Optimal harvest in an age structured model with different fishing selectivity

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Abstract
An age structured model of a fishery is studied where two fishing fleets, or fishing agents, are targeting two different mature age classes of the fish stock. The agents are using different fishing gear with different fishing selectivity. The model includes young and old mature fish that can be harvested, in addition to an age class of immature fish. The paper describes the optimal harvesting policy under different assumptions on the objectives of the social planner and on fishing selectivity. First, biomass yield is maximized under perfect fishing selectivity, second, equilibrium profit (rent) is maximized under perfect fishing selectivity, and third, equilibrium profit is maximized under imperfect fishing selectivity. The paper provides results that differ significantly from the standard lumped parameter (also surplus production, or biomass) model.

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1. Introduction

The economics of age and stage structured fishery models has played a minor role in the fishery economics literature. The modest interest is surprising in light of the last decade's concern about overfishing, the increasing tendency to catch small and immature fish, and the problem of 'fishing down the food chain' (Pauly et al. 1998). The discard problem is also a part of this picture (see, e.g., Anderson 1994 and Vestergaard 1996). Another recent cause for concern is that the ever increasing fishing pressure may cause various systematic changes in the internal structure and evolution of fish populations (e.g., Anderson et al. 2008) which indeed may have important economic effects. One obvious reason for this modest interest in age structured models is the difficulty of analyzing such models. On the one hand, it is relatively straightforward to formulate a reasonable age-structured model and numerically simulate the effects of variations in fishing mortality between age classes and over time (e.g., Caswell 2001). On the other hand, it is notoriously difficult to understand the various biological as well as economic forces at work in these models. There are also several technical difficulties and interpretation problems when one aims to optimize the economic yield, or biomass, over time in such models. As the present paper will demonstrate, even when an age structured model is formulated in its most simple form and studied within an equilibrium fishery context, no clear-cut results can be given even concerning the qualitative structure of optimal age-structured harvesting, e.g., whether harvesting all or only some of year classes represent an optimal harvest policy. Rather, it is shown that optimal harvesting essentially depends on the various biological (recruitment and survival) and economic (cost and price) parameters of the fishery under consideration. Harvesting selectivity also plays an important role, and the importance of differences in fishing selectivity is highlighted.

Colin Clark has a chapter on age structured models in the 1976 edition of his milestone book based on Beverton and Holt (1957). This chapter is more or less left unchanged in the 1990 edition (Clark 1990, Ch. 9)[1]. He first studies the condition for fishing a single age class, or cohort, independently of other cohorts, where the goal is to find the optimal time to harvest the entire cohort. The solution of this problem is similar to what one finds in a single-felling forestry model (the Fisher rule). He next formulates a multi cohort model with fixed

[1] In the third (2010) edition, the chapter (Ch. 6 in this edition) has been shortened substantially, in particular concerning the analysis of the multi cohort model.

(exogenous) recruitment and non-selective fishing mortality. When the management goal still is present value profit maximizing, he finds the solution to be a sequence of impulse controls. This second model is closely related to the remarkable Hannesson (1975) paper.

Other early studies include Walters (1969) who constructs a complete age structured model with endogenous recruitment governed by the Beverton-Holt recruitment function (Beverton and Holt 1957). However, no fishing costs are included, and the management goal is to maximize the yield (in tonnes) over a given time period. The model is solved numerically. Another early contribution is Reed (1980) who analytically studies the maximum sustainable yield problem. He finds that optimal harvesting includes at most two age classes (see also Getz 1985). Getz and Haight (1988) review various stage structured models and where the maximum sustainable yield problem is formulated as well, while Caswell (2001) gives a broad overview of various types of stage and age structured models (linear as well as nonlinear), but without any substantial economic content. Olli Tahvonen (2009) has recently derived both analytical and numerical results on optimal harvesting in a dynamic setting under various simplifying assumptions. When assuming non selective technology, he finds the optimal solution to be impulse control. As recruitment here is endogenous, this result generalises the above mentioned pulse harvesting found in Hannesson (1975) and Clark (1990). The intuitive reason for this result is that the pulse-fishing strategy synchronizes the age structure of the fish stock and thereby allows the manager to target fish of optimal size (Hannesson 1975, Tahvonen 2010).

In this paper several aspects of the optimal harvest of a stage structured model of a fishery are studied. The analysis is restricted to an equilibrium fishery problem such that natural growth of the various age classes is exactly balanced by fishing mortality. Different agents, or fishing fleets, targeting the different harvestable age classes of the fish population are included, and we are studying the situation of perfect as well as imperfect fishing selectivity. Perfect selectivity is a situation where the agents have full control over their catches of different age classes, while it under imperfect selectivity is some bycatches of non-targeted age classes[2]. The aim of the paper is to analyze optimal solutions, and hence the social planner situation only is studied. This means that actual management schemes, like regulated open access, is

[2] FAO defines bycatch as: 'Bycatch will be used to refer to that part of the catch which is not the primary target of the fishing effort. It consists of both fish which is retained and marketed (incidental catch) and that which is discarded or released' (Clucas 1997).

not considered. The biological model includes young and old mature fish that can be harvested, together with recruits. Recruitment is determined by the size of the spawning population. This is the simplest possible formulation of a ‘complete’ age structured model; that is, there is a harvest trade-off and recruitment is endogenously determined. The problem is, then, under various management goals of the social planner, to find the optimal harvest composition of the young and old mature fish stock in biological equilibrium. The paper describes and analyses the optimal harvesting policy under different assumptions. The paper provides four results, and where the first result is related to the existing literature (Reed 1980). Two of the other results are contrasted to the standard lumped parameter (surplus production) model as we are not aware of any other works within the economics of age structured models to relate these results to.

The paper is organized as follows. In the next section, the three stage population model is formulated, with recruitment governed by the Beverton-Holt function. In section three we study the maximum sustainable yield solution of the model. The harvest functions of the two agents are introduced in section four. In section five the model is solved as a rent maximizing problem with perfect fishing selectivity while in section six we analyze what happens under non selective harvesting technology. Section seven gives some numerical illustrations while the last section eight summarizes our study.

2. Population model

As already indicated, our study of optimal harvest of different age classes of a fish population is tackled by using a quite simple model with just three cohorts of the fish population. The youngest class is immature fish, whereas the two older classes are harvestable and contribute both to the spawning stock. Recruitment is endogenous and density dependent, and the old matures are assumed to have higher fertility than the young matures. Natural mortalities, or equivalently the survival rates, are assumed to be fixed and density independent for all three age classes.

In time discrete models like this, one has to decide whether fishing takes place before, or after, natural mortality, and also whether fishing takes place before or after spawning. Obviously, just as in the Beverton-Holt (1957) model, one may even have that fishing, as well as natural mortality, take place simultaneously throughout the year (cf. e.g., Hannesson 1975 and Getz 1985). However, such formulation creates unnecessary complications within our stylized framework, and we simply assume that fishing takes place instantaneously, and it occurs after spawning, but before natural mortality. This last assumption has a small effect on the principal working of the model (see Section two).

The fish population in number of individuals at time t (year) is structured as recruits $X_{0,t}$ ($yr < 1$), young mature fish $X_{1,t}$ ($1 \le yr < 2$) and old mature fish $X_{2,t}$ ($2 \le yr$). The number of recruits is governed by the recruitment function:

(1) $X_{0,t} = R(X_{1,t}, X_{2,t})$

where $R(\cdot,\cdot)$ may be a one-peaked value function (e.g., of the Ricker type) or it may be increasing and concave in both mature population sizes (e.g., of the Beverton-Holt type). The Beverton-Holt type is used here. Therefore, the recruitment function obeys $R(0,0) = 0$ and $\partial R / \partial X_{i,t} = R_i^{'} > 0$, together with $R_i^{''} < 0$ ($i = 1,2$). As higher fertility of the old than the young matures is assumed (similar as in Reed 1980 and others), we also have $R_2^{'} > R_1^{'}$.

The number of young mature fish follows next as:

(2) $X_{1,t+1} = s_0 X_{0,t}$,

where s_0 is the fixed natural survival rate. Finally, the number of old mature fish is described by:

(3) $X_{2,t+1} = s_1(1 - f_{1,t})X_{1,t} + s_2(1 - f_{2,t})X_{2,t}$

where $f_{1,t}$ and $f_{2,t}$ are the total fishing mortalities of the young and old mature stage, respectively, while s_1 and s_2 are the natural survival rates of these mature stages, assumed to be fixed and density independent. This last equation completes the population model. It has a recursive structure, and when combining (1) and (2) we find:

(4) $X_{1,t+1} = s_0 R(X_{1,t}, X_{2,t})$.

Therefore, equations (3) and (4) represent a reduced form model in two stages, where both equations are first order difference equations. This form is used when analyzing exploitation.

The population equilibrium for *fixed* fishing mortalities is defined by $X_{i,t+1} = X_{i,t} = X_i$ $(i = 1,2)$ such that:

(3') $X_2 = s_1(1 - f_1)X_1 + s_2(1 - f_2)X_2$

and

(4') $X_1 = s_0 R(X_1, X_2)$.

It what follows, equation (3') is notified as the *spawning constraint*, while equation (4') represents the *recruitment constraint*. Notice that an internal equilibrium holds for $0 \le f_1 < 1$ only; that is, not all the young mature fish can be harvested to sustain the fish stock.

