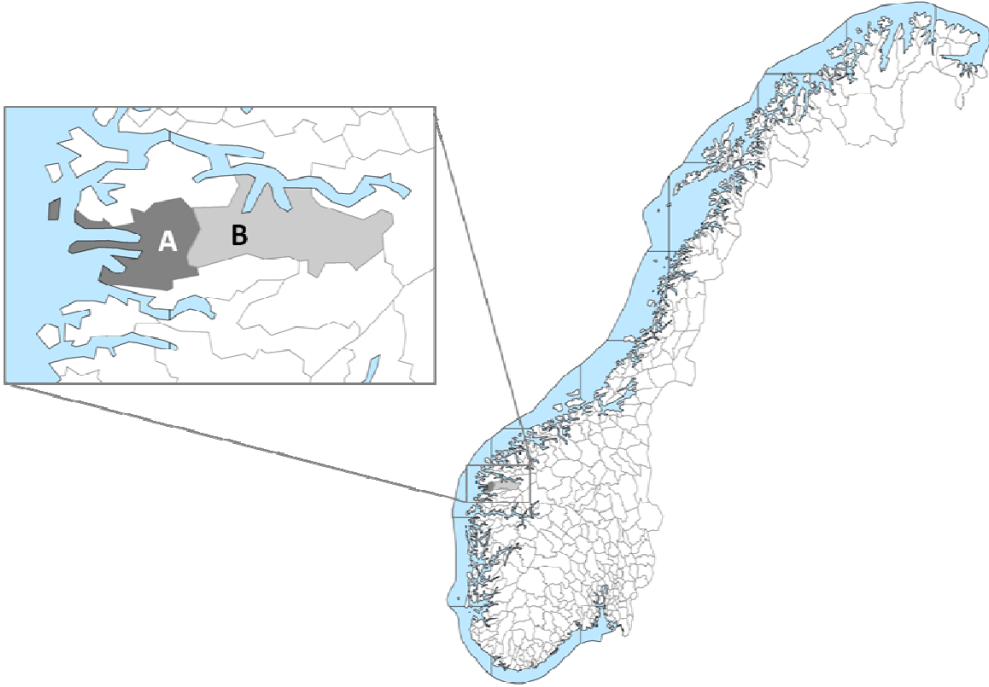
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857 **Figure 1**
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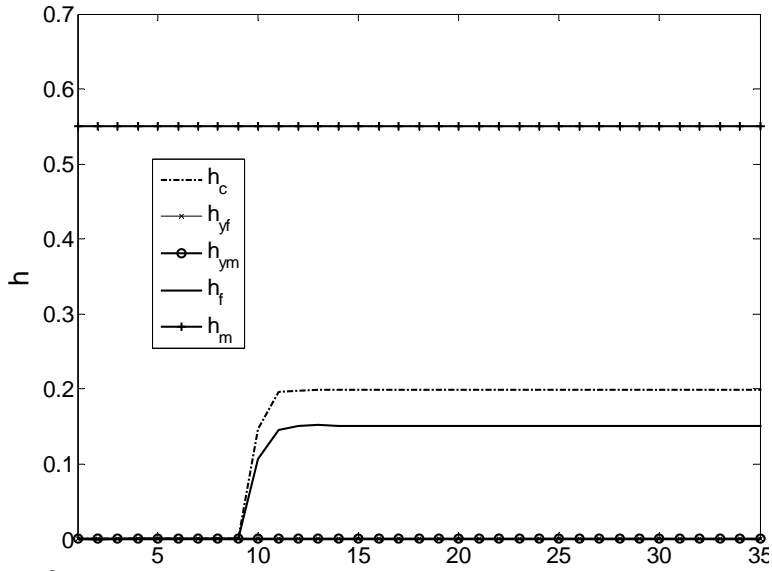
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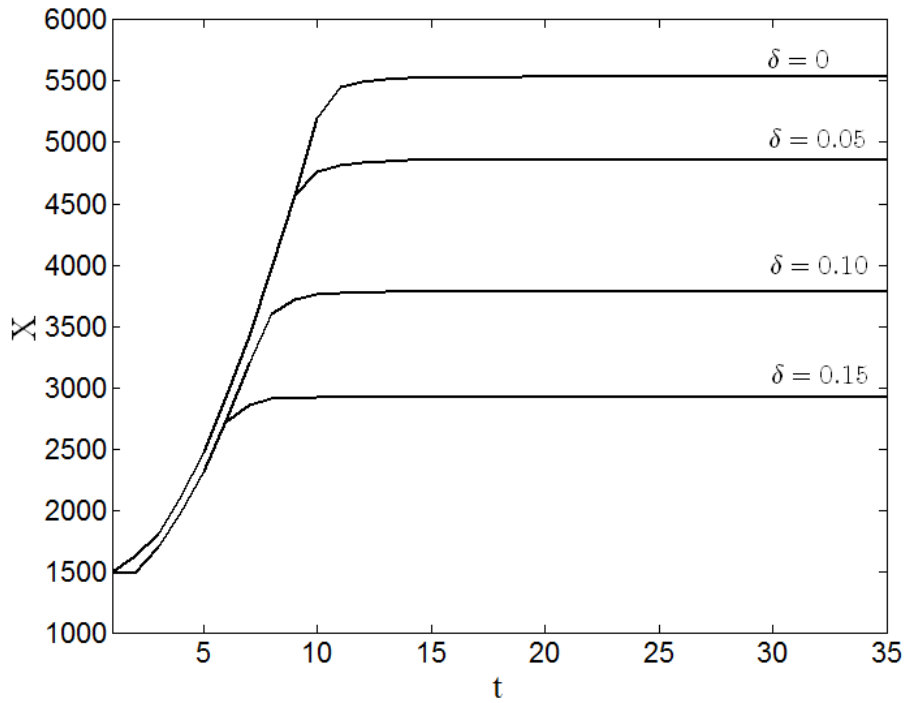
861 **Figure 2**

