Optimal Dutch Disease*

Egil Matsen† Ragnar Torvik‡

April 27, 2004

Abstract

Growth models of the Dutch disease, such as those of Krugman (1987), Matsuyama (1992), Sachs and Warner (1995) and Gylfason et al. (1999), explain why resource abundance may reduce growth. The literature, however, also raises a new question: if the use of resource wealth hurts productivity growth, how should such wealth be optimally managed? This question forms the topic of the present paper, in which we extend the growth literature on the Dutch disease from a positive to a normative setting. We show that the assumptions in the previous literature imply that the optimal share of national wealth consumed in each period needs to be adjusted down. Some Dutch disease, however, is always optimal. Thus lower growth in resource abundant countries may not be a problem in itself, but may be part of an optimal growth path. The optimal spending path of the resource wealth may be increasing or decreasing over time. What might be contrary to intuition; the bigger is the growth generating traded sector, the more of the resource income should be spend in early periods.

Keywords: Growth. Foreign Exchange Gifts. Resource Wealth. Optimal Saving. Current Account Dynamics.
JEL: F43, O41, Q32.

*We thank two anonymous referees, Thorvaldur Gylfason, Fredrik Wulfsberg, seminar participants at the 2003 EEA meeting, Harvard University, Norges Bank, the Norwegian University of Science and Technology, and the University of Bergen for helpful comments and suggestions. The views expressed are those of the authors, and not necessarily those of Norges Bank.

†Norwegian University of Science and Technology, and Norges Bank. Adress: NTNU, Department of Economics, Dragvoll, NO-7491 Trondheim, Norway. Phone: +47 7359 7852. Fax: +47 7359 6954. e-mail: egil.matsen@svt.ntnu.no

‡Norwegian University of Science and Technology, and Norges Bank, e-mail: ragnar.torvik@svt.ntnu.no

1 Introduction

There is now a large body of literature claiming that resource abundance lowers growth. Such findings in the case studies by Gelb (1988) have later been confirmed in other case studies by Karl (1997) and Auty (1999, 2001) as well as in econometric studies by Sachs and Warner (1995, 1997, 2001), Gylfason et al. (1999) and Busby et al. (2002).[1] The most widespread theoretical explanation of this apparent puzzle is found in models of the Dutch disease[2], where resource abundance shifts factors of production away from sectors generating learning by doing (LBD).[3] Studies by van Wijnbergen (1984), Krugman (1987), Matsuyama (1992), Sachs and Warner (1995) and Gylfason et al. (1999) all find that when the exploitation of more natural resources shrinks the traded (or industrial) sector, LBD and thus productivity growth is reduced. This literature has been most influential in explaining why resource wealth may lower growth. Little attention has, however, been given to the question of how resource wealth *should* be managed given that the use of such wealth lowers productivity growth. This is the topic of the present paper.

The seminal contribution on the Dutch disease with endogenous productivity is the two period model by van Wijnbergen (1984), where the second period productivity in the traded sector depends on the first period production of traded goods. Although van Wijnbergen does not directly discuss how the resource wealth should be optimally managed, the paper includes normative analysis on the design of subsidies. The later growth literature on the topic has, however, neglected the normative aspects. Krugman (1987), Sachs and Warner (1995), Gylfason et al. (1999) and Torvik (2001) consider an exogenous flow of resource income in each period and trace out the growth effects.[4] The present paper extends this growth literature from a positive to a normative setting. To do so we simply adopt the same assumptions regarding productivity growth as in the earlier literature and then derive

[1] For a paper that questions the empirical connection between resource abundance and growth, see Stijns (2002).

[2] Normally the term 'Dutch disease' refers to adverse effects on the traded sector when resource income pushes domestic demand up. The term has also been used to refer to the possible negative growth effects following the reallocation of production factors. As we will show, however, even in the case where growth decreases this may be the optimal response to resource abundance. Despite this, we choose to use the term 'disease' as this is firmly established among economists.

[3] Other explanations include theories of rent-seeking (Lane and Tornell, 1996; Tornell and Lane, 1999; Baland and Francois, 2000; Torvik, 2002; Mehlum et al., 2002) and political economy theories of why resource abundance invites bad policy choices (Ross, 1999, 2001; Robinson et al., 2002; Damania and Bulte, 2003).

[4] The resource abundance effect in each period is also exogenous in Matsuyama (1992), represented by the productivity of land. See also Rodriguez and Sachs (1999) for a model with exogenous productivity growth. Unlike us, they also assume that the current account is exogenous.

the implications for optimal consumption, management of resource wealth, and growth.

