The Supply of Public Sector Services when They Include Quantity and Quality Dimensions

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Most publicly provided services are multi-dimensional. This paper distinguishes between quantity and quality. In a model of majority voting with two competing political parties, the income effects with respect to quantity, quality, and other elements in the utility functions of the voters have ambiguous signs because the budget constraint of the public sector is non-linear. In addition, matching grants may have smaller effects than in traditional models with linear budget constraints. Data from Norwegian high schools are utilized in an empirical example. The income elasticity with respect to the number of students seems to be negative, the income elasticity of quality seems to be positive, and a matching grant related to the number of students has no effect on student enrollment. (JEL: H 40, I 22)

1. Introduction

The traditional public finance literature assumes that the public sector supplies one single public good. In reality, publicly provided services are more like private goods and they contain several dimensions that make measurement of supply complex. The designs of public services are important in an evaluation of the public sector. In situations of financial stress due to budget deficits, the pressure to reduce public expenditures often results in a debate about whether to cut the availability of public services or to make them less favorable for the users. When the public sector expands, there is a tendency to increase the physical size of some services at the same time as quality increases in other services.

This paper discusses public sector decision-making when public services are multi-dimensional. It is useful to distinguish between quantity and quality, a distinction that dates back to at least Hirsch (1968). He defines a basic service unit as the physical output of a specified quality level. This may be dif-
ficult to measure empirically, but measurement of quantity is an underlying premise for all comparison of specific public services across governments. In many cases it is useful to interpret quantity as the number of users of the service considered. In schooling this is the number of students and in Medicare it is the number of people included in the system. In the Nordic countries, the number of users is a political decision-variable in a wide range of highly subsidized services with limited private alternatives such as kindergartens, high schools (which are discussed closer below), care for the elderly, and hospitals. For these services, we often observe excess demand; there is rationing by waiting lists, or some of the applicants formally qualified to receive the service are not given access. In other cases the basic service unit depends on other characteristics. For infrastructure, for example, quantity is related to the length of the highway network and the number of airports.

The other dimension of public services, quality, is even harder to measure because quality assessment may require subjective judgment, and perception of quality can vary between users, service providers, voters, and politicians. But clearly, parents care about teacher competence, drivers care about road standards, and hospital patients are concerned about the probability of unsuccessful treatment. The different dimensions of public services are perhaps most clearly visible when the governments use competitive tendering and contracting-out. The formal contracts between governments and service producers specify pre-determined levels of both quantity and quality together with the financial payments, see Domberger and Rimmer (1994).

The consequences of a trade-off between quantity and quality of public services are to my knowledge not discussed in formal optimization models of public sector decision-making. For design decisions, the budget constraints are nonlinear. Total expenditures $Z$ on a particular public service can be written $Z = NQ$, where $N$ is quantity measured as the number of users of the service and $Q$ is spending per user. Since changes in quality have financial implications in a utility maximization optimum, $Q$ may be considered as an index of quality. This formulation implies that the politicians are faced with a budget constraint that is multiplicative in the choice variables.

In an interesting article, Sandmo (1973) suggests an alternative way to think about public service production. He argues that the consumers produce the final good consumed by the use of both private goods and public goods as inputs in the production process. Sandmo uses highways as an example. The final consumption good of a particular consumer may be seen as a trip between two points, and the inputs are consumer time and gasoline together with the road itself. Within this setup, quantity of highways can be thought of as the sum of individual decisions about journey miles. But nevertheless, several dimensions of the highways will influence the journey decision, and those dimensions will enter the production functions of the consumers.
Nonlinear constraints are prevalent in several areas of economic decisions. The distinction between quantity and quality was introduced in consumer choice models in the seminal papers of Houthakker (1952) and Theil (1952). The consequences of nonlinear budget constraints are further explored by Borjas (1979), Edlefsen (1981), and Blomquist (1989). One particular important result is that increased income will raise the expenditure on a commodity under normality conditions, but it cannot be established whether both the quantity and quality of the commodity will increase. A multiplicative budget constraint is also prevalent in the literature on fertility. To have children includes both the number of children and the quality of child bearing. Becker and Lewis (1973) and Becker and Tomes (1976) distinguish between “true” and “observed” income effects. The distinction is fruitful because when the budget constraint is multiplicative, the “shadow” price of quantity depends on quality, and the “shadow” price of quality depends on the quantity chosen. The true income effects are defined as the effects when shadow prices are kept constant, and they are therefore positive under normality conditions. However, the observed income effects in addition include shadow price changes, which might make observed income effects negative.

For example Willis (1973) presents evidence of a negative income elasticity with respect to the number of children in the U.S., while Rosenzweig and Wolpin (1980) conclude that there is a negative relationship between quantity and quality of children in India.

A specification of political institutions is necessary to have an equilibrium model of the political decision-making when a multi-dimensional publicly provided service is financed by taxes because the political decision-problem is multi-dimensional, see for example Persson and Tabellini’s (1999) overview of political decision-making models. I will use a model with two competing political parties along the lines of Lindbeck and Weibull (1987) and Dixit and Londregan (1995; 1996). The political parties commit to policy platforms ahead of elections. The electorate consists of identifiable groups of voters, and the voters within each group are heterogeneous in their non-economic preferences of the two parties. When they vote, these preferences are traded off against the economic benefits of the parties’ policy platforms.

\footnote{Stiglitz (1987) discusses instances in which a price “conveys information and affects behavior. Quality depends on price” (p. 3). Examples are capital markets with uncertainty about bankruptcy, insurance markets with uncertainty about risks, and labor markets with uncertainty about worker productivity. Models where the productivity of workers depends on the wage paid are especially well developed. Both the wage and employment are firm specific choice variables, and they enter the cost function multiplicatively. This nonlinearity is the driving force of a special result in this literature. It is in general unclear whether a firm should increase both the wage and employment as a response to increased product demand. Borjas (1980) and Johansen and Strøm (1997) discuss such models in a public sector institutional setting.}
In contrast to Lindbeck and Weibull (1987) and Dixit and Londregan (1995; 1996) who use the model to discuss income redistribution, the policy instruments in the present paper are publicly provided services and a linear income tax. The outcome of the model is as if the political parties maximize a well–behaved “welfare” function, separable across voter groups, where the weights attached to each group are endogenously determined.

The model is presented in section 2. Section 3 shows that the income effects of quantity and quality of a public service are ambiguous under normality conditions. The nonlinearity introduced in the budget constraint also implies that the sign of the income effects with respect to other elements in the utility functions of the voters, such as private consumption, are ambiguous. Thus, once the distinction between quantity and quality is important for one public sector service, the comparative static results for other goods also give few predictions. Nonlinear budget constraints may have consequences for the central government regulation of the local public sector. The model includes a federal matching grant related to quantity. Even though the income effect of quantity is uncertain, the effect of this type of matching grant is positive. However, due to the possible negative income effect, the effect of the quantity subsidy may be very small. A matching grant may also plausibly lead to a “corner” solution of the model.

Since the signs of the comparative static effects are ambiguous, empirical evidence is crucial to predict design decisions. Section 4 of the paper presents evidence from Norwegian high school education. For these schools, which are free of charge and the responsibility of the counties, student enrollment and education quality factors are decided simultaneously. The central government used a matching grant related to the number of students to an increasing degree from the late 1980s. Nevertheless, the central government was not satisfied with the student enrollment, and in a reform in 1994, each 16–year–old was given the right to start on high school education. The mechanisms in the model in the present paper may be one explanation of the regulatory changes. The empirical results indicate that the income effect with respect to the number of students was negative, while two different quality measures both were positively related to income. I cannot reject that the matching grant related to the number of students had no effect on student enrollment. Section 5 concludes.

2. The Model

The voters are divided into $G$ groups, distinguished by any criteria such as age, health, geographical location, or income. Following Lindbeck and Weibull (1987) and Dixit and Londregan (1995; 1996), I assume that the voters within
each group are homogenous in all aspects except for their “non–economic” preferences of the political parties. Attachments to non–economic factors $\Phi$ are referred to as ideological preferences, and may include considerations of human values, religious questions, or simply personal characteristics of the leadership of the political parties. The “extended” utility $W^v$ of voter $v$ in group $i$ consists of both ideology preferences and the preferences related to the economic policy outcome $U$:

$$W^v = U + \Phi^v$$

$U$ depends on the level and design of public sector services in addition to private consumption. Consider a specific public sector service that is directed against only one group of the electorate. It is possible to control the access to this publicly provided private good. The decision problem is partly to determine total access, denoted quantity $N$, and quality $\Theta = \Theta(Q)$ of the publicly provided private good. $Q$ is spending per unit quantity, $\Theta'(Q) > 0$ and $\Theta''(Q) < 0$. The quasi–concave economic utility function of the members of the group receiving the publicly provided private good, say group 1, is

$$U_1 = u_1(N, \Theta(Q)) = U_1(N, Q);$$

where subscripts denote partial derivatives. Increased quantity increases the probability of being a user of the public service, and increased quality increases the utility of using the service. Income and private consumption of group 1 are normalized to zero, and the membership is normalized to unity. Thus, $N$ is the share of the members that have access to the service. This utility function is further discussed below.