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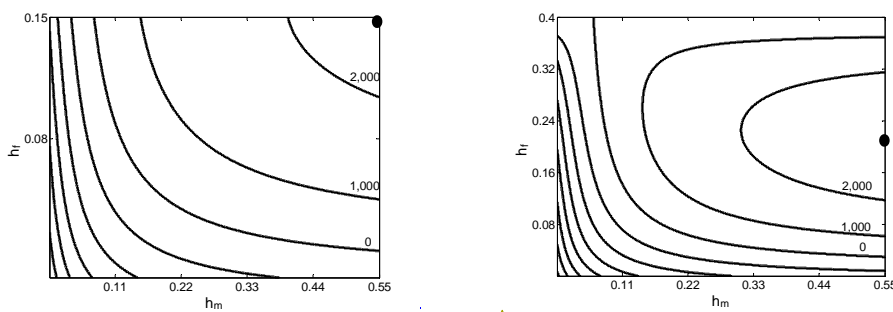


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Figure 3



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871 **Figure 4**
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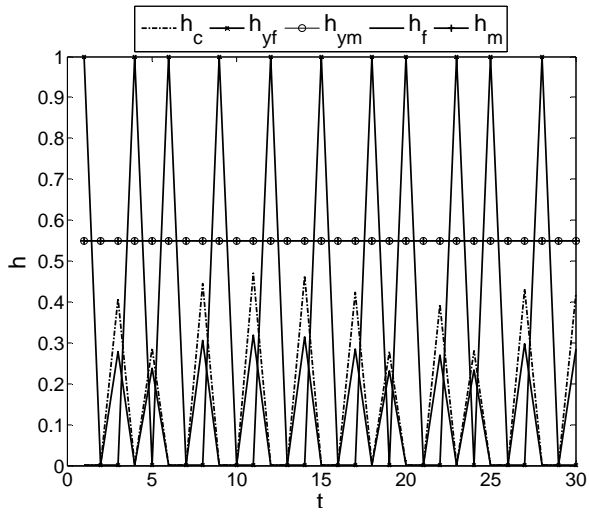
Case 1 Case 4