The Beverton-Holt recruitment function is specified as $R(X_{1,t}, X_{2,t}) = \frac{a(X_{1,t} + \alpha X_{2,t})}{b + (X_{1,t} + \alpha X_{2,t})}$ with a as the scaling parameter (maximum number of recruits per spawning fish), b as the shape parameter and $\alpha > 1$ as the parameter indicating higher fertility of the old mature stage (often related to weight differences; see, e.g., Getz and Haight 1989, p. 154). With this specification the recruitment constraint (4') may be written as $X_2 = \frac{X_1}{\alpha}\left[\frac{b}{s_0\, a - X_1} - 1\right]$. For $s_0 a - b < X_1 < s_0 a$, it describes the number of old mature fish as a positive, increasing and convex function of the number of young mature fish. As the spawning constraint is linear, this guarantees the uniqueness of the equilibrium. Both the recruitment and the spawning constraint are depicted in Figure 1[3]. In line with intuition, we find that higher fishing mortalities shift down the spawning constraint (3') and hence lead to smaller equilibrium stocks, while higher survival rates yield more fish. A relatively higher fertility of the old mature stock through an increased value of α also means more fish because the curvature of recruitment constraint (4') increases. A higher α will, however, not change the ratio of old to young mature fish (the age structure) in equilibrium, as this ratio simply is given by the slope of the spawning constraint, i.e., by $A = \frac{s_1(1 - f_1)}{1 - s_2(1 - f_2)}$. Therefore, we find that higher values of the survival coefficients making the spawning constraint steeper will increase the proportion of old matures. Higher fishing mortalities of both the old mature and the young mature work in the opposite direction and increase the equilibrium proportion of young mature. The reason why higher f_1 increases the proportion of the young mature population is therefore that the resulting decrease in this age class spills over to an even larger decrease in the old mature

[3] Arrows indicating the dynamics outside the equilibrium are also depicted in this figure. Given the current stock of young mature, the stock of young mature will increase (decrease) if the old mature stock is above (below) the recruitment constraint. Given the current stock of old mature, the stock of old mature will increase (decrease) if the young mature stock is to the left (right) of the spawning constraint.

population. In a harvest program with, say $f_2 = 1$ and $0 \le f_1 < 1$, the stock composition reads $A = s_1(1 - f_1)$ which simplifies further to s_1 if the young mature stock is left unexploited.

Figure 1 about here

It is possible to explicitly find the stock equilibrium values for the Beverton-Holt recruitment function. The results are $X_1 = s_0 a - \frac{b}{1 + \alpha A}$ and $X_2 = A X_1 = A\left[s_0 a - \frac{b}{1 + \alpha A}\right]$. From these expressions we find that $\frac{b - s_0 a}{s_0 \alpha a} < \frac{s_1(1 - f_1)}{1 - s_2(1 - f_2)} = A$ must hold to guarantee an interior solution[4].

3. The maximum sustainable biomass yield harvesting

We now proceed to analyze exploitation of the age-structured fish population, and just to fix ideas and as a background for the economic analysis, we start to look at the 'classical' problem of finding fishing mortalities maximizing the equilibrium biomass yield (cf. Reed 1980). With w_1 and w_2 as the fixed weights (kg per fish) of the young and old mature group, respectively, and where $w_2 > w_1$, the equilibrium biomass harvested (in kg) is described as $Y = w_1 f_1 X_1 + w_2 f_2 X_2$ as fishing, by assumption, takes place before natural mortality. The maximum sustainable yield problem is then defined by finding fishing mortalities that maximize Y subject to the spawning constraint (3') and the recruitment constraint (4'). The Lagrangian function of this problem may be written as $L = w_1 f_1 X_1 + w_2 f_2 X_2 - \lambda[X_1 - s_0 R(X_1, X_2)] - \mu[X_2 - s_1(1 - f_1)X_1 - s_2(1 - f_2)X_2]$ where $\lambda > 0$ and $\mu > 0$ are the recruitment constraint and the spawning constraint shadow prices, respectively. Following the Kuhn-Tucker theorem the first order necessary conditions are (assuming $X_i > 0$, $i = 1, 2$):

(5) $\partial L / \partial f_1 = (w_1 - \mu s_1) X_1 \le 0$; $0 \le f_1 < 1$,

[4] If this Beverton-Holt recruitment function is replaced with a peak valued recruitment function (e.g., the Ricker function), the recruitment constraint will no longer be a strictly increasing convex function as described in Figure 1. Two intersections with the spawning constraint would then be possible. However, it is not very plausible that a stock level above the peak value of the recruitment function could be optimal as just fishing (flow) values, and no positive stock values (like the fish stock intrinsic value), are included in the subsequent optimization problems.

(6) $\partial L/\partial f_2 = (w_2 - \mu s_2)X_2 \gtreqless 0$; $0 \le f_2 \le 1$,

(7) $\partial L/\partial X_1 = w_1 f_1 + \lambda[s_0 R_1' - 1] + \mu s_1(1 - f_1) = 0$

and

(8) $\partial L/\partial X_2 = w_2 f_2 + \lambda s_0 R_2' + \mu[s_2(1 - f_2) - 1] = 0$.

Condition (5) indicates that the fishing mortality of the young mature stock should take up the point where the marginal biomass gain is equal to, or below, the marginal biomass loss of the old mature stage, evaluated at the spawning constraint shadow price. As the fishing mortality of this stock must be below one to prevent stock depletion (section two above) the marginal biomass gain can not exceed the marginal biomass loss. Condition (6) is analogous for the old mature stock, except that the marginal biomass gain may exceed the marginal biomass loss if fishing mortality equals one. Equations (7) and (8) steer the shadow price values and say basically that the number of fish should be maintained such that the biomass gains equalize the shadow costs and where the shadow costs of both the mature classes, and hence also recruitment, should be taken into account.

From the control conditions it is evident that the biological 'discounted' biomass content w_i / s_i $(i = 1, 2)$ steer the fishing mortalities and the fishing composition, and that different fertilities of the two mature stocks (modelled by $\alpha > 1$) play no direct role. Because the ratio w_i / s_i differs among the two age classes, conditions (5) and (6) can not be satisfied simultaneously as equations. The natural survival rates of the old and young mature fish will in most instances not differ too much, and if they differ, they will typically be dominated by the weight difference[5]. Therefore, we assume $w_2 / s_2 > w_1 / s_1$ in the subsequent analysis[6]. The

[5] In stock assessments, the natural survival rate is often assumed to be constant across age-classes. However, in reality, the rate might vary to a certain degree with age and length (see, e.g., Gislason et.al. 2008). In the ICES report for the North Sea cod http://www.ices.dk/reports/ACOM/2010/WGNSSK/Sec%2014%20Cod.pdf , we find detailed estimates of weight and survival rates (Tables 14.4 and 14.7b). These data generally demonstrates that the weight/survival ratio increases quite significantly with age. For example, the 2009 data indicates that the weight/survival ratio was about 1.3 (kg) for age class one, 1.8 for age class two, 4.4 for age class three, 5.7 for age class four, 8.4 for age class five and 11.5 for age class six.

[6] It can be verified that this assumption has no principal effect on our reasoning as the opposite $w_1 / s_1 > w_2 / s_2$ simply reverses the different optimal harvest options. However, if the biological discounted biomass content is similar the structure of the solution will be different. We then find that that the manager simply will be indifferent between harvesting the two stocks when the sustainable yield is maximized.

maximum sustainable biomass yield fishing mortality is accordingly higher for the old than the young mature stock, $f_2 > f_1$. This is stated as:

Result 1. The biological discounted biomass content w_i / s_i $(i = 1, 2)$ *steers the maximum sustainable yield fishing mortalities. Fertility plays no direct role. When the biological discounted biomass of the old mature stock is higher than that of the young stock, a higher fishing mortality of the old than the young mature stock maximizes the sustainable yield.*

Our result is in line with Reed (1980) who finds that it is optimal to target one or two year classes. Further, if two age classes are harvested, the older one should be harvested completely. However, it should be noted that while we use a straightforward application of the Lagrangian method, Reed applies a rather complex mathematical method to deduce his result. Notice also that if the timing assumption of the harvest is changed (Section two) so that harvesting takes place *after* natural mortality, the equilibrium biomass harvested is defined as $Y = w_1 f_1 s_1 X_1 + w_2 f_2 s_2 X_2$. Because the biological constraints are left unchanged, the first order conditions (5) and (6) will change slightly as only weights, and not biological discounted weights, steers the optimal harvest policy; that is, conditions (5) and (6) change to $(w_i - \mu) X_i \leq 0$, $i = 1, 2$. However, as the survival rates among the mature fish do not differ too much, the yield maximizing fishing policy will typically be unaffected.