Our paper also relates to the influential contributions on intergenerational allocation of exhaustible resources by Solow (1974, 1986) and Hartwick (1977). In fact, the present paper can be viewed as an attempt to integrate the Dutch disease literature with the normative approach of Solow and Hartwick. Given the influential contributions on the linkage between LBD and the Dutch disease, the implications of this literature for the optimal management of resource wealth should clearly be of some interest. We show that the LBD mechanism in the earlier literature implies that the optimal share of national wealth consumed in each period needs to be adjusted downward. A positive fraction of the resource wealth, however, should be consumed in each period. Thus, lower growth in resource abundant countries may not be a problem in itself, but may be part of an optimal growth path – some Dutch disease is always optimal. When the market interest rate equals the social rate of time preference, open economy models with zero or exogenous growth imply a flat optimal consumption path. The optimal solution of the present model, however, in this case implies a rising consumption path. The optimal Dutch disease is thus sufficiently weak for each generation to consume more than the preceding generation. The spending path of the resource wealth may be increasing or decreasing over time. A positive growth potential with LBD pulls in the direction of large transfers to early generations, while a negative effect on productivity growth from using the resource wealth pulls in the other direction. The higher the share of non-traded goods in consumption, the weaker is the first effect and the stronger is the second. Thus, the more important that non-traded goods are as a proportion of consumption, and the less important traded goods are, the more likely it is that the optimal spending path of the resource wealth is increasing over time.

We believe the present model is also relevant for a current debate on the need for fiscal rules in resource wealth management (see Katz and Bartsch, 2003). Whereas this debate mainly is about the desirability of accumulating funds to avoid "boom-and bust" cycles, our model sheds lights on how spending rules should be formulated once funds are established. In particular, our model prescribes a careful spending policy, in the sense that endogenous effects on productivity growth implies higher saving of the resource wealth than what e.g. the permanent income hypothesis would imply.[5]

The rest of the paper is organized as follows. The model is presented

[5]Norway can serve as an example (close to home for the authors) of a resource rich country that has established a formal spending rule of its oil wealth. The rule says that (as an average over the business cycle) 4 % of accumulated financial assets could be spent every year. Notice that this implies an increasing spending path, as the resource wealth is gradually transformed from oil reserves to financial assets. See Hannesson (2001, ch. 7) and Røed Larsen (2003) for a more thorough description of the Norwegian spending rule.

in Section 2. Section 3 derives optimal consumption, while the implications for optimal current account and output growth are discussed in Section 4. Section 5 discuss some positive implications and extensions of our normative model. Section 6 concludes the paper.

2 The model

Following other models of the Dutch disease, we consider a small open economy that produces traded (T) and non-traded (N) goods. The single most important assumption in the models concerns what factor drives productivity growth. With the exception of Torvik (2001), the literature assumes that productivity growth is generated through LBD in the traded sector only. van Wijnbergen (1984), Krugman (1987), Matsuyama (1992) and Gylfason et al. (1999) assume that LBD only benefits the sector where it is generated, while productivity in the rest of the economy is constant. Thus, these studies involve models of unbalanced growth. Sachs and Warner (1995), on the other hand, have balanced growth, as they assume that the learning benefits the traded and non-traded sector in the same way.[6] Here we adopt the same LBD mechanism as Sachs and Warner (1995) because, in addition to its influence on the recent literature on the topic, the unbalanced growth mechanisms in the other papers contain predictions that might seem problematic.[7] Denoting the (fraction of the total) labor force employed in the traded sector in period t by η_t, the dynamics of productivity H are:

$$\frac{H_{t+1} - H_t}{H_t} = \alpha \eta_t, \tag{1}$$

where the parameter $\alpha \geq 0$ measures the strength of the LBD effect. As in the earlier literature, the LBD effect is external to firms, the underlying assumption being that each firm is too small to take its own contribution to LBD into account.

Normalizing the size of the labor force to unity, the production functions in the two sectors are given by:

$$\begin{aligned} X_{Nt} &= H_t(1 - \eta_t) && (2) \\ X_{Tt} &= H_t \eta_t && (3) \end{aligned}$$

where X_{Nt} and X_{Tt} denote production of non-traded and traded goods, respectively. As the production at each point in time has constant returns to scale, the real exchange rate (i.e. the relative price of non-tradables in

[6] A discussion of the Dutch disease literature can be found in Torvik (2001), who develops a more general model of learning by doing, and derives conditions for when resource abundance does or does not reduce growth.