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880 **Figure 5**
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Figure 6

917 **Supplement**918 *The maximization problem*

919 The Lagrangian of the problem of maximizing the present value benefit when only meat
 920 hunting value is included given the biological constraints plus constraints (12) and (13) may
 921 be written as

$$\begin{aligned}
 L = \sum_{t=0}^{\infty} \rho^t \{ & p_c h_{c,t} X_{c,t} + [p_{yf} h_{yf,t} + p_{ym} h_{ym,t} (1-\psi) / \psi] X_{yf,t} + p_f h_{f,t} X_{f,t} + p_m h_{m,t} X_{m,t} \\
 & - \rho \gamma_{t+1} [X_{c,t+1} - r_{yf} (X_{yf,t} + X_{f,t}) X_{yf,t} - r_f (X_{f,t} + X_{yf,t}) X_{f,t}] \\
 & - \rho \eta_{t+1} [X_{yf,t+1} - \psi s_c (1-h_{c,t}) X_{c,t}] \\
 922 & - \rho \lambda_{t+1} [X_{f,t+1} - s_{yf} (1-h_{yf,t}) X_{yf,t} - s_f (1-h_{f,t}) X_{f,t}] \\
 & - \rho \mu_{t+1} [X_{m,t+1} - [s_{ym} (1-h_{ym,t}) (1-\psi) / \psi] X_{yf,t} - s_m (1-h_{m,t}) X_{m,t}] \\
 & - \rho \omega_{t+1} (h_{f,t} X_{f,t} - h_{c,t} X_{c,t}) - \rho \zeta_{t+1} (h_{m,t} - \bar{h}_m) - \rho \chi_{t+1} (h_{ym,t} - \bar{h}_m) \}
 \end{aligned}$$

923 $\gamma_t > 0, \eta_t > 0, \lambda_t > 0, \mu_t > 0$ are the shadow prices of the calf population constraint (1),
 924 female yearling constraint (2), adult female constraint (4) and adult male population
 925 constraint (7), respectively. $\omega_t \geq 0$ is the shadow price of the female – calf harvesting
 926 restriction (12), and $\zeta_t \geq 0$ and $\chi_t > 0$ are the shadow prices of the adult male and male
 927 yearlings harvesting restrictions (13). Notice that the population constraint shadow prices, in
 928 contrast to the two harvesting constraints (12) and (13), always will be strictly positive
 929 because these constraints always bind (Kuhn-Tucker theorem).

930
 931 The first-order necessary control conditions of this maximizing problem are stated with the
 932 actual complementary slackness conditions and where the possibility for keeping each of the
 933 stages unexploited is considered while harvesting whole subpopulations are not considered as
 934 a possible option. These control conditions with $X_{i,t} > 0$ ($i = c, yf, ym, f, m$) read then:

$$935 \quad (A1) \quad \frac{\partial L}{\partial h_{c,t}} = X_{c,t} (p_c - \rho \eta_{t+1} \psi s_c + \rho \omega_{t+1}) \leq 0 ; 0 \leq h_{c,t} < 1,$$

$$936 \quad (A2) \quad \frac{\partial L}{\partial h_{yf,t}} = X_{yf,t} (p_{yf} - \rho \lambda_{t+1} s_{yf}) \leq 0 ; 0 \leq h_{yf,t} < 1,$$

$$937 \quad (A3) \quad \frac{\partial L}{\partial h_{ym,t}} = X_{ym,t} (p_{ym} - \rho \mu_{t+1} s_{ym}) \leq 0 ; 0 \leq h_{ym,t} < 1,$$

$$938 \quad (A4) \quad \frac{\partial L}{\partial h_{f,t}} = X_{f,t} (p_f - \rho \lambda_{t+1} s_f - \rho \omega_{t+1}) \leq 0 ; 0 \leq h_{f,t} < 1$$