For comparison with the standard approach in public finance, the model includes a public good $X$. For presentation simplicity, I assume that $X$ is only available for the $G - 1$ tax paying voter groups. The economic utility functions of these groups are

$$U_i = U_i(X, C);$$

where $C$ is private consumption. I assume a linear income tax system, $C = (1 - t)Y$, where $t$ is the tax rate and $Y$ is the exogenous income level. Compared to Lindbeck and Weibull (1987) and Dixit and Londregan (1995; 1996), the possibility to redistribute across groups is limited. It is not possible to transfer money to any group, and it is not possible to increase the public sector income without a higher tax on all tax paying groups.

Following a tradition since Downs (1957), it is assumed that the political parties maximize the number of votes. The economic utility levels of group $i$ according to the policy platform of the political parties denoted $L$ and $R$
are $U^{iL}$ and $U^{iR}$. A voter votes for party $L$ if $U^{iL} - U^{iR} > Φ^{iv}$. Even though party $L$ offers a more favorable policy platform to a group than party $R$, only a fraction of the group will vote on party $L$, and vice versa. The cumulative distribution function $F'(Φ^i)$, with density $f'(Φ^i)$, denotes the proportion of the group members to the left of $Φ^i$. The number of votes on party $L$ is $M^i F'(Φ^i)$, where $M^i$ is group size. Adding over groups, the total vote for party $L$ and $R$, respectively, are

$$V^L = \sum_{i=1}^{G} M^i F'(Φ^i),$$

(4)

$$V^R = \sum_{i=1}^{G} M^i [1 - F'(Φ^i)].$$

(5)

$V^L$ and $V^R$ are the objective functions of the political parties. The policy platforms are quadruples\(^3\), $(N, Q, X, t)$.

The main features of the model are valid for all types of governments. In the following, however, I will have a local government in mind because the quantity–quality tradeoff has interesting implications for the regulation of the local public sector. For local governments, matching grants may be non-symmetrically related to quantity and quality. Assume that the local government gets a matching grant $g$ related to the quantity of the publicly provided private good; $g$ is a quantity subsidy. Increasing the quality of the service does not influence the money received from this type of matching grant. Let $Z$, $q$, and $p$ denote, respectively, an unconditional grant from the central government, the price of the publicly provided private good, and the price of the public good. The budget constraint is

$$t Y + Z = qQN - gN + pX = (qQ - g)N + pX,$$

(6)

where $\sum_{i=1}^{G} M^i Y^i$ is the total income level of the society\(^4\). The budget constraint is multiplicative in the choice variables $N$ and $Q$.

The optimization problem of the political parties does not include party-specific variables, the two parties have the same decision problem. In Nash equilibrium, the policy that maximizes the number of votes for one party must necessarily also maximize the number of votes for the other party. Thus, it

\(^3\)The actual policy may, of course, differ from the policy platforms set prior to elections. The model does not discuss how the political platform of the winning party is transformed into economic outcome. In the model, however, the relationship between quantity and quality is solely decided by the preferences of voter group 1. Thus, the qualitative results for the quantity–quality relationship are valid of all models relying on well-behaved objective functions.

\(^4\)The price of the publicly provided private good, $q$, may be interpreted in several ways. For example, if there is a matching grant $m$ related to $NQ$, and $Q$ is employment per unit quantity with wage $w$, $q = w(1 - m)$. 
is sufficient to analyze the optimal policy for one party to characterize the
equilibrium. The first order conditions for party $L$, taking the policy of party
$R$ as given, are

\[ \frac{\partial V_L}{\partial N} = f^i(0)U_N^i - \lambda(qQ - g) - \mu = 0, \quad (7) \]
\[ \frac{\partial V_L}{\partial Q} = f^i(0)U_Q^i - \lambda qN = 0, \quad (8) \]
\[ \frac{\partial V_L}{\partial X} = \sum_{i=2}^{G} f^i(0)M^iU_X^i - \lambda p = 0, \quad (9) \]
\[ \frac{\partial V_L}{\partial t} = - \sum_{i=2}^{G} f^i(0)Y^iU_C^i + \lambda = 0. \quad (10) \]

$\lambda$ and $\mu$ are the Lagrange multipliers related to the budget constraint and
$N \leq N$, respectively, and $y_i = M^iY^i/\Upsilon$ is the income share of group $i$. Since
the policy platforms of the parties are equal in Nash equilibrium, all density
functions are evaluated for $\Phi^i = 0$. The optimization problem can be
interpreted as if the parties maximized a well–behaved “welfare” function
separable across groups. Groups with a high density of voters in the center
of the ideological spectrum ($\Phi^i = 0$) and high marginal utilities have large
weights in the “welfare” function.

Observe from (7) and (8) that when $\mu = 0 (N < 1)$, $(qQ - g)$ is the “shad-
low” price of $N$ and $qN$ is the “shadow” price of $Q$. The important feature
of the model is that the shadow price with respect to quantity depends on
quality, and the shadow price of quality depends on quantity. A rise in quan-
tity is more expensive when quality is high than when quality is low, and vice
versa. The relative shadow price between $N$ and $Q$ in optimum is

\[ \frac{U_N^i}{U_Q^i} = \frac{qQ - g + \mu/\lambda}{qN}. \quad (11) \]

For $\mu = 0$, the relationship between quantity and quality depends only on the
preferences of the group eligible of the publicly provided private good and
the parameters in the budget constraint directly related to this good. Indeed,
when $g = 0$, the ratio between the marginal utilities is equal to the inverse
ratio of the variables themselves. When $\mu > 0$, the model essentially includes
two public services $Q$ and $X$, and has a linear budget constraint.

Eq. (9) is a modified Samuelson condition for optimal supply of public
goods. In optimum, the sum of the marginal utilities of the public good,
weighted by $f^i(0)$, is equal to the marginal cost in utility terms of providing
the public good. The optimality condition for the tax rate is complicated by
the fact that the private income level varies across groups. Eq. (10) says that
a weighted average of the voters’ marginal utilities of private consumption is equal to the marginal utility of the public sector income $\lambda$.

3. Income and Price Effects

The comparative static results are presented by two Propositions. It is assumed that the second order condition is fulfilled. Define total private consumption by $\Gamma = \sum_{i=1}^{G} M_i C_i$. The use of the following Lemma simplifies the comparative static analysis.

**Lemma 1** If $y^i$ is independent of $\Upsilon$, $d\Lambda/d\Upsilon = d\Lambda/\Upsilon$, where $\Lambda = N, Q, X, \Gamma$.

The proof of Lemma 1 and the subsequent Propositions are in the Appendix. In a median voter model, the workhorse in the public finance literature, the effects of the income of the median voter and an unconditional grant are equal if the median voter income is equal to the mean income, see for example Fisher (1982). Lemma 1 has the same result because a rise in $\Upsilon$ is equivalent to a rise in the mean income level when $y^i$ is independent of $\Upsilon$.

The effects of $\Upsilon$ and $Z$ will be denoted the income effect.

I will only consider the case when the normality conditions are fulfilled. Then the income effects are positive in traditional models with linear budget constraints. This is not the case in the present model.

**Proposition 1**

(i) The income effects with respect to all decision variables are ambiguous.

(ii) The income effect with respect to $NZ$ is positive.

For quantity and quality, Appendix 6.2 establishes that the normality conditions cannot determine the signs of the income effects. Figure 1 illustrates the case of a negative income effect of quantity in the “two–good” case, $t = X = 0$. The initial outcome is at point $A$ at the tangency of the indifference curve $U^0 U^0$ and the rectangular hyperbola $Z^0 Z^0 = qNQ$. In the figure, the outcome after the rise in grants to $Z^0 Z^0$ is point $C$. Quantity decreases while quality increases substantially. Becker and Lewis (1973) define a movement from $A$ to $C$ as the “observed” income effects, distinguishing between “observed” and “true” income effects. True income effects emerge when the shadow prices are kept constant. In this case, the new outcome will be point $B$ if the outcome is restricted to be on $U^0 U^0$. The true income effect of quantity

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5 Notice that this is not a trivial assumption since the budget constraint is quasi–concave in the choice variables of the publicly provided private good $X$. Whether the second order condition is fulfilled depends on the curvature of the utility function of group 1 and the budget constraint as explored in the Appendix.
True and Observed Income Effects ($g = 0$) is positive even though the observed effect is negative. The relative shadow price does not remain constant when $Z$ increases unless, in the case $g = 0$, the true income effects change $N$ and $Q$ equiproportionately. In figure 1, the true income effects increase $N$ less than $Q$. As a result, the relative shadow price between $N$ and $Q$ increases, which induces a substitution away from $N$. If there is an underlying tendency for the quantity response to be smaller than the quality response, the implication of a multiplicative budget constraint is a smaller quantity response.