Generally, there are three possibilities to meet the above first order necessary conditions (5) and (6) with $w_2 / s_2 > w_1 / s_1$: i) $f_2 = 1$ and $0 < f_1 < 1$, ii) $f_2 = 1$ and $f_1 = 0$, and iii) $0 < f_2 < 1$ and $f_1 = 0$. See also Table 1. The spawning constraint (3') will for obvious reasons be steeper in case iii) than in case ii) which again will be steeper than in case i). Therefore, the size of both mature stocks will be highest with harvest option iii) and lowest if case i) represents the optimal policy. In case i), the spawning constraint shadow price is determined by (5) as $\mu = w_1 / s_1$, while equations (7) and (8) together with the population equilibrium equations (3') and (4') determine the stock sizes, the young adult fishing mortality and the young adult stock shadow price. Inserting $\mu = w_1 / s_1$ into equation (7) yields $\lambda = \frac{w_1}{(1 - s_0 R_1')}$ which, as expected, is positive because $(1 - s_0 R_1') > 0$ as the recruitment constraint (4') intersects with the spawning constraint (3') from below (cf. Figure 1). Rewriting equation (8) and inserting

$\mu = w_1 / s_1$, the recruitment constraint shadow price is $\lambda = \dfrac{w_1 / s_1 - w_2}{s_0 R_2 '}$. Therefore, if $w_1 / s_1 - w_2 < 0$, or equivalently s_1 is higher than the relative weight between the two stages, case i) can not represent the solution to the maximum sustainable yield problem. Arguing along the same lines, we further find that if s_2 is 'high' case ii) becomes less likely as the valid interval $w_1 / s_1 < \mu < w_2 / s_2$ making the shadow price $\lambda = \dfrac{\mu - w_2}{s_0 R_2 '}$ positive through (8) becomes smaller. In case iii), the adult shadow price is determined by (6) as $\mu = w_2 / s_2$. Equation (8) now reads $\lambda = \dfrac{w_2 / s_2 - w_2}{s_0 R_2 '} > 0$.

What the above discussion basically boils down to is that values of s_1 and s_2 making the gap $w_2 / s_2 > w_1 / s_1$ larger indicate that case ii) is more likely to take place, while values that make it smaller means that one of the two other harvest options is more likely to be optimal. Moreover, for given survival coefficients, we find that case ii) will more likely be the optimal option when the gap widens through a higher weight discrepancy. The reason why case i) can represent the optimal policy may be explained by the fact that when the survival rate of young fish is low then it is beneficial to catch them before they die. On the other hand, for a high survival rate of young fish combined with a relatively low survival rate of the old, it will be beneficial to fish as much as possible of the old mature stock.

4. Fishing mortalities

The fish stock is exploited by two fishing fleets (or 'agents'), utilizing different gears, and where each of the fleets is targeting a particular age class of the fish. Such harvesting scheme fits reality in many instances. For example, in the Norwegian cod fishery we find that the trawlers are targeting the small cod, i.e., the young mature group, and the coastal fleet, using passive gear, is targeting the large cod, i.e., the old mature group (see, e.g., Armstrong 1999). To a certain extent, the fleets might be able to influence their catch composition. For example, the mesh size may be increased, other gears may be adopted to leave the younger and smaller fish less exploited, the fleets may choose between various fishing grounds, and so forth (e.g., Beverton and Holt 1957, Clark 1990)[7]. However, in most instances the catches are composed of species from different cohorts and there is hence 'bycatch' (cf. also footnote 2). Bycatch is included in our model. By convention, it is assumed that fleet one (agent one) targets the young mature fish (stock one) while agent two targets the old mature fish (stock two). Fishing mortality is governed by standard Schaefer harvest functions with fixed 'catchability' coefficients.

When E_1 is effort use of fleet one, targeting the young mature fish, the catch function for this fleet writes:

(9) $$H_1 = h_1 X_1 = q_1 E_1 X_1$$

and

(10) $$B_2 = b_2 X_2 = \tilde{q}_2 E_1 X_2 ,$$

when again noting that fishing takes place (instantaneously) before natural mortality. H_1 and h_1 are the catch (in number of fish) and fishing mortality, respectively, of the young mature fish while B_2 and b_2 are the unintended catch (bycatch) and fishing mortality, respectively, of the old mature fish. q_1 and $\tilde{q}_2$ denote the (fixed) catchability (productivity) coefficients. Following this framework, the ratio of fishing mortalities of the target stock and the bycatch stock is simply determined as $h_1 / b_2 = q_1 / \tilde{q}_2$. In a similar manner with E_2 as the fleet two effort use, the catch functions of this fleet targeting the old mature fish write:

(11) $$H_2 = h_2 X_2 = q_2 E_2 X_2$$

And

(12) $$B_1 = b_1 X_1 = \tilde{q}_1 E_2 X_1 .$$

It is possible to define target catch and bycatch. Fishing mortality of target fish is higher than the fishing mortality of bycatch fish. Hence, for equally sized stocks, the catch of the targeted fish stock should be higher than that of the bycatch stock; that is, $q_1 > \tilde{q}_2$ for fleet 1 and $q_2 > \tilde{q}_1$ for fleet 2. Intuitively, one may also think that $\tilde{q}_1 < q_1$ and $\tilde{q}_2 < q_2$ should hold. For the young mature fish this means lower catch per unit effort (*CPUE*) for the unintended catch

[7] More recently, we find that Squires and Kirkley (1991), Turner (1997) and Singh and Weninger (2009) develop models where the fishermen to some extent can adjust their catch composition in response to changes in exogenous variables, e.g., the fish price.

than the target catch, $B_1 / E_2 < H_1 / E_1$. However, this will not necessarily hold. If, say, the trawler fleet targets the old mature fish while gill-netters target the young stock, one may suspect that catch per unit effort is larger for the trawlers than the gill-netters also for the young mature age group (see also section six below).

5. Exploitation I: Perfect fishing selectivity

As mentioned, economic exploitation is analyzed within an overall management scheme where the social planner aims to maximize profit of both fleets in biological equilibrium. The situation where the bycatch of both stages are negligibly small is first considered; that is, $\tilde{q}_2 = \tilde{q}_1 = 0$. With p_1 and p_2 as the fish prices (Euro/kg), assumed to be fixed and independent of the harvest and where the old mature fish typically is more valuable than the young fish, $p_2 \geq p_1$ (see, e.g., Armstrong 1999) and c_1 and c_2 as the constant unit effort cost of the two fleets, the current joint profit to be maximized is described by

$\pi = (p_1 w_1 q_1 E_1 X_1 - c_1 E_1) + (p_2 w_2 q_2 E_2 X_2 - c_2 E_2)$.

Total fishing mortality, defined by $f_i = h_i + b_i \ (i = 1,2)$, simplifies to $f_i = h_i$ with perfect selectivity. The spawning constraint (3') then writes $X_2 = s_1(1 - q_1 E_1)X_1 + s_2(1 - q_2 E_2)X_2$ when inserting for the catch functions (9) and (11). The Lagrangian of this profit maximizing problem may now be written as $L = (p_1 w_1 q_1 E_1 X_1 - c_1 E_1 + p_2 w_2 q_2 E_2 X_2 - c_2 E_2)$ $-\lambda[X_1 - s_0 R(X_1, X_2)] - \mu[X_2 - s_1(1 - q_1 E_1)X_1 - s_2(1 - q_2 E_2)X_2]$. Assuming positive stocks of both age classes ($X_1 > 0$ and $X_2 > 0$), the first order necessary conditions are[8]:

(13) $\partial L / \partial E_1 = p_1 w_1 q_1 X_1 - c_1 - \mu s_1 q_1 X_1 \leq 0$; $0 \leq E_1 < 1/q_1$,

(14) $\partial L / \partial E_2 = p_2 w_2 q_2 X_2 - c_2 - \mu s_2 q_2 X_2 \gtreqless 0$; $0 \leq E_2 \leq 1/q_2$,

(15) $\partial L / \partial X_1 = p_1 w_1 q_1 E_1 + \lambda[s_0 R_1' - 1] + \mu s_1(1 - q_1 E_1) = 0$

and

(16) $\partial L / \partial X_2 = p_2 w_2 q_2 E_2 + \lambda s_0 R_2' + \mu[s_2(1 - q_2 E_2) - 1] = 0$.

It is still assumed $w_2 / s_2 > w_1 / s_1$ and because $p_2 \geq p_1$, we have $p_2 w_2 / s_2 > p_1 w_1 / s_1$.

[8] While leaving both stocks unexploited may be beneficial due to, say, high harvesting costs, stock depletion can never be beneficial under this maximum economic yield scenario with zero discount rate.

If the harvest cost discrepancy $c_1 / q_1 > c_2 / q_2$ holds, we may then intuitively suspect that the solution of this problem will be very similar to the maximum sustainable yield problem. In the opposite situation with $c_2 / q_2 > c_1 / q_1$, we may suspect that an internal solution $0 < E_i < 1/q_i$ $(i = 1, 2)$ can be a possible optimal option. When combining conditions (13) and (14) a possible internal solution is characterized by $(p_1 w_1 / s_1 - c_1 / q_1 s_1 X_1) = (p_2 w_2 / s_2 - c_2 / q_2 s_2 X_2)$, indicating equal biological discounted marginal profit (Euro per fish) among the two fish fleets (or fish stocks). It can easily be verified that this equation describes X_2 as an increasing concave function of X_1, and its intersection with the recruitment constraint (4') hence yields an unique internal solution. As shown in the Appendix, however, this is not an internal maximum solution; it is a saddle point solution.

Therefore, possible optimal solutions satisfying conditions (13) – (16) indicate different biological discounted marginal profit among the two fish stocks. Altogether, four possibilities exist. First, we have the same three cases as in the maximum sustainable yield problem; i) $E_2 = 1/q_2$ and $0 < E_1 < 1/q_1$ ii) $E_2 = 1/q_2$ and $E_1 = 0$ and iii) $0 < E_2 < 1/q_2$ and $E_1 = 0$. A fourth possible option is fleet 1 only fishing, but not depleting the fish population. We thus have case iv) $E_2 = 0$ and $0 < E_1 < 1/q_1$. In the first three cases, conditions (13) and (14) hold as $(p_2 w_2 / s_2 - c_2 / q_2 s_2 X_2) > (p_1 w_1 / s_1 - c_1 / q_1 s_1 X_1)$ while this inequality is reversed in case iv). See Table 1. This is stated as:

Result 2. Any possible optimal sustainable rent harvesting policy under perfect fishing selectivity implies different biological discounted marginal profit (Euro per fish) among the two harvestable fish stocks. Targeting both stocks, the old mature stock only or the young mature stock only may all represent possible optimal harvesting schemes.