[7] For instance, although it is not discussed by the author, the model in Krugman (1987) implies that the real exchange rate approaches infinity.

terms of tradables) is uniquely determined by the supply side, as in Corden and Neary (1982, Section IV), for example. The equal productivity in (2) and (3) implies that the real exchange rate is simply equal to 1. By (2) and (3) total production (GDP) in period t is:

$$X_t = X_{Nt} + X_{Tt} = H_t \tag{4}$$

Consumers live for one period (which we think of as a generation). There is a representative individual in each generation. This consumer's labor supply is fixed, he or she has no bequest motive, and allocates spending on non-traded and traded goods according to a Cobb-Douglas felicity function. Let $\gamma \in (0, 1)$ be the weight on traded goods in the felicity function. The demand for non-traded goods is thus:

$$C_{Nt} = (1 - \gamma)Y_t = X_{Nt}, \tag{5}$$

where Y_t is disposable income for generation t and the last equality shows that in equilibrium domestic demand of non-traded goods must be matched by domestic production of such goods.

Notice that in the absence of a public sector (and thus intergenerational transfers) as well as of a foreign exchange gift we have $Y_t = H_t$, since the relative price of the two goods is one. As there is no private saving, the demand for traded goods is $C_{Tt} = \gamma Y_t$. It then follows from (2) and (5) that $\eta_t = \gamma$, implying that the output growth rate in this case is $\alpha\gamma$.

2.1 The social planner's problem

The social planner's horizon is M periods, where $M > 1$. Thus there are two inefficiencies in the model: the representative individuals have too short planning horizons (as in for instance Diamond (1965) and Obstfeld and Rogoff, 1996, Chapt. 3), and they ignore LBD in their allocation decisions. Potentially, therefore, there is a role for the government in the model, even in the absence of resource wealth. In general, however, we assume that the country receives resource wealth in the form of a foreign exchange gift W_1 at the beginning of period 1. The planner then decides (in period 1) how to allocate this gift over time, and we let R_t be net lump-sum transfers to generation t.[8]

[8] The assumption that the planner has a longer horizon than the consumers alive at each point in time is standard in the literature on intergenerational resource allocation, see for instance Solow (1974), and contributes to overutilization of resources. In our setup we will have overutilization even in the case where the consumers and the planner have the same time horizon, as the social return on saving exceeds the private return due to LBD externalities (see below). With equal horizons, however, the planner is not able to affect the consumption path by transfers, as private agents will perfectly counteract this by borrowing and lending abroad. Thus, although a horizon longer than one generation (due to for instance intergenerational altruism) makes the problem of overconsumption of

The objective is to maximize:

$$U = \sum_{t=1}^{M} \left(\frac{1}{1+\delta}\right)^{t-1} \left[\gamma \log C_{Tt} + (1-\gamma)\log C_{Nt}\right],$$

where δ is the social rate of time preference. This formulation implies that the planner's elasticity of intertemporal substitution is constant and equal to one.

It is convenient to rewrite the objective function in terms of aggregate consumption. From the static demand functions and the fact that disposable income with transfers is $Y_t = H_t + R_t$, aggregate consumption in period t is:

$$C_t = C_{Tt} + C_{Nt} = \gamma Y_t + (1-\gamma)Y_t = R_t + H_t$$

We can now rewrite the inside of the social planner's objective function as:

$$\gamma \log C_{Tt} + (1-\gamma)\log C_{Nt} = \log C_t + \gamma \log \gamma + (1-\gamma)\log(1-\gamma)$$

Ignoring the constant terms, the social welfare function can thus be written as:

$$U = \sum_{t=1}^{M} \left(\frac{1}{1+\delta}\right)^{t-1} \log C_t \tag{6}$$

It is important to keep in mind that $C_t = R_t + H_t$, since R is the policy instrument in the model.

In choosing the optimal path for R_t, the planner takes into account the fact that spending the gift in period t affects future productivity. Using (2) and (5), we find that traded sector employment is given by:

$$\eta_t = \gamma - (1-\gamma)\frac{R_t}{H_t} \tag{7}$$

(7) shows the static effect that is often termed the Dutch disease. Transferring resource income R to generation t increases demand for traded and non-traded goods. As increased demand for non-traded goods must be met by domestic production, resources are drawn out of the traded sector and into the non-traded sector. The effect is stronger the more important non-tradables are in consumption, and the larger transfers are relative to production. Corden and Neary (1982), Corden (1984), Neary and Purvis (1983) and Neary and van Wijnbergen (1986) provide detailed discussions of this and other effects of resource income in models without productivity growth.

resource wealth smaller, it also reduces the possibilities of transfers to adress the problem. In the rest of the paper we assume that private agents have shorther horizon than the planner, so that transfers affect consumption.