Blomquist (1989) discusses how the properties of the traditional consumer choice model with linear budget constraint carries over to the case with a general nonlinear budget constraint. He expresses the comparative statics in terms of the comparative statics of the traditional model and the curvature properties of the actual budget constraint. In a two-good case, say $X_1$ and $X_2$, the following conditions are sufficient for positive income effects: i) the budget constraint is quasi–convex; ii) the relative shadow price between $X_1$ and $X_2$ decreases when $X_2$ increases. None of these conditions are fulfilled in the case of a multiplicative budget constraint.

The analysis above shows that it is impossible to predict the outcome of the public sector decision–making when quantity and quality aspects are involved without having more structure on the utility functions than traditionally imposed. A simple and reasonable utility function in many cases is presented here, while a more complex utility function is presented in section 4. Each individual within group 1 derives utility from the publicly
provided public good only if it is given access to the service. Denote this utility \( h(Q) \). The probability of being a user is equal to \( N \). Normalizing the utility level of the group members without access to zero, the expected economic utility is

\[
U^1 = Nh(Q); \quad h'(Q) > 0, \quad h''(Q) < 0. \tag{12}
\]

In this case, the normality conditions hold and the second order condition is fulfilled. When \( \mu = 0 \), the first order conditions (7) and (8) can be written

\[
\frac{h'(Q)}{h(Q)} = \frac{q}{qO - g}. \tag{13}
\]

The only exogenous variables influencing the optimal level of quality are \( q \) and \( g \). Indeed, if \( g = 0 \), (13) implies that \( Q \) is fixed independent of the rest of the model. Increased income will only affect the quantity of the service. This result has an interesting parallel in the so-called efficiency wage literature. In efficiency wage models, the quality of the workers is positively related to the wage level. Solow (1979) shows that increased product demand has no effect on wages if “the wage enters the production function in a labor augmenting way” (p. 81), i.e. the production \( Y \) is given by \( Y = Y(Lh(w)) \), where \( L \) is employment and \( w \) is the wage level\(^6\). This specification of the production function is widely used, and implies that the wage is independent of variation in output. It follows from the fact that the price of quality (wage in efficiency wage models) depends on the quantity level (employment in efficiency wage models).

The income effect with respect to \( X, \alpha \), and \( \Gamma \) are harder to illustrate graphically because the ambiguity results require a trade-off between quantity and quality of the publicly provided private good. Consider first the case when the utility function of group 1 is given by (12). In this case, (8) can be written

\[
\lambda = f_1(0) \frac{h'(Q)}{q}. \tag{14}
\]

Since \( Q \) is independent of the income level, the marginal utility of public sector income \( \lambda \) is also independent of the income level. Of the parameters in the budget constraint, only \( q \) and \( g \) have an impact on \( \lambda \). Given this result, it follows from (9) and (10) that \( X, \alpha \), and \( \Gamma \) are also independent of income. Increased private sector income has no effect on private consumption and

\(^6\) Notice that in the present model, the signs of the income effects with respect to \( N \) and \( Q \) are independent of whether the marginal utility of \( N \) is constant (as for the utility function (12)) or the marginal utility of \( N \) is diminishing as is the case for the utility function comparable to Solow’s (1979) production function, \( U^1 = U^1(Nh(Q)) \) and \( U^{1r} < 0 \).

\(^7\) It is not a general result that \( d\lambda/dZ = 0 \) when \( dQ/dZ = 0 \). When the marginal utility of \( N \) is diminishing, for example when the utility function has the form \( U^1 = U^1(Nh(Q)) \) and \( U^{1r} < 0 \), \( dQ/dZ = 0 \) and \( d\lambda/dZ < 0 \).
the public good $X$. The Appendix proofs that a necessary condition for “perverse” income effects (including increasing marginal utility of public sector income) is either a negative income effect with respect to quantity or quality.

The possibility of increasing marginal utility of public sector income gives some interesting implications. In general, local governments and public agencies have incentives to manipulate the decision-makers at the central level in order to increase the grant; the local utility level is positively related to the budget size. When the marginal utility of income also is positively related to the budget, the incentives to work for a rise in the budget increases when the budget increases. Non-diminishing marginal utility of income may be one explanation of the causal observation that pressure and resource use to increase budgets is not negatively related to budget size.

Proposition 2 presents the price effects.

**Proposition 2**

(i) $\frac{dN}{dg} > 0$ when $\mu = 0$.

(ii) The partial effects of $g$ with respect to $Q$, $X$, $\Gamma$, and $t$ are ambiguous.

(iii) The partial effects of $q$ with respect to all decision variables are ambiguous. If $g = 0$, then $\frac{dN}{dq} \neq \frac{dN}{dZ}$ and $\frac{dQ}{dq} \neq \frac{dQ}{dZ}$.

(iv) $\frac{dX}{dp} < 0$.

(v) The partial effects of $p$ with respect to $N$, $Q$, $\Gamma$, and $t$ are ambiguous, and $\frac{dN}{dp}/(dQdp) = (dN/dZ)/(dQ/dZ)$.

The effects of the quantity subsidy $g$ on $Q$, $X$, $\Gamma$, and $t$ are ambiguous because the substitution effects are negative. For quantity, the positive substitution effect dominates a possible negative income effect. Irrespective of the size of a negative income effect of quantity, the effect of $g$ is positive by the normality conditions when $N < 1$. Hence, the Giffen paradox will not be observed for normal goods in the traditional sense.

Since the quantity subsidy has a positive effect on quantity, this type of matching grant can be effective in stimulating quantity. However, the effect may be very low since the income effect may be negative. Thus, matching grants may not be a good policy instrument to influence the local public sector outcome in a specific direction. On the other hand, matching grants may also be extremely effective as illustrated in figure 2. This is essentially the same

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8 The situation that there may not be diminishing marginal utility of income is an interesting feature of models with nonlinear budget constraints recently pointed out by Borley (1995). He proves the possibility of a convex indirect utility function in a general consumer choice model with a convex budget constraint. The shadow price of a good depends on the level of the good itself (and not the level of other goods). In this case, there may be a positive income effect on the marginal utility of income even though the income effects with respect to all goods in the model are positive.
A higher $g$ increases the curvature of the budget constraint. For a sufficiently high $g$, the outcome will be the corner solution $N = 1$ as illustrated as point $D$ in figure 2. This can also be achieved by law. If the regulator prefers interior solutions, and the local governments are heterogeneous, the grant level may have to be set very low to avoid some corner solutions. An alternative policy instrument for the central government is to set a minimum value of $Q$, $Q^m > g/q$. The outcome is then point $E$ in figure 2. Services with a quantity–quality trade-off may be one explanation of the extensive use of minimum standards in the developed countries. In the Nordic countries, minimum standards are extensively used for local public sector services where the trade-off between quantity and quality is most striking. Another strategy for the central government is to use a closed–end matching grant, where local governments receive grants only up to a maximum amount of quantity. Even in the simple model in the present paper, the regulator may have to use several instruments to get the desired outcome at the local level.

With a corner solution, the comparative static effects will differ from Proposition 2. The effect of $g$ with respect to quantity is equal to zero when $N = 1$. The effect can also be close to zero in the case of an interior solution. However, the observed income effect differs in these two cases. For $N = 1$, the observed income effect is equal to zero, while in the latter case, it is negative.
Parts (iii)–(v) of Proposition 2 present the results for $q$ and $p$. A rise in $q$ reduces $NQ$ via the traditional income and substitution effects. However, whether both $N$ and $Q$ are reduced is uncertain by the same mechanisms as the ambiguous income effects. When $g = 0$, the signs of the income effects determine the signs of the effects of $q$. A variant of the Giffen paradox cannot be ruled out even when the normality conditions hold. A positive effect of $q$ with respect to $N$ or $Q$ is not an example of a “pure” Giffen paradox since $q$ is a combined price for two “goods”. Result (iv) follows from the fact that the “pure” Giffen paradox is not a possible outcome of the model. The result that $(dN/dp)/(dQ/dp) = (dN/dZ)/(dQ/dZ)$ follows from the separability across voter groups in the political parties’ objective functions.

4. An Empirical Application: Norwegian High Schools

The institutional set-up of the Norwegian public sector high schools is attractive for an empirical example of the quantity–quality trade-off, in particular for the period prior to a major reform in 1994. First, private alternatives are limited. In 1990, private schools, all heavily subsidized by the central government, covered only 5% of the students. Second, the high schools are the responsibility of the counties, and the counties have no freedom on the revenue side of their budget, which implies that the income effect can be estimated by using the total revenues of the counties.