939 and

$$940 \quad (A5) \quad \frac{\partial L}{\partial h_{m,t}} = X_{m,t} (p_m - \rho \mu_{t+1} s_m - \rho \zeta_{t+1} / X_{m,t}) \leq 0 ; 0 \leq h_{m,t} < 1.$$

941 The portfolio conditions $\partial L / \partial X_{i,t} = 0$ ($i = c, yf, f, m$) are rather messy and are not stated
 942 here.

943
 944 Condition (A1) says that calf harvest should take up to the point where marginal harvest value
 945 is equal to, or below, the cost in term of reduced yearling growth of both categories evaluated

946 at their shadow prices while also taking the discounting into account. The female - calf
 947 harvesting restriction shadow price is also included in this condition. When this condition
 948 holds as an inequality, the marginal benefit is below its marginal cost and harvesting of this
 949 stage is thus not profitable, $h_{c,t} = 0$. In a similar manner, condition (A2) and (A3) indicate
 950 that harvesting of the yearling female (male) category should take place up to the point where
 951 the marginal benefit is equal to, or below, the cost in terms of reduced population of adult
 952 females (males). The female and male adult conditions (A4) and (A5), respectively, can be
 953 given a similar interpretations, but the shadow cost of the female - calf harvesting constraint is
 954 included in the female condition (A4) while the shadow cost of the male harvesting rate is
 955 included in condition (A5).

956 These first order control conditions together the portfolio conditions and the biological
 957 constraints comprise a complex dynamic system. It is therefore difficult, if not impossible, to
 958 assess the dynamics analytically. However, based on the control conditions, it is possible to
 959 draw some important conclusions about the harvesting composition. To see this, we look
 960 away from the constraints (12) and (13). The above conditions are then simplified to:

962 (A1') $\frac{\partial L}{\partial h_{c,t}} = X_{c,t}(p_c - \rho\eta_{t+1}\psi s_c) \leq 0 ; 0 \leq h_{c,t} < 1,$

963 (A2') $\frac{\partial L}{\partial h_{yf,t}} = X_{yf,t}(p_{yf} - \rho\lambda_{t+1}s_{yf}) \leq 0 ; 0 \leq h_{yf,t} < 1,$

964 (A3') $\frac{\partial L}{\partial h_{ym,t}} = X_{ym,t}(p_{ym} - \rho\mu_{t+1}s_{ym}) \leq 0 ; 0 \leq h_{ym,t} < 1,$

965 (A4') $\frac{\partial L}{\partial h_{f,t}} = X_{f,t}(p_f - \rho\lambda_{t+1}s_f) \leq 0 ; 0 \leq h_{f,t} < 1$

966 and

967 (A5') $\frac{\partial L}{\partial h_{m,t}} = X_{m,t}(p_m - \rho\mu_{t+1}s_m) \leq 0 ; 0 \leq h_{m,t} < 1.$

968 From these conditions it is straightforward to find that harvesting female yearlings and
 969 females at the same time cannot represent a possible optimal policy. To see this, assume first
 970 that female yearling harvest is optimal. (A2') (with $X_{yf,t} > 0$) holds then as $p_{yf} = \rho\lambda_{t+1}s_{yf}$. In
 971 a similar manner, assume next that female harvest is optimal. With $X_{f,t} > 0$, (A4') then reads
 972 $p_f = \rho\lambda_{t+1}s_f$. Combining these two equation yields then $p_{yf}/s_{yf} = p_f/s_f$. As only
 973 parameters are included here, this equation holds only by accident, Therefore, harvesting both
 974 these two stages at the same time cannot represent an optimal solution when the constraints
 975 (12) and (13) are not binding and when the harvest recreation value is not included. When
 976 combining (A3') and (A5') we can draw a similar type of conclusion; that is, harvesting male
 977 yearlings and males at the same time is not in accordance with the optimality conditions. In
 978 the main text (section 5.1) more discussions based on (A1') – (A5') are found. For example,
 979 harvesting female yearlings together with zero adult female harvesting yields
 980 $p_{yf}/s_{yf} > p_f/s_f$, which represents a contradiction due to the structure of the parameter
 981 values.