Table 1 about here

In case ii) with $E_2 = 1/q_2$ and $E_1 = 0$ the spawning constraint (3')
$X_2 = AX_1 = \dfrac{s_1(1 - q_1 E_1)}{1 - s_2(1 - q_2 E_2)} X_1$ simplifies to $X_2 = s_1 X_1$. The above harvest condition following (13) and (14), $(p_2 w_2 / s_2 - c_2 / q_2 s_2 X_2) > (p_1 w_1 / s_1 - c_1 / q_1 s_1 X_1)$, may therefore also be written as $(p_2 w_2 / s_2 - p_1 w_1 / s_1) > (1/ s_1 X_1)(c_2 / q_2 s_2 - c_1 / q_1)$. With $p_2 w_2 / s_2 > p_1 w_1 / s_1$ this condition

is for sure satisfied with $c_1 / q_1 \geq (c_2 / q_2) / s_2$. Therefore, not surprisingly, we find that case ii) is a possible optimal harvest option when the fleet one harvest cost is strictly above that of fleet two, $c_1 / q_1 > c_2 / q_2$. However, this harvest option may also be possible with $c_2 / q_2 > c_1 / q_1$ if $p_2 w_2 / s_2$ is 'substantially' higher than $p_1 w_1 / s_1$. In case i) with $E_2 = 1/q_2$ and $0 < E_1 < 1/q_1$ and where the spawning constraint (3') reads $X_2 = s_1(1 - q_1 E_1) X_1$, the harvest condition following (13) and (14) can be written as $(p_2 w_2 / s_2 - p_1 w_1 / s_1) > (1/ s_1 X_1)(c_2 / q_2 s_2 (1 - q_1 E_1) - c_1 / q_1)$. This inequality is for sure met with $c_1 / q_1 \geq (c_2 / q_2) / s_2 (1 - q_1 E_1)$, which is more restrictive than the above related case ii) condition. Notice that just as in case ii), case i) can also be satisfied if $c_2 / q_2 > c_1 / q_1$.

Harvesting the old mature stock completely may therefore be optimal if the harvest cost of the fleet targeting the young fish is strictly above the cost of the fleet targeting the old mature fish, as well as the opposite. However, a large marginal profit difference, either through a substantial marginal fish value discrepancy together with small cost differences, or lower cost for the fleet targeting the old mature stock than the young mature stock, will work in the direction that case ii) becomes more likely as the optimal harvesting scheme. With values of s_1 and s_2 making the biological discounted marginal profit gap larger, we will just as in the maximum sustainable yield fishery (see above) also find that case ii) is more likely to be the optimal option.

Arguing along the same line as above, we find that case iii) with $0 < E_2 < 1/q_2$ and $E_1 = 0$ for sure is satisfied with $c_1 / q_1 \geq (c_2 / q_2)[1 - s_2(1 - q_2 E_2)] / s_2$. This condition can be met if $c_1 / q_1 > c_2 / q_2$, but also the opposite as $[1 - s_2(1 - q_2 E_2)] / s_2 < 1$ may hold. Therefore, targeting the old mature stock only, but not harvesting the stock completely, can be a possible optimal policy even if $c_2 / q_2 > c_1 / q_1$. When still using conditions (13) and (14), we find case iv) with only young mature fishing to be an optimal option if $(p_2 w_2 / s_2 - p_1 w_1 / s_1) < (1/ s_1 X_1)[c_2 (1 - s_2) / q_2 s_2 (1 - q_1 E_1) - c_1 / q_1]$. Therefore, with $c_1 / q_1 \geq (c_2 / q_2)(1 - s_2) / s_2 (1 - q_1 E_1)$ young mature fishing only can *not* represent the optimal fishing policy.

The above analysis shows that changes in effort costs c_i ($i = 1, 2$) may shift the optimal harvest policy from targeting one stock to targeting the other stock, or targeting both stocks. However, in what degree and to what extent is far from clear. Changes in technology q_i and fish prices p_i may also shift the optimal harvest policy from one corner solution to another corner solution. Notice that these discontinuities are not an obvious result as the model is nonlinear due to the nonlinear and concave recruitment function. The solution of this maximum rent problem coincides with the above section three maximum sustainable yield problem if, say, case ii) with $E_2 = 1/q_2$ and $E_1 = 0$, or $f_2 = 1$ and $f_1 = 0$, represents the optimal harvest policy. The optimal stock sizes in both problems are then determined by the spawning constraint (3') as $X_2 = s_1 X_1$ together with the recruitment constraint (4'). In this case the optimal size of the standing biomass, defined as $Q = w_1 X_1 + w_2 X_2$, is similar under the two different maximizing objectives; maximizing biomass and maximizing rent (profit). Such possible outcome contrasts the prediction from the standard lumped parameter (surplus production) model (e.g., Clark 1990) where the stock (biomass) *always* will be higher in the maximum equilibrium rent problem (with zero discount rate, MEY) than in the maximum sustainable yield problem (MSY).

We may also find that higher fishing costs can be associated with smaller stocks and lower standing biomass in the rent maximizing problem than in the maximum sustainable yield problem. This happens under certain conditions if the optimal harvest policy shifts from, say, case ii) with $E_2 = 1/q_2$ and $E_1 = 0$ to case iv) with $E_2 = 0$ and $0 < E_1 < 1/q_1$ due to a sharp fleet two cost increase. While the spawning constraint (3') is defined as $X_2 = s_1 X_1$ in case ii), it becomes $X_2 = \frac{s_1(1 - q_1 E_1)}{(1 - s_2)} X_1$ in case iv). If $\frac{s_1(1 - q_1 E_1)}{(1 - s_2)} < s_1$, or equivalently $E_1 q_1 > s_2$, the spawning constraint hence intersects with the recruitment constraint (4') for smaller number of both stocks. Therefore, when a 'high' c_2 changes the rent maximizing harvesting policy from case ii) to case iv) such that the young mature fishing mortality exceeds the old mature survival coefficient, higher costs are not working in a stock conserving manner.

Such outcome also contrasts the prediction from the standard lumped parameter model where higher costs *always* work in a stock conserving manner. Arguing along the same lines, we may also find that more valuable fish stocks through higher prices can be associated with a more stock conserving harvesting policy. Perhaps even more important, improved fishing efficiency can work in the same stock conserving manner. For example, technological shifts, or gear changes, that make harvest of the young stock more efficient and hence reduce the harvest cost c_1 / q_1, case iv) with $E_2 = 0$ and $0 < E_1 < 1/q_1$ may replace case ii) with $E_2 = 1/q_2$ and $E_1 = 0$ as the optimal harvest policy. As explained above, such harvest policy change can be accompanied by a higher standing biomass. This is stated as:

Result 3. Lower effort costs, higher fishing prices and more efficient harvesting technology may lead to a maximum economic rent policy with a larger standing biomass size in the age structured model.

Still in contrast with the lumped parameter model, we can also find that changing costs and prices as well as changing harvesting technology may keep the stock unchanged within the same harvesting scheme (same corner solution). This is evident in case ii) with $E_2 = 1/q_2$ and $E_1 = 0$. As just seen, the stock sizes in this case are determined by the spawning constraint (3') as $X_2 = s_1 X_1$ together with the recruitment constraint (4'). Therefore, small changes in q_2, as well as q_1, will keep the total standing and harvested biomass constant. The stock composition will stay unchanged as well. On the other hand, profitability, in this case described by $\pi = p_2 w_2 X_2 - c_2 / q_2$, increases with a higher q_2 value just as in the lumped parameter model. However, only the direct effect is present. The indirect and counterbalancing effect through a lower stock size (first term RHS), as we find in the biomass model, is not present.

6. Exploitation II: Imperfect selectivity

We now proceed to analyse the more difficult rent maximum problem with imperfect harvesting selectivity and bycatch, and fishing mortalities described in section four such that $\tilde{q}_2 > 0$ and $\tilde{q}_1 > 0$. When inserting for all catch functions (9) – (12), the spawning constraint (3') now reads $X_2 = s_1(1 - q_1 E_1 - \tilde{q}_1 E_2) X_1 + s_2(1 - q_2 E_2 - \tilde{q}_2 E_1) X_2$, or $X_2 = \dfrac{s_1(1 - q_1 E_1 - \tilde{q}_1 E_2)}{1 - s_2(1 - q_2 E_2 - \tilde{q}_2 E_1)} X_1 = A' X_1$. Introduction of bycatch for fixed fishing effort, i.e., higher fishing mortalities, indicate a less steep spawning constraint and it hence intersects with the recruitment constraint (4') further down. Therefore, not surprisingly, with smaller stocks, but possibly surprisingly, a stock composition with a higher proportion of young mature.

Fishing effort is restricted through the conditions:

(17) $0 \le (q_1 E_1 + \tilde{q}_1 E_2) < 1$

and

(18) $0 \le (q_2 E_2 + \tilde{q}_2 E_1) \le 1$.

The allowable fishing effort is thus constrained by the lines $O-a-b-c-O$ in Figure 2 under the current assumptions of $q_1 > \tilde{q}_2$ and $q_2 > \tilde{q}_1$ (section four above). Therefore, fishing effort should be strictly inside segment $b-c$ (but see below) while possibly being located on the segment $a-b$. Notice that the above restrictions of the catchability coefficients imply $q_1 / \tilde{q}_1 > \tilde{q}_2 / q_2$, or $q_1 / \tilde{q}_2 > \tilde{q}_1 / q_2$.