Finally, and most important, while the size of the enrollment is outside the control of the authorities in most school systems, this is not the case for the Norwegian public sector high schools in the empirical period. The county authorities formally determine the number of study places. Even though quantity measured by enrollment is a political decision variable, this institutional setting does not, of course, automatically imply that the number of high school students is supply determined by the counties rather than demand determined by the youth. One critical implication of enrollment being supply determined is that some individuals formally qualified for high school education are denied access. The available information clearly indicates that this has been the case at least since the middle of the 1980s.

9 The Norwegian high school system is described in more detail by Bonesrønning and Rattsø (1994), Briseid (1995), and Falch and Rattsø (1999).
10 To be eligible to public subsidy, the private schools must, compared to the public sector schools, have an alternative pedagogical approach. They are therefore mostly religious schools.
11 In NOU (1991), the share of the high school applicants denied access is estimated to about 6% each year in the period 1986–1989. According to the Ministry of Education (KUF), the number of applicants was 32% above the number of study places in 1989 and gradually declined to 21% in 1993, and, for example in 1990, varied across the counties...
Enrollment has been based on performance in the lower secondary school, other education, and work experience. While the general track, enrolling about 60% of the students, consisted of three consecutive years of study, the students in the vocational tracks had to apply for continuation each year prior to 1994. According to Briseid (1995), there was shortage of study places particularly in the last two years of vocational education. After a high school reform in 1994, 16 years-olds have a statutory right to at least three years of high school education. But still the enrollment is not completely demand determined because the counties seem to restrict the enrollment of older students. Due to the reform, the empirical period in the present analysis is 1976–1993.

The fact that high school enrollment was restricted does not imply that supply factors are more important than demand factors in the enrollment determination. But if enrollment is at least partly supply-determined, the income effect may, according to the theoretical model, be either positive or negative, determined by the utility function of the group to which students belong, basically the youth. The gain of access to education is a wage premium. Assume a dual labor market and a positive relationship between the wage premium of high school education and the quality of the education. In addition, the wage premium depends on the supply of educated labor. When a larger share of the youth attends high schools, the supply increases and the wage premium decreases. The value of the discounted wage of educated and non-educated labor measured in utility terms may be written $\omega(N, Q)$ and $\omega(N)$, respectively, where $w_e(N) < 0$, $w_Q > 0$, and $w_N > 0$. The expected economic utility of the youth is

$$U = N\omega(N, Q) + (1 - N)\omega(N) = U(N, Q),$$

and the sign of the comparative static effects of the model is ambiguous.

From 18 to 41%. These numbers overestimate the excess demand of study places because some applicants who are offered a study place choose not to enroll. In 1990, KUF estimated that 90% of the 16-years-old applicants, which is the students finishing the compulsory lower secondary school, enrolled either in a public or private high school.

12. To investigate how effectively the high school access was restricted prior to the 1994 reform, one possibility could be to compare the pre- and post-1994 enrollment ratios. Such a comparison is complicated by the fact that enrollment only became a statutory right for applicants just finishing the lower secondary school. In addition, such a before–after analysis is problematic because the unemployment rate started to decline in 1994, increasing the probability of finding a job, and thereby probably reducing the attractiveness of high school education.


14. Notice that the voting age is 18 years. The relevant preferences for voting behavior therefore partly reflect the parents’ preferences.
4.1. The Budget Constraint

The grant system of the counties is complicated and involves matching grants related to several different issues. Unfortunately, data are only available for the value of total matching grants. A grant reform in 1986 abolished most of the matching grants, and while matching grants up to 1986 were mainly related to total school spending, grants were to an increasing degree related to the number of students in the period 1986–93. Partly as a response to rising unemployment among youth, the central government wanted to encourage student enrollment by paying subsidies for new classes.\footnote{In 1985, matching grants accounted for 54\% of current school spending at county mean, compared to 7\% in 1986. Measured in 1993–NOK, the matching grant per student increased from 3,850 in 1986 to 11,440 in 1993.}

The counties are responsible for several services in addition to high schools, notably hospitals.\footnote{At sample mean during the empirical period, the hospital service and high school education account for about 55\% and 20\% of total county current spending, respectively. Since 2002, the central government is responsible for the hospital services.} The budget constraint can be written

\[
Z^* = \frac{Sp}{Po} - mS \frac{Sp}{Po} - g^* \frac{St}{Po} + X - mX X, \tag{16}
\]

where \(Sp\) is high school spending, \(St\) is the number of students, \(Po\) is population, \(X\) is hospital spending per capita, \(mS\) is the high school matching grant prior to 1986, \(g^*\) is the high school matching grant from 1986, and \(mX\) is the matching grant to hospitals. The revenue per capita \(Z^*\) (excluding matching grants) is determined by national general grants and a fixed income tax revenue sharing. \(Z^*\) is therefore exogenous for the county and can be considered an unconditional grant.

Eq. (16) can be written

\[
Z^* = \left( q \frac{Sp}{St} - g^* \right) \frac{St}{Yo} \frac{Yo}{Po} + p^* X, \tag{17}
\]

where \(q = (1 - mS)\), \(p^* = (1 - mX)\), and \(Yo\) is the number of youth (16–19 years). The county spending share for high schools \(q\) and hospitals \(p^*\) are the prices of the services. Because the theoretical model does not provide any hypothesis of the effects of variables like \(q\), \(Yo/Po\), and \(p^*\), the chosen strategy is to simplify the model by dividing through (17) by \(qYo/Po:\)

\[
Z = \left( \frac{Sp}{St} - g \right) \frac{St}{Yo} + p X, \tag{18}
\]

where \(Z = Z^*(Po/Yo)/q, p = p^*(Po/Yo)/q, \) and \(g = g^*/q\). Notice that

\[
g = \begin{cases} 
0 & \text{in 1976–1985} \\
g^* & \text{in 1986–1993}
\end{cases}
\tag{19}
\]

Thus, the effect of \(g\) is equal to the effect of the student subsidy \(g^*\). The restrictions on the effects of \(q\) and \(Yo/Po\) implied by (18) are testable.
4.2. Data and Econometric Specification

The quantity of the high school service is measured by the number of students per youth $N = St/Yo$. A measure of quality is not straightforward. The main finding in the literature on educational production functions is that school resources as school spending and class size have at best a very small effect on student achievement, see for example Hanushek (1986), Grogger (1996), and Heckman (2000). Regarding Norwegian high schools, two aspects may relate resource use per student to quality. First, both the youth and the parents seem to prefer a decentralized school structure, which implies small schools, small classes, and high teacher intensity. Second, because the teacher intensity is about twice as high in the vocational study tracks compared to the general study track, increased share of students in the vocational study tracks has been considered as increased quality.

The quality measures I apply are spending per student, $Q_1 = Sp/St$, and the teacher–student ratio, $Q_2$. Since education is labor intensive, teachers seem to be the most important input in school production. The teacher–student ratio is closely related to class size, but class size is hard to define because the students belong to different groups in different subjects. Table 1 presents the correlation matrix for the dependent variables. The correlation coefficients between $N$ and both quality measures are positive, but close to zero for $Q_1$. Table 2 presents descriptive statistics for the dependent variables and the variables in the budget constraint. The variance of the variables is decomposed into the variance between the counties (using mean values during the empirical period) and within the counties. For $N$, the variance between the counties is smaller than the variance due to increased enrollment rates over time, while for the quality variables, the variance is of the same magnitude in the two dimensions.