Remark 1 *Since $\eta_t \in (0,1)$, equation (7) implies the following restrictions on the ratio of transfers to GDP:*

$$-1 < \frac{R_t}{H_t} < \frac{\gamma}{1-\gamma}, \forall t$$

The first inequality simply states that negative transfers (i.e. taxes) cannot be higher than 100 % of GDP, while the second inequality says that the transfer-GDP ratio must be lower than the ratio of tradables to non-tradables in aggregate consumption. All the solutions presented below are assumed to obey these restrictions.

Substituting (7) into (1), we find that productivity (and GDP) in period $t+1$ is:

$$H_{t+1} = H_t(1+\alpha\gamma) - \alpha(1-\gamma)R_t \tag{8}$$

(8) shows the dynamic effect often associated with the Dutch disease. As in van Wijnbergen (1984), Krugman (1987), Sachs and Warner (1995) and Gylfason et al. (1999) generation t's spending of the foreign exchange gift R has a negative effect on future productivity because employment in the traded sector, and thus productivity growth, is reduced. The effect is stronger the stronger is the LBD effect and the more important are non-tradables in aggregate consumption. The reason for the latter is that a large proportion of non-tradables in consumption greatly reduces traded sector employment when demand increases.

So far our model has added nothing important to the earlier endogenous growth models of the Dutch disease. As in the models of Krugman (1987), Sachs and Warner (1995), Gylfason et al. (1999) and Torvik (2001), we have simply shown that when assuming that LBD is generated in the traded sector, the use of resource income lowers growth. In the remainder of the paper, however, we depart from the earlier growth models. While these models assume exogenous resource income at each point in time as well as an exogenous current account, our aim is to find the optimal intertemporal use of resource income and the implied optimal current account and growth dynamics. We thus extend the endogenous growth models of the Dutch disease from a positive to a normative setting.

To derive the intertemporal budget constraint, we make use of the economy's current account. The stock of foreign assets in period t is denoted W_t. We assume that the foreign exchange gift is the only initial foreign. When there is a constant exogenous real interest rate r, the current account in period t can be written as:

$$\begin{aligned} CA_t &= W_{t+1} - W_t = X_{Tt} - C_{Tt} + X_{Nt} - C_{Nt} + rW_t \\ &= \eta_t H_t - \gamma(H_t + R_t) + rW_t \\ &= \gamma H_t - (1-\gamma)R_t - \gamma(H_t + R_t) + rW_t \\ &= rW_t - R_t \end{aligned} \tag{9}$$

The second row follows from using the demand function for traded goods (3), and the equilibrium condition (5). The third row follows from using (7). Equation (9) highlights the fact that the planner's problem may be viewed as the task of choosing the optimal current account over time. By repeated iterative substitutions for $W_{t+1}, W_{t+2}, ..$ in (9) (in the manner of Obstfeld and Rogoff (1996, ch. 2.1)), we arrive at the economy's intertemporal budget constraint:

$$\sum_{t=1}^{M} \left(\frac{1}{1+r}\right)^{t-1} R_t = (1+r)W_1 \tag{10}$$

In (10), we have also imposed the terminal condition $W_{M+1} = 0$; the planner will use all the resources his or her budget constraint allows.

2.2 National wealth

As stated above, the planner's problem is to maximize (6) subject to (8), (9) and the terminal condition. This problem is more easily analyzed, however, by merging (8) and (9) into one constraint, describing the dynamics of national wealth. At the start of period $t+1$, the planner's measure of national wealth NW is:

$$NW_{t+1} = (1+r)W_{t+1} + \sum_{s=t+1}^{M} \left(\frac{1}{1+r}\right)^{s-(t+1)} H_s \tag{11}$$

It includes (financial/natural resource) wealth W accumulated through period t plus the present value of current and future income. For later use we rewrite (11) in more familiar form of (national) wealth dynamics:

$$\begin{aligned} NW_{t+1} &= (1+r)[(1+r)W_t - R_t] + (1+r)\sum_{s=t}^{M}\left(\frac{1}{1+r}\right)^{s-t} H_s - (1+r)H_t \\ &= (1+r)\left[(1+r)W_t + \sum_{s=t}^{M}\left(\frac{1}{1+r}\right)^{s-t} H_s - C_t\right] \\ &= (1+r)\left(NW_t - C_t\right). \end{aligned} \tag{12}$$