Figure 2 about here

The profit to be maximized in this fishery is defined by $\pi = (p_1 w_1 q_1 E_1 X_1 + p_2 w_2 \tilde{q}_2 E_1 X_2 - c_1 E_1) + (p_2 w_2 q_2 E_2 X_2 + p_1 w_1 \tilde{q}_1 E_2 X_1 - c_2 E_2)$. The Lagrangian function is written as $L = (p_1 w_1 q_1 E_1 X_1 + p_2 w_2 q_2 E_2 X_2 + p_1 w_1 \tilde{q}_1 E_2 X_1 + p_2 w_2 \tilde{q}_2 E_1 X_2 - c_1 E_1 - c_2 E_2)$ $-\lambda[X_1 - s_0 R(X_1, X_2)] - \mu[X_2 - s_1(1 - q_1 E_1 - \tilde{q}_1 E_2) X_1 - s_2(1 - q_2 E_2 - \tilde{q}_2 E_1) X_2]$ $-\eta[(q_2 E_2 + \tilde{q}_2 E_1) - 1]$ where $\eta \ge 0$ is the old mature fish stock fishing mortality constraint shadow price (restriction 18). The first order necessary conditions are:

(19) $\partial L / \partial E_1 = p_1 w_1 q_1 X_1 + p_2 w_2 \tilde{q}_2 X_2 - c_1 - \mu(s_1 q_1 X_1 + s_2 \tilde{q}_2 X_2) - \eta \tilde{q}_2 \le 0$; $E_1 \ge 0$,

(20) $\partial L / \partial E_2 = p_2 w_2 q_2 X_2 + p_1 w_1 \tilde{q}_1 X_1 - c_2 - \mu(s_1 \tilde{q}_1 X_1 + s_2 q_2 X_2) - \eta q_2 \le 0$; $E_2 \ge 0$,

(21) $\partial L / \partial X_1 = p_1 w_1 q_1 E_1 + p_1 w_1 \tilde{q}_1 E_2 + \lambda[s_0 R_1' - 1] + \mu s_1(1 - q_1 E_1 - \tilde{q}_1 E_2) = 0$

and

(22) $\partial L / \partial X_2 = p_2 w_2 q_2 E_2 + p_2 w_2 \tilde{q}_2 E_1 + \lambda s_0 R_2' + \mu[s_2(1 - q_2 E_2 - \tilde{q}_2 E_1) - 1] = 0$.

In addition, we have $\eta[1 - (q_2 E_2 + \tilde{q}_2 E_1)] = 0$, and where $\eta = 0$ holds when the old mature stock fishing mortality is below one.

As in the perfect selectivity fishery, an interior extremum of the profit function is a saddle point solution, rather than a maximum (see Appendix). There are, however, now more corner solution candidates than in the perfect selectivity harvesting situation, as all combinations of E_1 and E_2 restricted by the lines $O-a-b-c-O$ in Figure 2 as described above, but not any interior points, may represent feasible solutions. These include the section five cases ii) $E_2 = 1/q_2$ and $E_1 = 0$, iii) $0 < E_2 < 1/q_2$ and $E_1 = 0$, and iv) $E_2 = 0$ and $0 < E_1 < 1/q_1$, but not case i) $E_2 = 1/q_2$ and $0 < E_1 < 1/q_1$. This case is now replaced by all combinations along the segment $a-b$. As mentioned, solutions along the segment $b-c$ including point b, but also point c, are not feasible.

In the Appendix it is shown that case ii) with $E_1 = 0$ and $E_2 = 1/q_2$, and therefore $\eta > 0$, represents a possible optimal solution if

$\dfrac{p_2 w_2 q_2 X_2 + p_1 w_1 \tilde{q}_1 X_1 - c_2}{(s_1 \tilde{q}_1 X_1 + s_2 q_2 X_2)} > \dfrac{p_1 w_1 q_1 X_1 + p_2 w_2 \tilde{q}_2 X_2 - c_1}{(s_1 q_1 X_1 + s_2 \tilde{q}_2 X_2)}$. Again, this is a marginal discounted profit condition (Euro per fish), but now weighted by the catch composition and where the survival rates of both stocks are included. With $p_2 w_2 / s_2 > p_1 w_1 / s_1$ it may be further shown (see Appendix) that this condition for sure is satisfied if

$\dfrac{c_1}{(s_1 q_1 X_1 + s_2 \tilde{q}_2 X_2)} \geq \dfrac{c_2}{(s_1 \tilde{q}_1 X_1 + s_2 q_2 X_2)}$. When inserting for the spawning constraint $X_2 = AX_1$ and doing a small rearrangement, this inequality may also be written as

$c_1 / q_1 \geq (c_2 / q_2)[\dfrac{s_1 + s_2(\tilde{q}_2 / q_1)A}{s_1 \tilde{q}_1 / q_2 + s_2 A}]$. With $A = s_1(1 - \tilde{q}_1 / q_2)$ in this case ii), we hence have

$c_1 / q_1 \geq (c_2 / q_2)[\dfrac{1 + s_2(\tilde{q}_2 / q_1)(1 - \tilde{q}_1 / q_2)}{\tilde{q}_1 / q_2 + s_2(1 - \tilde{q}_1 / q_2)}]$. As the RHS bracket term $[.]$ exceeds one, it implies $c_1 / q_1 > c_2 / q_2$. Therefore, also with imperfect harvesting selectivity, harvesting the old mature stock completely may be an optimal option when the targeted harvest cost of fleet one is higher than that of fleet two. However, just as with perfect selectivity, this case ii) can also be met if the fleet two cost is highest. With knife-edge harvesting selectivity (see, e.g., Clark 1990, Ch. 9), no fishing of the young mature fish by fleet two takes place. When $\tilde{q}_1 = 0$ the above case ii) cost condition thus simplifies to $c_1 / q_1 \geq (c_2 / q_2)\dfrac{1 + s_2(\tilde{q}_2 / q_2)}{s_2}$. Therefore, with knife-edge fishing selectivity and no fleet two bycatch income, harvesting the old mature stock completely may also be an optimal option, provided that $c_1 / q_1 > c_2 / q_2$ holds.

Case iii) with $0 < E_2 < 1/q_2$ and $E_1 = 0$ and hence $\eta = 0$ is for sure satisfied with the same type marginal cost condition as above, $c_1 / q_1 \geq (c_2 / q_2)[\frac{s_1 + s_2(\tilde{q}_2 / q_1)A}{s_1\tilde{q}_1 / q_2 + s_2 A}]$ (see Appendix), but with different stock composition because the slope of the spawning constraint now reads $A = \frac{s_1(1-\tilde{q}_1 E_2)}{1 - s_2(1 - q_2 E_2)}$. As the RHS bracket term [.] here may be above as well as below one, this harvesting scheme can be met by $c_1 / q_1 > c_2 / q_2$ as well as the opposite. Therefore, also with imperfect fishing selectivity, fishing the old mature stock only may be an optimal option even if the fleet two target harvest cost is above that of fleet one. This may even hold if there is no bycatch.

The above analysis demonstrates, somewhat surprisingly, that cost conditions indicating the various profit maximizing harvesting schemes may be quite similar to what was found under perfect targeting. However, imperfect harvesting technology may have crucial effects on the size of the standing biomass and stock composition. To see this, let us consider scheme ii) with $E_2 = 1/q_2$ and $E_1 = 0$ so that harvesting the old mature stock completely with fleet two only is optimal. In this case, as just seen, the spawning constraint (3') reads $X_2 = s_1(1-\tilde{q}_1 / q_2)X_1$. Compared to the same optimal harvest option without bycatch, i.e., when $\tilde{q}_1 = 0$, the spawning constraint now intersects with the recruitment constraint (4') further down. With bycatch as described here, it is hence beneficial to harvest more and keep smaller sizes of both subpopulations. The stock composition is also different as the proportion of young mature is higher than in the fishery with perfect targeting. Therefore, we find here that the reduction in the young mature stock spills over to an even larger reduction in the old mature population (see also section two above),

It is also possible to say something about the profitability effect of bycatch. In harvesting scheme ii) the optimal profit is described by $\pi = p_2 w_2 X_2 + p_1 w_1(\tilde{q}_1 / q_2)X_1 - c_2 / q_2$. When inserting for the spawning constraint, the profit can further be written as $\pi = p_2 w_2 s_1(1-\tilde{q}_1 / q_2)X_1 + p_1 w_1(\tilde{q}_1 / q_2)X_1 - c_2 / q_2$. Differentiating and evaluating at $\tilde{q}_1 = 0$ yields $\partial\pi / \partial\tilde{q}_1 = p_2 w_2 s_1(\partial X_1 / \partial\tilde{q}_1) - (p_2 w_2 s_1 - p_1 w_1)(X_1 / q_2)$. Because $\partial X_1 / \partial\tilde{q}_1$ is negative, we hence find that bycatch for sure has a negative profitability effect if $(p_2 w_2 - p_1 w_1 / s_1) \geq 0$,

i.e., if the old mature stock value (Euro per fish) is not strictly below the biological discounted young mature stock value. For many fish stocks, this will typically hold[9]. We then state:
Result 4: Bycatch may reduce profitability in the age structured model

This result contradicts intuitive reasoning as the presence of bycatch compared to the perfect selective harvesting situation can be interpreted as if joint production replaces a single good production with zero additional production costs. We may also think of such transition as a costless technological improvement, i.e., the fishing fleet is able to harvest more with the same amount of effort. The possible negative profitability effect must therefore be explained by two effects where the above described pattern is the first effect. The second effect is that bycatch also means that a fixed proportion of unintended catch is landed for every ton of the targeted fish stock. This reduces the flexibility of targeting age classes separately and the possibility of controlling the fish stocks in an optimal way. In the above described case ii), this second effect dominates. In this case, we also find that catching the young mature as bycatch leads to reduction in recruitment and hence lower stock biomass also of the old mature. A corollary of Result 4 is therefore that better designed fishing gear that reduces unintended fishing may increase profitability. However, the opposite may certainly also hold when, say, changes in the bycatch pattern leads to shifts between different optimal harvesting schemes (numerical section below).