Table 1

<table>
<thead>
<tr>
<th></th>
<th>$N$</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>1</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>$Q_1$</td>
<td></td>
<td>1</td>
<td>0.59</td>
</tr>
<tr>
<td>$Q_2$</td>
<td></td>
<td>0.43</td>
<td>1</td>
</tr>
</tbody>
</table>

Since $g$ was equal to zero prior to 1986, table 2 offers separate statistics for the pre– and post–grant reform periods. While quantity has clearly increased over time, the quality variables are almost constant at mean. Revenue per
### Table 2

**Descriptive Statistics**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard deviation</td>
<td>Mean</td>
<td>St. dev.</td>
<td>Mean</td>
<td>St. dev.</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>0.658</td>
<td>0.141</td>
<td>0.132</td>
<td></td>
<td>0.767</td>
<td>0.119</td>
</tr>
<tr>
<td><strong>Q1</strong></td>
<td>50.179a</td>
<td>6.032a</td>
<td>6.182a</td>
<td></td>
<td>51.047a</td>
<td>5.106a</td>
</tr>
<tr>
<td><strong>Q2</strong></td>
<td>0.098</td>
<td>0.012</td>
<td>0.009</td>
<td></td>
<td>0.094</td>
<td>0.011</td>
</tr>
<tr>
<td><strong>Z</strong></td>
<td>8.136a</td>
<td>2.935a</td>
<td>1.167a</td>
<td></td>
<td>10.848a</td>
<td>2.021a</td>
</tr>
<tr>
<td><strong>Zu</strong></td>
<td>184,992a</td>
<td>45,325a</td>
<td>33,338a</td>
<td></td>
<td>170,892a</td>
<td>25,386a</td>
</tr>
<tr>
<td><strong>g</strong></td>
<td>3,108a</td>
<td>4,389a</td>
<td>4,312a</td>
<td></td>
<td>6,993a</td>
<td>4,020a</td>
</tr>
<tr>
<td><strong>p</strong></td>
<td>13.4</td>
<td>2.7</td>
<td>1.4</td>
<td></td>
<td>14.2</td>
<td>3.2</td>
</tr>
</tbody>
</table>

*a Measured in 1993–NOK. The exchange rates NOK/USD ≈ 9 and NOK/Euro ≈ 8.*

The starting point of the estimation is the model in Falch and Rattsø (1999), distinguishing between short and long run effects. I will concentrate the discussion on the income and price effects, and investigate different functional forms of the long-run income effects. The equations estimated are reduced forms and not Euler equations to emphasize the results from the comparative statics of the model.

\[
\Delta \ln N_{jt} = \alpha_0 j + \alpha_0 t + \alpha_1 \ln N_{jt-1} + f_1(Z_{jt-1}) + \alpha_4 \ln g_{jt-1} + \alpha_5 \Delta \ln V_{1jt} + \alpha_6 V_{2jt} + \varepsilon_{Njt},
\]

\[
\Delta \ln Q_{jt} = \beta_0 j + \beta_0 t + \beta_1 \ln Q_{jt-1} + f_2(Z_{jt-1}) + \beta_4 \ln g_{jt-1} + \beta_5 \ln p_{jt-1} + \beta_6 \Delta \ln V_{1jt} + \beta_7 V_{2jt} + \varepsilon_{Qjt}.
\]

\(\Delta\) is a differential operator, \(j\) denotes county, \(t\) denotes time, and \(\varepsilon_N\) and \(\varepsilon_Q\) are i.i.d. error terms. \(V_1\) is a vector of \(Z, g, p\), and \(V_2\) is a vector of control variables intended to capture other factors influencing the demand for high school education. \(V_2\) includes the payroll tax, the share of payroll tax, the share of payroll tax, and \(f_1(Z)/\alpha_1, f_2(Z)/\beta_1\) are long-term income elasticities.

\[f_1(\bar{Z})/\alpha_1, f_2(\bar{Z})/\beta_1\] are the long-term income elasticities at mean value of \(Z(\bar{Z})\).
population employed in manufacturing, the unemployment rate, the share of the students in private and state owned high schools, the share of the population above 80, and population, all variables at both differenced form and with one lag. The model includes county and time specific effects to reduce the possibility of omitted variable bias. Only the within-county variations that differ from the aggregate evolution over time are used to estimate the effects of the variables of interest.

4.3. Results

Columns A in table 3 report the results for the log–linear specification of the models estimated by ordinary least squares. The long-term income elasticity is above 0.30 for both quality measures. The income effect of quantity, however, is negative with an elasticity of $-0.13$. It seems that the enrollment ratio in Norwegian high schools decreases when the revenue of the counties increases. The effect is, however, not significant at conventional levels.

18 A closer description of the data set is available from the author on request.
19 There are some specification changes compared to Falch and Rattsø (1999). First, spending per student can be written

$$Q_1 = (w(1 + a)Q_2 + A/St),$$

where $w$ is teacher wage, $a$ is payroll tax, and $A$ is non-wage spending. Falch and Rattsø (1999) investigate determinants of $Q_2$ and $A/St$, while $w(1 + a)$ is treated as an exogenously given national decision variable. While the teacher wage is determined solely by seniority and the amount of formal education, the educational level (and thereby the wage) varies across study tracks. Thus, the counties may influence the mean wage $w$, and since $w$ is a part of $Q_1$, I replace $w(1 + a)$ with $(1 + a)$ in the present analysis. Payroll taxes vary to an increasing degree across the counties throughout the sample period. Second, I impose the restrictions on the effects of $q$ and $Yo/Yo$ implied by (18). Third, I specify matching grants as grants per student in the post–1986 period. The qualitative difference between grants related to school spending and grants related to quantity is an insight from the present theoretical work. Lastly, while Falch and Rattsø (1999) highlight the political processes within the county councils, political variables are excluded from the present paper.

20 LM-tests for first order autocorrelation based on a model including dummy variables for the counties clearly indicates presence of autocorrelation in the models for $N$ and $Q_2$. Thus, the errors are corrected by the Newey-West method for first order autocorrelation and heteroscedasticity. The revealed autocorrelation may still be of some concern because it may imply that there are important dynamic features not captured by the models. The model for $Q_2$ may be seen as a labor demand model, and in the case of adjustment costs, an empirical dynamic labor demand model may include two lags in the dependent variable. It is, however, outside the scope of the present paper to take additional possible dynamic features carefully into account. Table 3 includes tests for normality of the residuals, and all test statistics are highly significant. It turns out the rejections of normality are due to some extreme values. When observations for which the estimated errors exceed three standard deviations of the residual are excluded, normality is not rejected at 10%-level in any of the models, and the estimated standard errors and coefficients of the
For all the dependent variables, the short-term income elasticities are highly significant with the same sign as the long-run effects.

The implication of the results from the log–linear model is a student enrollment ratio close to zero for very high income levels. To investigate whether the income elasticity is nonlinear, I estimate equations that include $1/Z$ and equations that include both $\ln Z$ and $(\ln Z)^3$. The results, reported in columns B and C in table 3 and illustrated in figures 3–5, indicate decreasing income elasticities. For quantity, it is not rejected that the nonlinear models encompass the linear model at 5%-level. But both the hypothesis that the reciprocal specification encompasses the cubic specification and that the cubic specification encompasses the reciprocal specification are rejected.

The flexible specification in column D includes $\ln Z$, $(\ln Z)^3$, and $1/Z$. With this specification, the income elasticity is significantly negative for mean value of $Z$. The elasticity is smallest one standard deviation below mean, see figure 3, but becomes positive for revenue levels more than 1.7 standard deviations above mean. For the quality variables, the differences between the specifications are small.

Because the “observed” income effect of quantity seems to be negative, the “true” income effect is likely to be small. Consequently, the effect of the student subsidy $g$ is likely to be small. The results indicate that $g$ has a small and insignificant effect on quantity both in the short and long run. Even though this can be a result of a corner solution, a negative income effect is not in accordance with this interpretation. The effect of increased student subsidy is higher school quality. However, at mean values of $Q_1$ and $g$, only about 35% of an increase in the student subsidy ends up in more spending per student. Thus, other services expand as well.

The cross–price elasticity on quantity and quality, the elasticity of the price of hospital services $p$, have opposite signs. This is in accordance with part (v) of Proposition 2. It seems that increased costs in hospital production decrease high school spending, and via the income effect, this has a negative impact on quality and a positive impact on quantity. Notice, however, that only the short-run effects with respect to $N$ and $Q_2$ are significant.

It must be emphasized that even though a negative income effect is possible in the theoretical model presented, it is somewhat surprising, and there may be other reasons for the empirical results found. I do not perform models are almost unchanged. In the models for $N$, $Q_1$, and $Q_2$, three, four, and two observations are excluded by this procedure, respectively.