Next, we observe that repeated iterative substitutions in (8) implies that GDP in period $s > t$ can be written as:

$$H_s = (1+\alpha\gamma)^{s-t} H_t - \alpha(1-\gamma)\sum_{i=t}^{s-1}(1+\alpha\gamma)^{s-1-i} R_i.$$

Using this and equation (9) in (11), we can express national wealth in period

$t+1$ as:

$$\begin{aligned} NW_{t+1} &= (1+r)\left[(1+r)W_t - R_t\right] + (1+r)\sum_{s=t+1}^{M}\left(\frac{1+\alpha\gamma}{1+r}\right)^{s-t} H_t \\ &\quad -\alpha(1-\gamma)\sum_{s=t+1}^{M}\left(\frac{1}{1+r}\right)^{s-(t+1)}\left[(1+\alpha\gamma)^{s-(t+1)}R_t\right. \\ &\quad \left.+\sum_{i=t+1}^{M}(1+\alpha\gamma)^{s-1-i}R_i\right]. \end{aligned} \tag{13}$$

This *single* dynamic constraint now replaces the two constraints (8) and (9) in the planner's maximization problem. We notice that the period t spending of the foreign exchange gift enters the constraint via two terms. The first term represents the ordinary effect of lower future financial/natural resource wealth, while the second term represents the negative effect on future income through lower productivity growth. Given this formulation of the budget constraint, we can also restate the terminal condition as $NW_{M+1} = 0$.

3 Optimal aggregate consumption

We shall first present the solution for optimal aggregate consumption. As will become clear below, our model has interesting implications for the optimal intertemporal consumption allocation compared to models either without growth or with exogenous growth. A non-growing economy can be studied within our framework when there is no LBD, i.e. when $\alpha = 0$. A model with exogenous growth can be analyzed by considering the borderline case of $\gamma = 1$. Our country would then produce and consume tradables only, in effect giving us a one-sector model with an exogenous output growth rate $= \alpha$. The planner chooses $\{R_t\}$ to maximize (6) subject to (13) and the terminal condition. In solving this problem, we make one assumption which is a sufficient condition for positive consumption in all periods (see below) and is standard in open economy growth models:

Assumption 1: $r > \alpha\gamma$.

In effect it states that the interest rate is higher than the economy's output growth in the absence of government intervention.

Proposition 1 *Let*

$$J(NW_t) = \max_{R_t}\sum_{t=1}^{M}\left(\frac{1}{1+\delta}\right)^{t-1}\log(R_t + H_t),$$

subject to (13) and the terminal condition. Then:

$$J(NW_t) = \Phi_t + \Theta_t \log NW_t,$$

where $\Theta_t = \frac{1+\delta}{\delta}\left[1 - \left(\frac{1}{1+\delta}\right)^{M-t}\right]$ *and* Φ_t *is an inessential function of time only. Optimal consumption is:*

$$C_t = h_t NW_t, \tag{14}$$

where

$$h_t \equiv \frac{1}{1 + \left[\frac{1+\delta}{\delta}\left(1 - \left(\frac{1}{1+\delta}\right)^{M-t+1}\right) - 1\right]\left[1 + \frac{\alpha(1-\gamma)}{r-\alpha\gamma}\left(1 - \left(\frac{1+\alpha\gamma}{1+r}\right)^{M-t}\right)\right]} \tag{15}$$

Proof. See the appendix. ■

By applying equation (12) and (14) it is now straightforward to demonstrate that aggregate consumption grows according to:

$$\frac{C_{t+1}}{C_t} = (1+r)\frac{h_{t+1}}{h_t}(1-h_t) \tag{16}$$

in optimum. Although the optimal consumption growth rate is generally time-varying and non-linear, an important intuition can be provided:

Corollary 1 *Compared to non-growing economies or economies with exogenous growth, learning by doing implies that it is optimal to consume a lower fraction of national wealth in any period, except for the last period* $t = M$*.*

Proof. (A) That $h_M = 1$ regardless of the size of α or γ follows directly from (15).[9] (B) In any period $t < M$, the last square bracket in the denominator of (15) is (i) larger than 1 if $\alpha > 0$ and $\gamma < 1$, and (ii) equal to 1 if $\alpha = 0$ or $\gamma = 1$. Hence $h_{t|\alpha=0} = h_{t|\gamma=1} > h_{t|\alpha>0,\gamma<1}$, $t < M$. ■

The intuition behind Corollary 1 is that consumption is more costly in our endogenous growth model. In our economy increased consumption in one period not only lowers future financial wealth, it also lowers future productivity growth. In other words, saving an extra euro in our model gives interest *plus* higher production in the future. Hence, it is optimal to save more than in economies either without growth or with exogenous growth. Moreover, the consumption-wealth ratio increases faster over time with LBD.