982
 983
 984 When including grazing damage, $D_t = d_c X_{c,t} + d_{fy} X_{fy,t} + d_{my} X_{my,t} + d_f X_{f,t} + d_m X_{m,t}$, but still
 985 not including the recreational hunting value, the landowner problem is to

986 $\max_{h_{c,t}, h_{yf,t}, h_{ym,t}, h_{f,t}, h_{m,t}} \sum_{t=0}^{t=\infty} \rho^t (V_t - D_t)$, again subject to the biological growth equations (1), (2), (4) and
 987 (7) together with constraints (12) and (13). It is easily recognized that the control conditions
 988 of this problem will be similar to that of (A1) – (A5) because the harvest rates are not included
 989 in the D_t expression; that is, the damage occurs before the hunting.

990
 991
 992 When the recreational value also is included, the problem is to maximize

$$993 \sum_{t=0}^{\infty} \rho^t (V_t - D_t + B_t) = \sum_{t=0}^{\infty} \rho^t \left\{ p_c h_{c,t} X_{c,t} + [p_{yf} h_{yf,t} + p_{ym} h_{ym,t} (1 - \psi) / \psi] X_{yf,t} + p_f h_{f,t} X_{f,t} + p_m h_{m,t} X_{m,t} \right. \\ \left. - (d_c X_{c,t} + d_{fy} X_{fy,t} + d_{my} X_{my,t} + d_f X_{f,t} + d_m X_{m,t}) + (z / \theta) (h_{c,t} + h_{yf,t} + h_{ym,t} + h_{f,t} + h_{m,t}) \right\}$$

994 to the same constraints as above.

995
 996 The first-order necessary control conditions with $X_{i,t} > 0$ ($i = c, yf, ym, f, m$) read now:

$$997 (A1'') \frac{\partial L}{\partial h_{c,t}} = X_{c,t} (p_c + z / \theta X_{c,t} - \rho \eta_{t+1} \psi s_c + \rho \omega_{t+1}) \leq 0 ; 0 \leq h_{c,t} < 1,$$

$$998 (A2'') \frac{\partial L}{\partial h_{yf,t}} = X_{yf,t} (p_{yf} + z / \theta X_{yf,t} - \rho \lambda_{t+1} s_{yf}) \leq 0 ; 0 \leq h_{yf,t} < 1,$$

$$999 (A3'') \frac{\partial L}{\partial h_{ym,t}} = X_{ym,t} (p_{ym} + z \psi / (1 - \psi) \theta X_{yf,t} - \rho \mu_{t+1} s_{ym}) \leq 0 ; 0 \leq h_{ym,t} < 1,$$

$$1000 (A4'') \frac{\partial L}{\partial h_{f,t}} = X_{f,t} (p_f + z / \theta X_{f,t} - \rho \lambda_{t+1} s_f - \rho \omega_{t+1}) \leq 0 ; 0 \leq h_{f,t} < 1$$

1001 and

$$1002 (A5'') \frac{\partial L}{\partial h_{m,t}} = X_{m,t} (p_m + z / \theta X_{m,t} - \rho \mu_{t+1} s_m - \rho \xi_{t+1} / X_{m,t}) \leq 0 ; 0 \leq h_{m,t} < 1.$$

1003 The important difference is now that the population sizes are included in these control
 1004 conditions.

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1005