21 Figures 3–5 include the sample values of the gross revenue per youth $Z$.

22 The result is in contrast to the positive effect of BNP per capita on school enrollment found in the cross–country studies of Schultz (1988) and Checchi (2000). In the inter-country case, however, enrollment is probably best seen as determined by the demand side (free access) and not from the supply side as in the present analysis.
Table 3
The Income and Price Effects

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>$\Delta \ln N_{jt}$</th>
<th>$\Delta \ln Q_{1jt}$</th>
<th>$\Delta \ln Q_{2jt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>$\ln N_{jt-1}$</td>
<td>-0.236</td>
<td>-0.236</td>
<td>-0.237</td>
</tr>
<tr>
<td></td>
<td>(6.32)**</td>
<td>(6.46)**</td>
<td>(6.44)**</td>
</tr>
<tr>
<td>$\ln Q_{1jt-1}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln Q_{2jt-1}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln Z_{jt-1}$</td>
<td>-0.030</td>
<td>-0.416</td>
<td>-3.461</td>
</tr>
<tr>
<td></td>
<td>(0.98)</td>
<td>(2.15)**</td>
<td>(2.70)**</td>
</tr>
<tr>
<td>$(\ln Z_{jt-1})^{0.01}$</td>
<td>-</td>
<td>0.087</td>
<td>0.686</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.97)**</td>
<td>(2.71)**</td>
</tr>
<tr>
<td>$1/Z_{jt-1}$</td>
<td>-</td>
<td>6083</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.80)**</td>
<td></td>
</tr>
<tr>
<td>$\ln g_{jt-1}$</td>
<td>-0.003</td>
<td>-0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.47)</td>
<td>(0.59)</td>
<td>(0.65)</td>
</tr>
<tr>
<td>$\ln p_{jt-1}$</td>
<td>0.003</td>
<td>0.003</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.12)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$\Delta \ln Z_{jt}$</td>
<td>-0.129</td>
<td>-0.130</td>
<td>-0.127</td>
</tr>
<tr>
<td></td>
<td>(3.17)**</td>
<td>(3.60)**</td>
<td>(3.11)**</td>
</tr>
<tr>
<td>$\Delta \ln g_{jt}$</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.34)</td>
<td>(0.31)</td>
<td>(0.32)</td>
</tr>
<tr>
<td>$\Delta \ln p_{jt}$</td>
<td>0.073</td>
<td>0.075</td>
<td>0.074</td>
</tr>
<tr>
<td></td>
<td>(2.24)**</td>
<td>(2.49)**</td>
<td>(2.25)**</td>
</tr>
</tbody>
</table>
### Table 3

**Continued**

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Δ ln Njt</th>
<th>Δ ln Q1jt</th>
<th>Δ ln Q2jt</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean long-term income</td>
<td>−0.127</td>
<td>0.304</td>
<td>0.365</td>
</tr>
<tr>
<td>elasticity (Z)</td>
<td>(0.99)</td>
<td>(1.49)</td>
<td>(2.75)</td>
</tr>
<tr>
<td>Long-term grant</td>
<td>−0.014</td>
<td>0.032</td>
<td>0.059</td>
</tr>
<tr>
<td>elasticity (g)</td>
<td>(0.47)</td>
<td>(0.61)</td>
<td>(1.94)</td>
</tr>
<tr>
<td>Long-term cross-price</td>
<td>0.012</td>
<td>−0.163</td>
<td>−0.121</td>
</tr>
<tr>
<td>elasticity (p)</td>
<td>(0.11)</td>
<td>(1.03)</td>
<td>(1.32)</td>
</tr>
<tr>
<td>NORM, ( \chi^2(2) )</td>
<td>15.68**</td>
<td>961.7**</td>
<td>961.7**</td>
</tr>
<tr>
<td>SSR</td>
<td>0.103</td>
<td>0.431</td>
<td>0.692</td>
</tr>
<tr>
<td>R²</td>
<td>0.777</td>
<td>0.878</td>
<td>0.691</td>
</tr>
<tr>
<td>Model B encompasses model k</td>
<td>0.039</td>
<td>0.060</td>
<td>2.025</td>
</tr>
<tr>
<td>Model k encompasses model B</td>
<td>2.227</td>
<td>2.722</td>
<td>2.227</td>
</tr>
</tbody>
</table>

Note: Absolute t-values in parentheses, corrected for heteroskedasticity and first order autocorrelation by the Newey-West method. NORM is the Jarque and Bera test of normality of the errors at asymptotic form, SSR is sum of squared residuals, and the encompassing tests are F-tests where k = A, C. * and ** denote significance at 10%- and 5%-level, respectively. In addition to the reported variables, the models include time-specific effects, county-specific effects, and the control variables described in the text.
any tests to discriminate between alternative hypotheses, but the robustness of the utility maximizing explanation can be evaluated. First, the results may be biased due to the inclusion of lagged dependent variables in the models, making the ordinary least square estimator biased in small samples. I believe the bias is rather small in a sample of 18 years, but this is an empirical question. As a robustness check on the empirical results, results from models on differenced form, excluding all lagged variables from the models, are reported in Appendix 6.4 (table 4)\textsuperscript{23}. The income effects have the same sign as in the previous models for all the dependent variables, and independent of whether fixed effects are included\textsuperscript{24}, they are significant at 5\%-level for $N$ and $Q_2$ and at 10\%-level for $Q_1$. The income elasticities are more precisely estimated within this model formulation probably because the short-run effects are more precisely estimated than the long-run effects.

Second, the long- and short-run effects always have the same sign, and the short-run effects are not greater than the long-run effects. Third, it is possible to test the validity of the restrictions implied by the budget constraint (18). Columns C in table 4 (Appendix 6.4) report the unrestricted models. In a log–linear specification, the coefficients of $q$ and $Yo/Po$ shall be equal to the sum of the coefficients of $Z^u$ and $p^u$, with opposite signs\textsuperscript{25}. The results are mixed in two cases, but the restrictions on the long-run parts of the models are never rejected at 10\%-level\textsuperscript{26}.

\textsuperscript{23} The standard errors of these models are not corrected for first order autocorrelation because the error term of a differenced model is autocorrelated by construction.

\textsuperscript{24} Including county specific effects in models at differenced form implies that the underlying models (at levels) include county-specific time trends in addition to the county-specific time–invariant terms.

\textsuperscript{25} The log–linear version of the quantity model (20) without the restrictions implied by (18) looks like

$$\ln N_{j,t-1} = a_0 + a_0 + a_2 \ln Z_{j,t-1} + \gamma_1 \ln q_{j,t-1} + \gamma_2 \ln (Yo/Po)_{t-1} + \alpha_4 \ln p_{j,t-1} + \gamma_3 V_3 + \epsilon_{Nj,t},$$

where $V_3$ is a vector of $\ln N_{j,t-1}$, $\ln q_{j,t-1}$, $\Delta V_{j,t}$, and $V_2$. Based on (18), the model can be written

$$\ln N_{j,t-1} = a_0 + a_0 + a_2 (\ln Z^u - \ln q - \ln (Yo/Po))_{t-1} + \alpha_4 (\ln p^u - \ln q - \ln (Yo/Po))_{t-1} + \gamma_3 V_3 + \epsilon_{Nj,t}.$$ 

Thus, for the long-run part of the model, the restrictions implied by (18) are $\gamma_1 = \gamma_2 = -\alpha_2 - \alpha_4$. There are, of course, similar restrictions in the short run.

\textsuperscript{26} For $Q_2$, the restrictions are not rejected either in the short or long run. For $Q_1$ and $N$, however, the results are mixed. Regarding student enrollment, this is because the youth share of the population $Yo/Po$ has a significant short-run effect of “wrong” sign. For $Q_1$, the long-term income elasticity is low when the restrictions are not imposed. At 10\%-level, the restrictions are neither rejected in the short-run or in the long-run part of the model. A combined test for both the long- and short-run parts of the model, however, rejects the restrictions at 5\%-level.
Fourth, the revenue variable may be endogenous, particularly in the quantity model. When a larger share of the youth enrolls in high schools, the tax base of the county diminishes, which creates a negative relationship between county revenues and student enrollment. Although this may bias the income
effect, it is unlikely that such a relationship can explain the size of the effect estimated\textsuperscript{27}.

Fifth, changes in unemployment may have major effects on the number of high school applicants, and because capacity adjustment in schools takes time and is costly, this may create excess demand for study places only when the unemployment is rising. I have investigated this hypothesis by estimating a model that includes separate effects of the income and price variables in the period 1988–1993, a period with increasing general unemployment rate each year. By this model formulation, the long- (short-) run income elasticities are estimated to be $-0.08$ ($-0.08$) and $-0.26$ ($-0.25$) for the period prior to 1988 and the period thereafter, respectively, indicating that supply factors are most important when unemployment is rising. However, the short-run elasticity is significant at 10%-level also prior to 1988, even though it is significantly smaller than in the latter period at 5%-level.

Lastly, it is in general possible that rich areas within the counties create their own private alternatives to the public sector supply as in the U.S. Such behavior is not observed, probably because the income inequality is much lower than in the U.S. Anyway, such behavior would most likely be captured

\textsuperscript{27}The income tax accounts for about 40% of the counties’ revenues, and the youth’s share of the workforce is less than 10%. Since the taxes of the youth consist of less than 4% of the counties’ revenues, the estimated income elasticity of $-0.13$ (both in the short and the long run) cannot be fully explained by a reverse causality of this kind.
by the number of students in private high schools per youth, included in the models as a control variable\textsuperscript{28}.

The estimated quantity equation indicates that supply factors influenced the determination of the high school enrollment, and the negative income effect indicates that the majority of the electorate had strong preferences for school quality compared to school quantity. But the empirical model is not able to evaluate the importance of supply factors relative to the importance of demand factors, mainly because the time-specific effects may capture important trends. The effect of the supply factors must be interpreted as for given demand for high school education, and the results do not imply, of course, that the enrollment decreases when demand increases.