7. Numerical illustration

The above theoretical reasoning will now be demonstrated numerically. This is merely an illustration and the chosen parameter values are not related to any particular fisheries. Table 1 shows these parameter values. In the baseline scenarios, the same natural survival coefficient for the young and old mature is assumed while the weight of the old mature is assumed to be 50 percent higher than the young mature. Similar catchability coefficients, unit effort cost of the two fleets and equal fishing prices are assumed. The bycatch catchability coefficients are similar as well, and are assumed to be one fifth of the targeted coefficients for both fleets.

Table 2 about here

[9] Again, we may refer to the North Sea cod data mentioned in footnote 5.

The results from the maximum sustainable yield problem (section three above) are reported first. As shown, the biological discounted biomass content steers the fishing mortality (Result 1). In the baseline scenario, we find old mature fishing only to be optimal with $f_2 = 1$. When the old mature survival coefficient s_2 increases making the difference $(w_2 / s_2 - w_1 / s_1)$ smaller, the optimal solution, as expected, eventually shifts from scheme ii) to iii). When reducing the young mature survival coefficient while keeping the old survival coefficient at its baseline value, we find case i) with fishing of the young mature age class together with complete harvesting the old mature stock as the optimal scheme. As shown (section three), case i) represents the optimal solution only if s_1 does not exceed a certain limit. When increasing the fertility parameter of the old mature stock while keeping all the other parameters unchanged, both stock sizes increase while the fishing mortalities, as expected, are left unchanged.

Figure 3 indicates the sensitivity of the maximum sustainable yield problem under the baseline parameter scenario solution where case ii) with $f_2 = 1$ and $f_1 = 0$ represents the optimal solution. Therefore, with $f_2 = 1$ and increasing values of f_1, the sustainable yield reduces (upper curve). With $f_2 = 0.5$, the sustainable yield stays more or less unchanged for a young fishing mortality up to 0.6 – 0.7 before it reduces moderately. With a zero old mature fishing mortality (lower curve), the maximum sustainable yield increases more or less over the whole range of a more intensive exploitation of the young mature group.

Figure 3 about here

In the maximum rent problem with perfect fishing selectivity (section five above) it is confirmed that any possible optimal scheme implies different biological discounted marginal profit for the two fishing fleets (Result 2). Fishing the old mature stock only is optimal in the baseline parameter value scenario with $c_2 / q_2 = c_1 / q_1$. See Table 3, last row. Sensitivity analysis under the baseline parameter values (not reported) indicates very much the same pattern as in the maximum sustainable yield problem shown in Figure 3. When shifting the fleet two harvest cost c_2 up while keeping all the other parameters constant at their baseline values, it becomes relatively more profitable to harvest young mature fish. For a certain threshold level of c_2, case iii) where the old mature stock is harvested only partially becomes optimal. See Figure 4, lower panel. When c_2 increases further, the optimum harvest policy switches to case iv) with no harvesting of old mature and partial harvesting of young mature fish. While the standing biomass monotonically increases with higher c_2 before this second threshold is reached, it suddenly falls for harvesting cost larger than this threshold (upper panel). Because nothing is harvested of old mature fish in case iv), a further increase in c_2 has no effect on the standing biomass. These shifts illustrate Result 3. Similar outcomes may be obtained when shifting the fleet one harvest cost as well as prices and catchability coefficients.

Figure 4 about here

In the baseline maximum rent problem with imperfect selectivity, we find zero fleet one harvesting and fleet two harvesting well below maximum fishing mortality, i.e., case iii), to be optimal (row one Table 3). As suggested (Result 4), profit is lower than with perfect fishing selectivity (last row) even if both stocks and the standing biomass are higher. With knife-edge selectivity ($\tilde{q}_2 = 0$), fleet two fishing only is still optimal, but now the old mature stock is fished completely. Therefore, this solution gives the same result as with perfect targeting under the baseline parameter assumption (last row). Finally (row three), it is seen that profit may decline even with higher stock sizes when the bycatch coefficients shift up. Here the optimal fishing effort levels change back to case iii).

Table 3 about here

8. Concluding remarks

In this paper we have considered a simple formulation of a 'complete' age structured fishery model; that is, there is a harvest trade-off included and recruitment is endogenously determined. Both the maximum sustainable yield problem and the maximum rent problem, the last with perfect as well as imperfect harvesting selectivity, are studied. The analysis shows that even in this simple framework and where biological equilibrium only is considered, very few straightforward results appear. On the one hand, we find that differences in weight and survival among the harvestable stocks as well as differences in costs and technology among the fleets targeting the stocks play an important role in explaining the socially (overall) optimal harvest pattern. On the other hand, surprisingly enough, differences in the reproductive potential, or fertility, among the mature stocks play no direct role.

In the maximum sustainable yield problem it is first shown that the weight- survival ratio, or the biological discounted biomass content, steers the optimal policy, and that higher fishing mortality of the old mature fish than the young mature fish will represent the optimal policy under the assumption that the biological discounted biomass content is higher for the old stock. When next analyzing the maximum rent problem with perfect harvesting selectivity, we find that any possible optimal fishing scheme implies different biological discounted rent among the two harvestable fish stocks. Targeting both stocks, the old mature stock only or the young mature stock only may all represent possible optimal policies. In this problem, the condition for harvesting the old mature stock only is found as well, and it is analyzed how continuous changes in costs and harvesting technology will change the optimal harvesting from targeting one age class, the other age class, or both age classes. Results that contrast well know outcomes from the standard lumped parameter (surplus production) model are also demonstrated. In the maximum rent problem with imperfect harvesting selectivity, we also find corner solutions only to be optimal, and it is shown that bycatch may reduce profitability within our social planner optimizing framework.

One important policy implication of our analysis is that a 'balanced' fishing policy that implies equal fishing mortality among the fishable stocks, will lead to economic losses. Any Total Allowable Catch (TAC) policy should hence be based on specified fishing mortalities for the various age classes which generally will be different. With imperfect targeting and bycatch an optimal TAC policy demands more information than without bycatch. Another important implication is that a policy enforcing a more selective fishing gear can reduce as well as increase total profitability of the fishery.

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APPENDIX

Interior solution maximum rent problem with perfect fishing selectivity

To prove that an interior solution is a saddle point in the maximum rent problem with perfect fishing selectivity (section five), we consider the unconstrained optimization problem where the derived explicit expressions for X_i ($i = 1,2$) (section two) are inserted into the profit function. The profit is then described by

$$\pi = p_1 w_1 q_1 E_1 (s_0 a - \frac{b}{1+\alpha A}) + p_2 w_2 q_2 E_2 A\left[s_0 a - \frac{b}{1+\alpha A} \right] - c_1 E_1 - c_2 E_2$$

with $A = \dfrac{s_1 (1 - q_1 E_1)}{1 - s_2 (1 - q_2 E_2)}$ (see also section two main text), and is expressed as a function of A and E_2. Doing some rearrangements we first find

$$\pi = \left[\frac{p_1 w_1}{s_1} \left[s_1 - \left(1 - s_2 + s_2 q_2 E_2 \right) A \right] + \frac{p_2 w_2}{s_2} s_2 q_2 E_2 A \right] \left[s_0 a - \frac{b}{1+\alpha A} \right] - \frac{c_1}{s_1 q_1} \left[s_1 - \left(1 - s_2 + s_2 q_2 E_2 \right) A \right] - c_2 E_2$$

.

which further may be written as

$$\pi = \left[\left[\frac{p_2 w_2}{s_2} - \frac{p_1 w_1}{s_1} \right] \left[s_0 a - \frac{b}{1+\alpha A} \right] + \frac{c_1}{s_1 q_1} \right] s_2 q_2 E_2 A - \frac{p_1 w_1}{s_1} b \frac{s_1 - \left(1 - s_2 \right) A}{1+\alpha A}$$

$+ p_1 w_1 s_0 a - \dfrac{c_1}{q_1} - \left[\dfrac{p_1 w_1}{s_1} s_0 a - \dfrac{c_1}{s_1 q_1} \right] \left(1 - s_2 \right) A - c_2 E_2$. The Hessian matrix is $H = \begin{pmatrix} \pi_{AA} & \pi_{AE_2} \\ \pi_{AE_2} & 0 \end{pmatrix}$

with $\partial^2 \pi / \partial A^2 = \pi_{AA} = -2 \dfrac{b\alpha}{\left(1+\alpha A \right)^3} \left[\dfrac{p_1 w_1}{s_1} \left(1 - s_2 \left(1 - q_2 E_2 \right) + \alpha s_1 \right) - p_2 w_2 q_2 E_2 \right]$, and

$\pi_{AE_2} = \left[\left(\dfrac{p_2 w_2}{s_2} - \dfrac{p_1 w_1}{s_1} \right) \left[s_0 a - \dfrac{b}{\left(1+\alpha A \right)^2} \right] + \dfrac{c_1}{s_1 q_1} \right] s_2 q_2 E_2$. Because $\det H < 0$, the interior solution is a saddle point.