Further intuition on the result of the optimal consumption growth can be provided by considering asymptotic properties of our model, i.e. when

[9] It also follows from combining (15) with the terminal condition $NW_{M+1} = 0$.

$M \to \infty$. When the planner has a very long time horizon, equation (15) gives:

$$\lim_{M\to\infty} h_t = \frac{\delta}{1+\delta+\frac{\alpha(1-\gamma)}{r-\alpha\gamma}}, \tag{17}$$

which is a constant. We note that when $\alpha = 0$ (zero growth) or when $\gamma = 1$ (exogenous growth), a constant share $\frac{\delta}{1+\delta}$ of national wealth should be consumed in each period. But with LBD, a lower constant share of national wealth should be consumed in each period. Furthermore, from (17) and (16) we have:

$$\lim_{M\to\infty} \frac{C_{t+1}-C_t}{C_t} = \frac{r\left(1+\frac{\alpha(1-\gamma)}{r-\alpha\gamma}\right)-\delta}{1+\delta+\frac{\alpha(1-\gamma)}{r-\alpha\gamma}}.$$

Thus with an infinite planning horizon, the optimal consumption growth rate is a constant. The first term in the numerator on the right-hand side of this expression can be interpreted as the effective interest rate with an infinite horizon in our model. It gives the marginal return from saving in the infinite horizon case. The planner would tilt the optimal consumption path up or down according to the difference between this adjusted interest rate and the rate of time preference. For instance, with $r = \delta$ it would be optimal with a flat consumption path in non-growing or exogenous growth economies, while in our model this parameter combination implies increasing optimal consumption over time. Again, this is because the effective interest rate is higher than r in our setup, increasing optimal saving.[10]

4 Optimal transfers and output growth

The optimal path for aggregate consumption discussed above has implications for how the foreign exchange gift should be phased into the economy. This section derives the optimal spending path, from which the paths for output and the current account follow. As the optimal consumption growth rate in general is time-varying and non-linear, the analytical solutions of the model become quite complex for horizons of more than two to three periods. To highlight the intuition behind our model we therefore proceed in two steps. First, we discuss the analytical solution in the two-period case in some detail. Second, we show numerical paths to highlight the intuition in the general case.

[10] With an infinite horizon our solution may be in conflict with $\eta_t \in (0,1)$, as in Matsuyama (1992). In that case one needs to maximize (6) subject to (13) *and* $\eta_t \in (0,1)$. We do not pursue this matter further.

4.1 An example with $M = 2$

With $M = 2$, from (15) we have $C_2 = NW_2$ and $C_1 = \frac{1+\delta}{2+\delta+\frac{\alpha(1-\gamma)}{1+r}} NW_1$. Then (16) gives us:

$$\frac{C_2}{C_1} = \frac{1+r}{1+\delta}\left(1 + \frac{\alpha(1-\gamma)}{1+r}\right). \tag{18}$$

LBD implies higher optimal consumption growth than in models with zero or exogenous growth. Since $C_t = H_t + R_t$, (18) can be expressed as:

$$R_2 + H_2 = (R_1 + H_1)\left[\frac{1+r}{1+\delta}\left(1 + \frac{\alpha(1-\gamma)}{1+r}\right)\right].$$

Substituting for H_2 from (8), we find that second period spending of the foreign exchange gift is:

$$\begin{aligned} R_2 \quad = \quad & \left[\frac{1+r}{1+\delta}\left(1 + \frac{\alpha(1-\gamma)}{1+r}\right) + \alpha(1-\gamma)\right] R_1 \\ & + \left[\frac{1+r}{1+\delta}\left(1 + \frac{\alpha(1-\gamma)}{1+r}\right) - (1+\alpha\gamma)\right] H_1. \end{aligned} \tag{19}$$

Let us pause here and temporarily assume that $r = \delta$:

- Without LBD ($\alpha = 0$) equation (19) would reduce to $R_2 = R_1$, which from (10) implies that $R_1 = \frac{(1+r)^2}{2+r} W_1$. This ensures that the two generations are given equal amounts of the foreign exchange gift.