The evidence indicates that grants were not useful instruments for the central government to increase high school enrollment in the Norwegian case. It seems that neither the unconditional grants nor the student subsidy had a positive impact on student enrollment. This may be one factor behind the high school reform in 1994 where each 16-years-old was given the right to high school education. The federal government argued that a reform was necessary to hinder that some individuals qualified for high school education were denied access. Within the simple model of two competing political parties presented, such behavior seems to imply that the preferences of the voters differ in local and federal elections. However, this model does not include the interaction between different levels of governments. While the youth may benefit from limited access to high school education given a fixed revenue at the local level, they could favor free access simply because that could increase the grant level from the central government. While there is a clear trade-off between student enrollment and school quality at the local level, the trade-off may be weaker when the determination of the grant policy is taken into account.

5. Conclusion

Economists tend to believe that positive income effects are universal. The only exception seems to be found in the fertility literature. This paper has illustrated that the mechanisms in fertility models apply also in the case of quantity–quality trade-off of publicly provided services. When public services are disaggregated into quantity and quality components, the budget constraint is multiplicative in these dimensions, and the traditional norm-

\textsuperscript{28} In addition, the negative income effect cannot simply be a result of the fact that the gross revenue per youth $Z$ is reduced over time while enrollment has increased over time (see table 2) because the model includes time-specific effects. This can also be seen by considering the unrestricted model in table 4 using revenue per capita $Z^t$, which increases over time.
mality conditions are not sufficient conditions for positive income effects. The possible occurrence of negative income effects makes predictions about public service design decisions difficult even in the case of a stable decision–making environment and well–behaved objectives of the decision–makers. To say something about the sign of the income effects theoretically, one needs stronger assumptions on the utility functions than in traditional models with linear budget constraints.

Much of the political debate is related to design decisions. This paper shows that simply increasing the revenue of the public sector does not imply that all dimensions of public sector services expand. Implicitly, the design of the services will be affected. The interaction between different components of public sector services as quantity and quality via the shadow prices imply that the costs of expanding one dimension depend on what happens with other dimensions. Empirical evidence is therefore crucial to understand design decisions.

There are to my knowledge no empirical examples of negative income effects of components of public services. The reason may be that empirical studies typically analyze total spending on particular public services or spending aggregated over several services. The empirical application in the present paper indicates a negative effect of budget size on student enrollment in Norwegian high school education. In addition, a matching grant related to the number of students had an insignificant effect, which may be a result of the negative income effect. However, whether negative income effects of components of publicly provided services are mainly a theoretical possibility is a question for further empirical research.

6. Appendix

I only need to consider the case when $\mu = 0$ to prove the propositions. Then it follows from the first order conditions (7)–(10) that the bordered Hessian of the maximization problem is

$$H = \begin{bmatrix}
0 & -(qQ - g) & -qN & -p & \Upsilon \\
-(qQ - g) & f^1U^1_{NN} & f^1U^1_{NQ} - \lambda q & 0 & 0 \\
-qN & f^1U^1_{NQ} - \lambda q & f^1U^1_{QQ} & 0 & 0 \\
-p & 0 & 0 & c_1 & -\Upsilon c_3 \\
1 & 0 & 0 & -c_3 & c_2
\end{bmatrix}, \quad (22)$$

where $c_1 = \sum_{i=2}^{G} f_i M'U'_{XY} < 0$, $c_2 = \sum_{i=2}^{G} f_i y'U'_{CC} < 0$, and $c_3 = \sum_{i=2}^{G} f_i y'U'_{XC}$. The sign of $H$ is not obvious because $\lambda$ enters the expression. It may be useful
to write out the determinant:
\[
D = -\left[(qN)^2f^1 U_{NN}^1 + (qQ - g)^2f^1 U_{QQ}^1 \\
- 2qN(qQ - g)(f^1 U_{NQ}^1 - \lambda q)\right][c_1c_2 - \Upsilon c_3^2] \\
- \left[(f^1)^2 U_{NN}^1, U_{QQ}^1 - (f^1 U_{NQ}^1 - \lambda q)^2\right][\Upsilon c_1 + p^2c_2 - 2p\Upsilon c_3].
\] (23)

While the last brackets in each term are positive when the utility functions are quasi–concave, the signs of the first brackets are in general ambiguous, reflected by the inclusion of \(\lambda\). Quasi–concave utility functions are not a sufficient condition for an interior solution in the case of a multiplicative budget constraint. When the utility function of group 1 is \(U^1 = Nh(Q)\) as in (14), the determinant is
\[
D = -(qQ - g)^2Nh'(Q)(c_1c_2 - \Upsilon c_3^2) > 0,
\] (24)
and the second order condition is fulfilled. Notice that in this case the second term in (23) is negative when the second order condition is fulfilled, but the first term has to be positive. This characteristic of the determinant will be utilized below. If the first term is equal to zero, the second term is negative and the second order condition is violated.

Total differentiation of the first order conditions yields
\[
\begin{bmatrix}
\frac{d\lambda}{dN} \\
\frac{d\lambda}{dQ} \\
\frac{d\lambda}{dX} \\
\frac{d\lambda}{dt}
\end{bmatrix} =
\begin{bmatrix}
-1 & -t & -N & NQ & X \\
0 & 0 & -\lambda & \lambda Q & 0 \\
0 & 0 & 0 & \lambda N & 0 \\
0 & -c_3(1-t) & 0 & 0 & \lambda \\
0 & c_2(1-t)/\Upsilon & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
dZ \\
d\Upsilon \\
dg \\
dq \\
dp
\end{bmatrix}.
\] (25)

In the following, I will assume that \(H\) is positive semidefinite and that the normality conditions are fulfilled.

6.1. Proof of Lemma 1

Because total revenue \(t\Upsilon = \Upsilon - \Gamma\), the budget constraint of the society can be written
\[
\Upsilon + Z = (qQ - g)N + pX + \Gamma.
\] (26)

When changes in \(\Upsilon\) have no distributional consequences (\(\gamma'\) is independent of \(\Upsilon\)), Lemma 1 follows directly from this identity. Clearly, the effects of \(Z\) and \(\Upsilon\) with respect to the tax rate \(t\) are unequal. The technical proof is easy, and is omitted.
6.2. Proof of Proposition 1

Consider first the income effects with respect to the quantity and quality of the publicly provided private service. (25) yields
\[
\frac{dN}{dZ} = -\frac{1}{D} \left((qQ - g)f^{1}U_{QQ}^{1} - qN(f^{1}U_{Q}^{1} - \lambda q)\right)(c_1c_2 - \gamma c_3^2)
\]
\[
= -f^{1} \frac{qN}{U_{Q}^{1}D} \left(U_{Q}^{1}U_{N}^{1} - U_{Q}^{1}U_{N}^{1} + U_{Q}^{1}U_{N}^{1} \right) (c_1c_2 - \gamma c_3^2) \tag{27}
\]
\[
\frac{dQ}{dZ} = -\frac{1}{D} \left[qNf^{1}U_{NN}^{1} - (qQ - g)(f^{1}U_{Q}^{1} - \lambda q)\right](c_1c_2 - \gamma c_3^2)
\]
\[
= -f^{1} \frac{qN}{U_{Q}^{1}D} \left(U_{NN}^{1}U_{Q}^{1} - U_{NN}^{1}U_{N}^{1} + U_{NN}^{1}U_{N}^{1} \right) (c_1c_2 - \gamma c_3^2) \tag{28}
\]

where the first order conditions (7) and (8) are utilized. The normality conditions, \(U_{Q}^{1}U_{N}^{1} - U_{Q}^{1}U_{N}^{1} < 0\) and \(U_{NN}^{1}U_{Q}^{1} - U_{NN}^{1}U_{Q}^{1} < 0\), are not sufficient for positive income effects. This proves the results for quantity and quality.

The income effect with respect to total spending on the publicly provided private service can be calculated using the results above. It is easiest to show the effect on net outlays \((qQ - g)N\). If there is a positive income effect with respect to net outlays, then the same must be true for gross outlays \(NQ\). It follows from (27) and (28) that
\[
\frac{d((qQ - g)N)}{dZ} = (qQ - g) \frac{dN}{dZ} + qN \frac{dQ}{dZ}
\]
\[
= -\frac{1}{D} \left((qQ - g)\right)^2 f^{1}U_{NN}^{1} + (qQ - g)^2 f^{1}U_{Q}^{1} - 2qN(qQ - g)(f^{1}U_{Q}^{1} - \lambda q)\right)(c_1c_2 - \gamma c_3^2) \tag{29}
\]

This is positive by the second order condition.