Interior solution maximum rent problem with imperfect fishing selectivity

To prove that an interior solution in the maximum rent problem with imperfect fishing selectivity (section six) is a saddle point, we again consider the unconstrained optimization problem where the derived explicit expressions for X_i ($i = 1,2$) (section two) are inserted into the profit function. The profit is then described by

$$\pi = p_1 w_1 (q_1 E_1 + \tilde{q}_1 E_2)(s_0 a - \frac{b}{1+\alpha A}) + p_2 w_2 (\tilde{q}_2 E_1 + q_2 E_2) A \left[s_0 a - \frac{b}{1+\alpha A} \right] - c_1 E_1 - c_2 E_2$$

with $A = \dfrac{s_1 (1 - q_1 E_1 - \tilde{q}_1 E_2)}{1 - s_2 (1 - \tilde{q}_2 E_1 - q_2 E_2)}$, such that $E_1 = \dfrac{s_1 - (1 - s_2) A - (s_1 \tilde{q}_1 + s_2 q_2 A) E_2}{s_1 q_1 + s_2 \tilde{q}_2 A}$. Just as above with perfect selectivity, it follows that the profit, expressed as a function of A and E_2, is linear in E_2. Hence, the determinant of the Hessian matrix is also now negative, so any interior solution must be a saddle point.

Unit cost conditions with imperfect fishing selectivity

The necessary control conditions (19) and (20) are first rewritten as

(19') $$\frac{p_1 w_1 q_1 X_1 + p_2 w_2 \tilde{q}_2 X_2 - c_1}{(s_1 q_1 X_1 + s_2 \tilde{q}_2 X_2)} \leq \mu + \frac{\eta \tilde{q}_2}{(s_1 q_1 X_1 + s_2 \tilde{q}_2 X_2)}$$

and

(20') $$\frac{p_2 w_2 q_2 X_2 + p_1 w_1 \tilde{q}_1 X_1 - c_2}{(s_1 \tilde{q}_1 X_1 + s_2 q_2 X_2)} \leq \mu + \frac{\eta q_2}{(s_1 \tilde{q}_1 X_1 + s_2 q_2 X_2)},$$

respectively. We first look at case iii) with zero fleet one harvesting, $E_1 = 0$, and old mature stock fishing mortality below one, $0 < E_2 < 1/q_2$. Constraint (18) is then not binding and $\eta = 0$. With (19') as an inequality and (20') as an equation, case iii) hence implies directly a higher biological discounted marginal profit of fleet two than fleet one. This may also be written as

$$\Delta = \frac{p_2 w_2 q_2 X_2 + p_1 w_1 \tilde{q}_1 X_1}{(s_1 \tilde{q}_1 X_1 + s_2 q_2 X_2)} - \frac{p_1 w_1 q_1 X_1 + p_2 w_2 \tilde{q}_2 X_2}{(s_1 q_1 X_1 + s_2 \tilde{q}_2 X_2)} > \frac{c_2}{(s_1 \tilde{q}_1 X_1 + s_2 q_2 X_2)} - \frac{c_1}{(s_1 q_1 X_1 + s_2 \tilde{q}_2 X_2)}.$$

Inserting for the spawning constraint $X_2 = AX_1$ (section two), we next find

$\Delta = \frac{p_2 w_2 q_2 A + p_1 w_1 \tilde{q}_1}{(s_1 \tilde{q}_1 + s_2 q_2 A)} - \frac{p_1 w_1 q_1 + p_2 w_2 \tilde{q}_2 A}{(s_1 q_1 + s_2 \tilde{q}_2 A)}$. Doing some small rearrangements, it can be shown easily that $\Delta > 0$ when $p_2 w_2 / s_2 > p_1 w_1 / s_1$, which holds per assumption. Therefore, with $\Delta > 0$ case iii) will be an optimal option for sure if $\frac{c_1}{(s_1 q_1 X_1 + s_2 \tilde{q}_2 X_2)} \geq \frac{c_2}{(s_1 \tilde{q}_1 X_1 + s_2 q_2 X_2)}$.

However, we may also find that $\Delta > 0$ holds if the sign of this last inequality is reversed.

We then consider case ii) $E_2 = 1/q_2$ and $E_1 = 0$. The first order conditions (19') and (20') still hold as an inequality and an equation, respectively. Because $\eta > 0$, these conditions indicate higher biological discounted marginal profit for fleet two than fleet one for sure only with

$\frac{\eta q_2}{(s_1 \tilde{q}_1 X_1 + s_2 q_2 X_2)} > \frac{\eta \tilde{q}_2}{(s_1 q_1 X_1 + s_2 \tilde{q}_2 X_2)}$. Inserting for the spawning constraint $X_2 = AX_1$, we find this inequality satisfied as long $q_1 / \tilde{q}_2 > \tilde{q}_1 / q_2$ which holds per assumption (see also Figure 2 main text). Therefore, also this case ii) is met if

$$\frac{c_1}{(s_1 q_1 X_1 + s_2 \tilde{q}_2 X_2)} \geq \frac{c_2}{(s_1 \tilde{q}_1 X_1 + s_2 q_2 X_2)}.$$

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Figure 1. Biological equilibrium with fixed fishing mortalities ($0 \le f_1 < 1$ and $0 \le f_2 \le 1$)**. Beverton-Holt recruitment function. Arrows indicate the dynamics outside equilibrium**

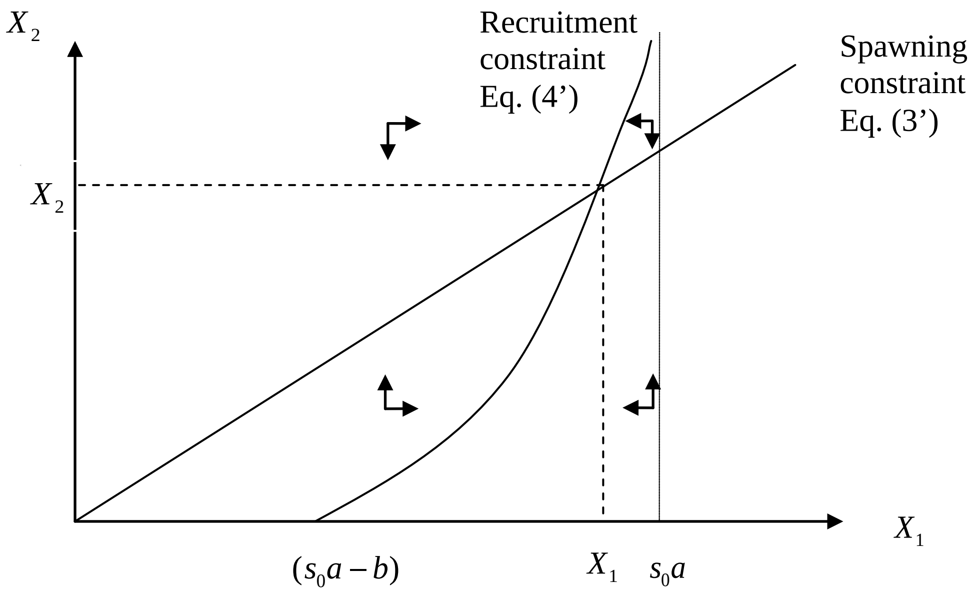

Figure 2. Feasible effort with imperfect fishing selectivity

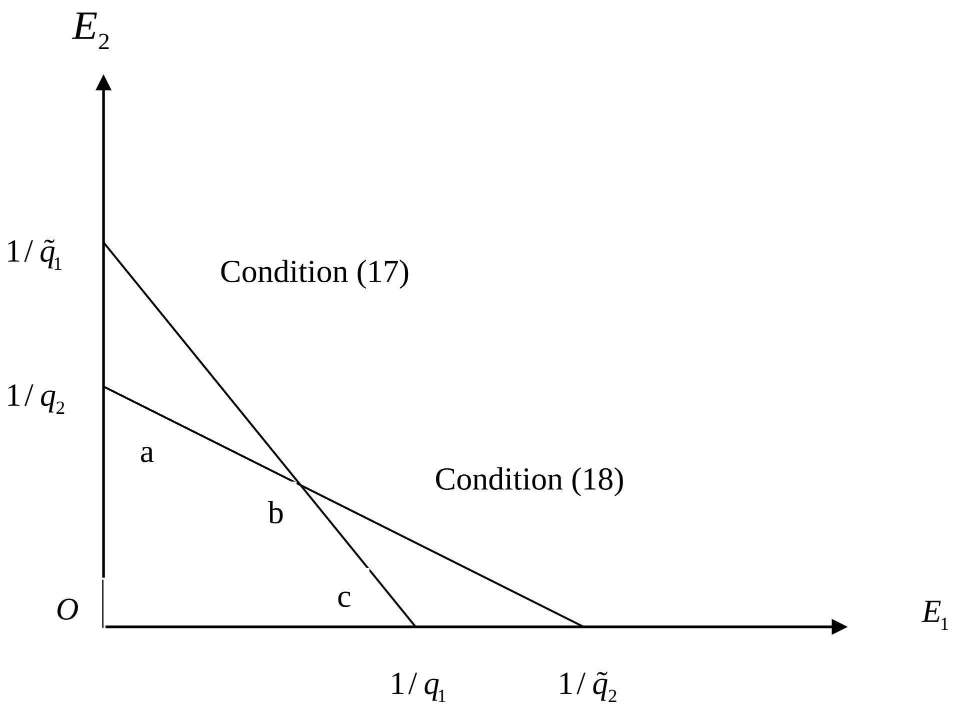

Figure 3. Sensitivity maximum sustainable yield problem. Baseline parameter values