- Within an exogenous growth framework ($\gamma = 1$), (19) gives $R_2 = R_1 - \alpha H_1$. Applying (10), we find $R_1 = \frac{(1+r)^2}{2+r} W_1 + \frac{1}{2+r}\alpha H_1$. The planner would now increase transfers to generation 1 with a share $1/(2+r)$ of the exogenous output growth from period 1 to 2.

- Using (19) in (10), our two-sector, LBD framework implies:

  $$R_1 = \frac{(1+r)^2}{2+r+\frac{2+r}{1+r}\alpha(1-\gamma)} W_1 + \frac{\alpha\gamma - \frac{\alpha(1-\gamma)}{1+r}}{2+r+\frac{2+r}{1+r}\alpha(1-\gamma)} H_1. \tag{20}$$

  The higher the foreign exchange gift W_1, the higher the transfers to generation 1 should be. With LBD, however, it is optimal to transfer a lower fraction of the foreign exchange gift than is otherwise the case.

  In the absence of a foreign exchange gift, transfers to the first generation are positive provided that $\gamma - \frac{(1-\gamma)}{1+r} > 0$, and negative if the opposite is the case. The intuition for this is that two effects pull in opposite directions. On the one hand, with a positive growth potential ($\alpha > 0$) the planner would like to transfer resources away from generation 2 towards generation 1. On the other hand, transferring resources to generation 1 is costly in terms of lower output growth. This cost is higher the more a given amount of transfers push down traded sector employment, and thus learning. The larger the share of non-traded goods in consumption $(1-\gamma)$, the more costly are transfers to generation one in terms of future output. Thus for a sufficiently high $(1-\gamma)$, transfers to the first generation are negative.

Whereas $r=\delta$ implies that the foreign exchange gift should be spread out in equal amounts in a non-growing economy, the first generation should receive more than the second with exogenous growth. With endogenous growth, this effect may very well be reversed. It is costly in terms of lower future output to spend the gift today, and so the planner may in fact transfer less to generation 1 compared to a non-growing economy.

Leaving the case of $r=\delta$, we can use (19) in (10) to find the general expression for optimal R_1:

$$R_1 = \frac{(1+r)^2}{1+r+\frac{1+r}{1+\delta}+\frac{2+\delta}{1+\delta}\alpha(1-\gamma)} W_1 + \frac{1+\alpha\gamma-\frac{\alpha(1-\gamma)}{1+r}-\frac{1+r}{1+\delta}}{1+r+\frac{1+r}{1+\delta}+\frac{2+\delta}{1+\delta}\alpha(1-\gamma)} H_1. \quad (21)$$

Without the foreign exchange gift R_1 is negative if the last numerator in (21) is negative. It then follows from (8) that the optimal output growth rate is higher than the 'market solution' implies. If the last numerator is positive, the optimal growth rate is lower than in the 'market solution'; more resources should be transferred to the first generation despite the lower growth that follows.

Equation (21) also shows us that R_1 is unambiguously increasing in W_1. Thus, the optimal output growth path decreases when the country receives a foreign exchange gift. In contrast to the positive growth models of the Dutch disease, such as Krugman (1987), Matsuyama (1992), Sachs and Warner (1995), Gylfason et al. (1999) and Torvik (2001), which tend to view lower growth as a problem resulting from foreign exchange gifts, we have shown that this is in fact an optimal response.

The implications for the current account are straightforward: ceteris paribus, LBD implies less consumption of the foreign exchange gift in period 1, giving a smaller current account deficit (larger surplus). Using (21) in (9), the current account in period 1 is:

$$CA_1 = \frac{r\left(\frac{1+r}{1+\delta}+\alpha(1-\gamma)\frac{2+\delta}{1+\delta}-1\right)-1}{1+r+\frac{1+r}{1+\delta}+\alpha(1-\gamma)\frac{2+\delta}{1+\delta}} W_1 + \frac{\frac{\alpha(1-\gamma)}{1+r}+\frac{1+r}{1+\delta}-(1+\alpha\gamma)}{1+r+\frac{1+r}{1+\delta}+\alpha(1-\gamma)\frac{2+\delta}{1+\delta}} H_1,$$

which in general has an ambiguous sign.