Consider next the income effect with respect to \(\lambda\). (25) yields
\[
\frac{d\lambda}{dZ} = -\frac{1}{D} \left(f^{1}U_{NN}^{1}U_{Q}^{1} - (f^{1}U_{Q}^{1} - \lambda q)\right)^2 (c_1c_2 - \gamma c_3^2) \tag{30}
\]

It follows from the discussion below (24) that this sign may be negative when the second order condition is fulfilled. Notice, however, that if both the income effects with respect to \(N\) and \(Q\) are positive,
\[
U_{N}^{1}(U_{Q}^{1})^{-1}(U_{Q}^{1}N^{-1} - U_{Q}^{1}) < -U_{NN}^{1}
\]
and
\[
U_{Q}^{1}(U_{N}^{1})^{-1}(U_{Q}^{1}N^{-1} - U_{Q}^{1}) < -U_{QQ}^{1}
\]
Thus, \(U_{NN}^{1}U_{Q}^{1} > (U_{Q}^{1}N^{-1} - U_{Q}^{1})^2\), and it follows from (30) that \(d\lambda/dZ < 0\). Only when either \(dN/dZ < 0\) or \(dQ/dZ < 0\) may \(d\lambda/dZ > 0\).
The sign of the marginal effect of $X$,
\[
\frac{dX}{dZ} = \frac{d\lambda}{dZ} \frac{pc_2 - \Upsilon c_3}{c_1c_2 - \Upsilon c_3^2},
\] (31)
is determined by the same condition as for $\lambda$. Regarding the tax rate, the effects of $Z$ and $\Upsilon$ differ. The effect of $Z$ is given by
\[
\frac{dt}{dZ} = \frac{d\lambda}{dZ} \frac{pc_2 - c_1}{c_1c_2 - \Upsilon c_3^2}.
\] (32)
This is negative under normality conditions if $d\lambda/dZ < 0$, reflecting a positive income effect with respect to private consumption. The effect of $\Upsilon$ with respect to $t$ is in general unknown, and reflects whether private consumption is income elastic ($dt/d\Upsilon < 0$) or income inelastic ($dt/d\Upsilon > 0$). Lastly, the income effect with respect to total private consumption $\Gamma$ is obviously determined by the sign of the income effect with respect to $t$.

6.3. Proof of Proposition 2

For the quantity subsidy $g$, it follows that
\[
\frac{dN}{dg} = -N \frac{dN}{dZ} + \frac{1}{D} \left\{ \lambda(qN)^2(c_1c_2 - \Upsilon c_3^2) 
+ \lambda f^1 U_{QQ}^1(\Upsilon c_1 + p^2 c_2 - 2p \Upsilon c_3) \right\}
= -f^1 \frac{1}{D} \left\{ N[(qQ - g)U_{QQ}^1 - qNU_{NQ}^1](c_1c_2 - \Upsilon c_3^2) 
+ f^1 \lambda U_{QQ}^1(\Upsilon c_1 + p^2 c_2 - 2p \Upsilon c_3) \right\} > 0.
\] (33)
This is always positive when the normality conditions are fulfilled, and proves (i). The effect of $g$ with respect to the other dependent variables consists of income and substitution effects with opposite signs. This proves (ii).

The effect on quantity of the price of the publicly provided private good $q$ is
\[
\frac{dN}{dq} = -NQ \frac{dN}{dZ} - \frac{\lambda}{D} qNg(c_1c_2 - \Upsilon c_3^2) 
- \frac{\lambda}{D} \left[ (qf^1 U_{QQ}^1 - N(f^1 U_{NQ}^1 - \lambda q))(\Upsilon c_1 + p^2 c_2 - 2p \Upsilon c_3) \right].
\] (34)
The first term is the income effect while the last two terms are substitution effects. The second term reflects that $q$ changes the relative price between $N$ and $Q$. The third term is a variant of a traditional substitution effect, but the sign of the term is ambiguous. In fact, the sign of the last substitution effect is equal to the sign of the income effect. There is a substitution away from $NQ$, but again it is unknown whether both $N$ and $Q$ are reduced. This is clearly
seen if $g = 0$. Then (34) can be written
\[
\frac{dN}{dq} = \frac{1}{D} \left[ Q f^1 U_{QQ}^1 - N (f^1 U_{NQ}^1 - \lambda q) \right] \left[ q N Q (c_1 c_2 - \gamma c_3) \
- \lambda (\gamma c_1 + p c_2 - 2 p \gamma c_3) \right].
\] (35)

The last bracket is negative under normality conditions. Thus, sign $dN/dq \neq$ sign $dN/dZ$. When $g = 0$, the result for quality is symmetric to the result for quantity. When $g > 0$, it follows from (11) that $q$ increases the shadow price of $N$ relative to the shadow price of $Q$. There is a substitution effect away from $N$ towards $Q$. From (34), however, it follows that increased $g$ has an uncertain effect on $dN/dq$. Thus, $dN/dq$ and $dQ/dq$ may be positive when the income effect is positive.

The effect of $q$ with respect to $X$, $t$, $\Gamma$, and $\lambda$ includes the traditional income and substitution effects, making the overall effects ambiguous. This proves (iii).

Writing out $dX/dp$, it follows that
\[
\frac{dX}{dp} = -X \frac{dX}{dZ} - \frac{\lambda}{D} \left[ c_2 \left[ (qN)^2 f^1 U_{NN}^1 + (qQ - g)^2 f^1 U_{QQ}^1 \right. \right.
\left. \left. - 2 q N (qQ - g) f^1 U_{NQ}^1 \right] \right.
\left. + \gamma \left[ (f^1)^2 U_{NN}^1 U_{QQ}^1 - (f^1 U_{NQ}^1 - \lambda q)^2 \right] \right].
\] (36)

The substitution effect consists of two parts. The first part is negative by the second order condition, while the sign of the second part of the substitution effect is equal to the sign of the income effect. Taken together, and utilizing (31), the price effect is negative by the second order condition. This proves (iv).

The effect of $p$ with respect to $N$, $Q$, $t$, $\Gamma$, and $\lambda$ includes the traditional income and substitution effects, making the overall effect ambiguous. It may be useful to write out $dN/dp$,
\[
\frac{dN}{dp} = -X \frac{dN}{dZ} + \frac{\lambda}{D} \left[ (qQ - g) f^1 U_{QQ}^1 - q N (f^1 U_{NQ}^1 - \lambda q) \right] (pc_2 - \gamma c_3)
\]
\[
= -\frac{dN}{dZ} \left( X - \frac{\lambda p c_2 - \gamma c_3}{c_1 c_2 - \gamma c_3} \right).\] (37)

Again the sign of the substitution effect is equal to the sign of the income effect, and the sign of the income effect determines the sign of $dN/dp$. Because the objective functions of the political parties are separable across groups, there is a similar expression for $dQ/dp$, which proves (v).
# Models on Differenced Form and Models without Restrictions on the Budget Constraint Components

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<th>$\Delta \ln N_{jt-1}$</th>
<th>$\Delta \ln Q_{1jt-1}$</th>
<th>$\Delta \ln Q_{2jt-1}$</th>
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<td>$\ln Q_{1jt-1}$</td>
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<tr>
<td>$\ln Q_{2jt-1}$</td>
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<td>$\ln g_{jt-1}$</td>
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</tr>
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<tr>
<td>$\ln(\text{Yo}/\text{Po})_{jt-1}$</td>
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<tr>
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<td>$-0.095$</td>
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<td>(2.12)**</td>
<td>(2.51)**</td>
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<td>$0.004$</td>
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<td>(0.16)</td>
<td>(0.65)</td>
<td>(2.26)**</td>
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### Table 4
Continued

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<td>∆ ln pjt</td>
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<td></td>
<td>(1.54)</td>
<td>(1.69)*</td>
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</tr>
<tr>
<td>∆ ln qjt</td>
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<td>–</td>
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<tr>
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<td>(1.39)</td>
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<td>∆ ln (Yo / Po)jt</td>
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<td>–</td>
<td>–0.446</td>
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<td></td>
<td>(2.02)**</td>
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<td>20.42**</td>
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<td>–</td>
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Note: The estimation covers the 18 counties in Norway during 1977–1993, 306 observations. Absolute t-values in parentheses, corrected by the Newey-West method for heteroskedasticity in columns A and B and for both heteroskedasticity and first order autocorrelation in columns C. * and ** denote significance at 10%- and 5%-level, respectively. NORM is the Jarque and Bera test of normality of the errors at asymptotic form and SSR is the sum of squared residuals. FFTOL is an F-test of the restrictions on long-run coefficients implied by equation (18), that is, whether models C in this table are allowable reductions of models A in table 3. Degrees of freedom in parentheses. FTOOL is an F-test of the restrictions on the short run coefficients implied by equation (18). FRHO tests both the long- and short-run restrictions simultaneously.

In addition to the reported variables, the following control variables are included in the model: The payroll tax, the share of the population employed in manufacturing, the unemployment rate, the share of the students in private and state owned schools, the share of the population above 80, and population. The control variables are included in differenced form in models A and B and in both differenced form and with one lag in model C.
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