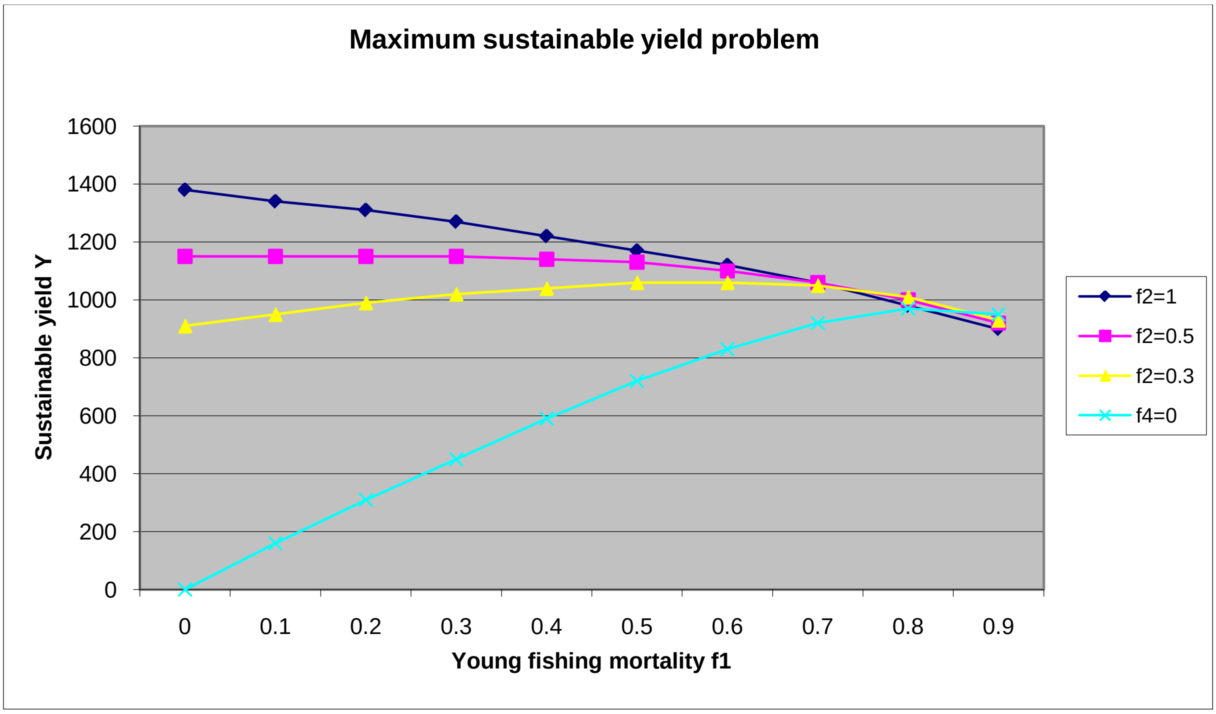

Figure 4. Upper panel: Optimal standing biomass (*Q*)- Lowere panel: Fishing mortalities (solid: old mature f_2, and dashed: young mature f_1). Base line parameter values and changes fleet two effort cost c_2

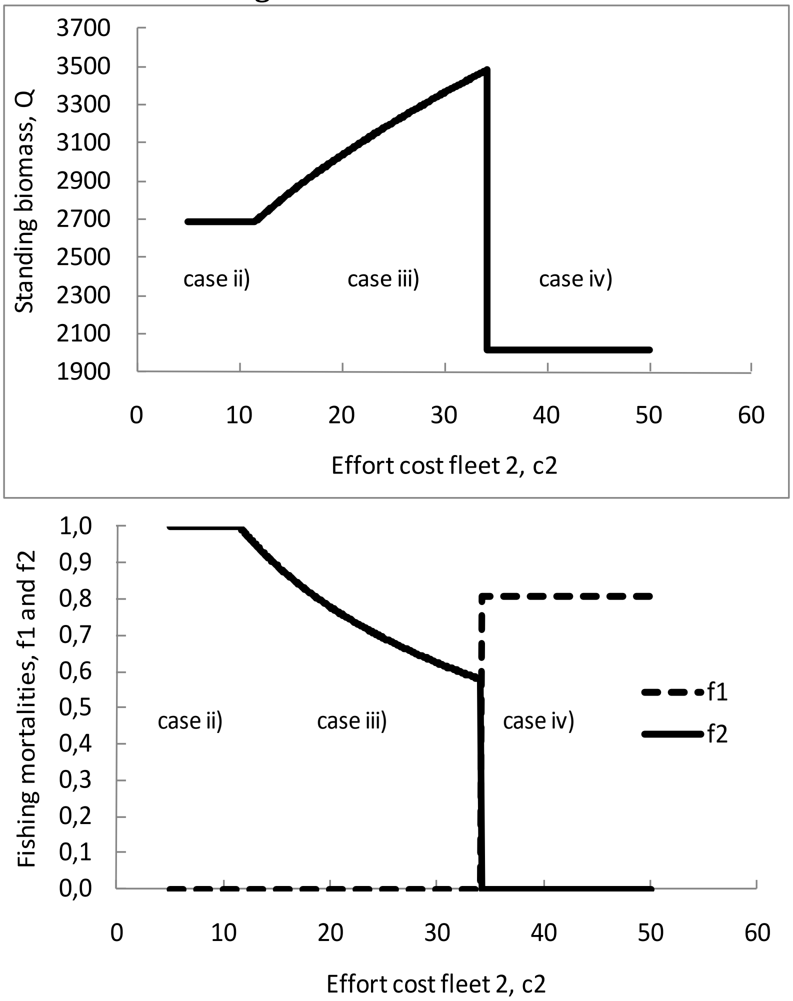

Table 1. Possible optimal solutions. Perfect fishing selectivity

Solution	Fishing effort (fishing mortality)	Marginal biological discounted profit condition
Case i)	$E_2 = 1/q_2$ ($f_2 = 1$) and $0 < E_1 < 1/q_1$ $(0 < f_1 < 1)$	$(p_2 w_2 / s_2 - c_2 / q_2 s_2 X_2) > (p_1 w_1 / s_1 - c_1 / q_1 s_1 X_1)$
Case ii)	$E_2 = 1/q_2$ ($f_2 = 1$) and $E_1 = 0$ $(f_1 = 0)$	$(p_2 w_2 / s_2 - c_2 / q_2 s_2 X_2) > (p_1 w_1 / s_1 - c_1 / q_1 s_1 X_1)$
Case iii)	$0 < E_2 < 1/q_2$ $(0 < f_2 < 1)$ and $E_1 = 0$ ($f_1 = 0$)	$(p_2 w_2 / s_2 - c_2 / q_2 s_2 X_2) > (p_1 w_1 / s_1 - c_1 / q_1 s_1 X_1)$
Case iv)	$E_2 = 0$ ($f_2 = 0$) and $0 < E_1 < 1/q_1$ $(0 < f_1 < 1)$	$(p_1 w_1 / s_1 - c_1 / q_1 s_1 X_1) > (p_2 w_2 / s_2 - c_2 / q_2 s_2 X_2)$

Table 2. Biological and economic baseline parameter values

Parameter	Description	Baseline value
s_0	Natural survival rate recruits	0.6
s_1	Natural survival rate young mature	0.7
s_2	Natural survival rate old mature	0.7
a	Scaling parameter recruitment function	1500 (number of fish)
b	Shape parameter recruitment function	500 (number of fish)
α	Fertility parameter	1.5
w_1	Weight young mature	2.0 (kg/fish)
w_2	Weight old mature	3.0 (kg/fish)
q_1	Catchability coefficient fleet one	0.05 (1/effort)
q_2	Catchability coefficient fleet two	0.05 (1/effort)
$\tilde{q}_2$	Catchability coefficient fleet one bycatch	0.01 (1/effort)
$\tilde{q}_1$	Catchability coefficient fleet one bycatch	0.01 (1/effort)
p_1	Fish price young mature	1 (Euro/kg)
p_2	Fish price old mature	1 (Euro/kg)
c_1	Effort cost fleet one	10 (Euro/effort)
c_2	Effort cost fleet two	10 (Euro/effort)

Table 3. The maximum rent problem with imperfect selectivity. f_i fishing mortality, E_i fishing effort, X_i number of fish ($i = 1$ young mature fish, $i = 2$ old mature fish), Y sustainable yield (kg) and Q standing biomass (kg), π profit (Euro)

	E_1 (f_1)	E_2 (f_2)	X_1	X_2	π	Y	Q
Baseline parameter values	0 (0)	13 (0.65)	674	542	1,071	1,069	2,974
$\tilde{q}_2 = 0$	0 (0)	20 (1.00)	656	459	1,177	1,377	2,689
$\tilde{q}_1 = 0.02$ $\tilde{q}_2 = 0.02$	0 (0)	10 (0.49)	685	601	1,016	879	3,175
$\tilde{q}_1 = 0$ $\tilde{q}_2 = 0$	0 (0)	20 (1.00)	656	459	1,177	1,377	2,689

Table note: Fishing mortalities in brackets. Last row: No bycatch and perfect selectivity