4.2 General case

To find the optimal spending of the foreign exchange gift when $M > 2$, we start by rewriting (16) as:

$$R_{t+1} + H_{t+1} = \left[(1+r)\frac{h_{t+1}}{h_t}(1-h_t)\right](R_t + H_t),$$

which in combination with (8) implies:

$$\begin{aligned} R_{t+1} &= \left[(1+r)\frac{h_{t+1}}{h_t}(1-h_t) + \alpha(1-\gamma)\right] R_t \\ &\quad - \left[1 + \alpha\gamma - (1+r)\frac{h_{t+1}}{h_t}(1-h_t)\right] H_t. \end{aligned} \tag{22}$$

Equations (8) and (22) comprise a system of difference equations that the two endogenous variables R and H have to fulfill in the optimum.

For horizons longer than two to three periods, the analytical solutions quickly become complex, and we illustrate the intuition with numerical simulations.

Parameters and initial state variable values

Each time period (generation) is 25 years and the planner has a planning horizon of 250 years, i.e. $M = 10$. In our benchmark simulations we set r and δ equal at 85.4 %. This corresponds to annual time preference rates and interest rates of 2.5 %. The traded goods expenditures share is set to $\gamma = 0.4$. We start out with a moderate LBD effect, using $\alpha = 0.1$ in our benchmark simulation. We normalize the first period's GDP, which is predetermined, to $H_1 = 100$. Finally, we assume that the country receives a substantial foreign exchange gift $W_1 = 25$, corresponding to about six years of initial period production.

Benchmark results

Chart 1 displays the optimal path of production, foreign exchange gift spending, foreign assets, and the current account, given the parameters and initial state variable values above.[11]

Chart 1 about here

Both output H and transfers R grow over time, but whereas output growth decreases through time, the growth in R increases (although this is barely visible in the chart, the effect is there). As it is optimal to spend relatively little of the foreign exchange gift in the first periods, the country initially builds up its foreign assets further. Not until period 7 does the planner start to run current account deficits CA. We notice that since

[11] To limit the number of paths, we leave out the path for aggregate consumption; it is simply the sum of H and R in each period.

R grows faster than output, equation (7) implies that employment in the traded sector optimally decreases over time.

To put these results into perspective, we display the corresponding paths in a non-growing economy ($\alpha = 0$) and an economy with exogenous growth ($\gamma = 1$) in charts 2 and 3 respectively. Without growth, all generations receive the same share of the foreign exchange gift, equal to the annuity value of the gift. As a result, the nation runs a current account deficit in each period, albeit at an increasing pace. (Up to and including period 6, the deficit is smaller than 1 % of GDP.) As there is a constant ratio between R and H, employment in the two sectors in this case is constant.

Chart 2 about here

Interestingly, chart 3 shows the opposite patterns for W, R, and CA compared to those in chart 1. With exogenous growth, the spending path for the foreign exchange gift should decrease over time. Foreign assets should decline at a rapid pace initially, and the current account should be negative until period 8 and then positive. We notice that this ensures equal consumption for each generation, whereas the endogenous growth framework in chart 1 implies increasing consumption over time. Again, this is because the optimal real interest rate for consumption decisions is in effect larger within our LBD framework.

Chart 3 about here

4.3 The slope of the spending path

We now turn to the factors affecting slope of the spending curve. Chart 4 displays the paths for output H (in the upper graphs) and transfers R (in the lower graphs) for different values of α. The higher is α, the more concave is the output path, and the more convex is the spending path of the foreign exchange gift. For higher values of α, the optimal R should start at a lower level and then increase faster the closer we are to the time horizon. The resulting output growth is one of fast initial growth that slows as we approach period M.

** Chart 4 about here **

Turning to the effect of the traded goods expenditure share γ, we have already seen from charts 1 and 3 above that different values can have important effects on the solution. While $\gamma = 0.4$ implies an increasing spending path, $\gamma = 1$ gives a negatively sloped optimal spending path. The opposite slopes of the spending paths reflect a fundamental trade-off that the planner faces in our model: *on the one hand output growth generally implies that the early generations should receive a larger share of the foreign exchange gift (as in an exogenous growth model), but on the other hand, spending should be postponed because of its adverse effect on future productivity.* The effect that pulls in the direction of large transfers to early generations is *stronger* the higher is γ, as a large expenditure share on traded goods implies a large traded sector and thus a high growth potential for any given level of total demand. The effect that pulls in the direction of postponing spending, on the other hand, is *weaker* the larger is γ. This is because a large expenditure share on traded goods ensures that little of an extra euro in demand is directed towards the non-traded sector. That is, higher demand does not greatly reduce traded sector employment (and thus productivity growth). Therefore, there is little gain in future productivity from postponing spending.

Thus, there is some value of γ where the two effects cancel, giving a constant optimal spending path. Holding other parameters fixed, $\gamma \approx 0.466$ gives a constant spending path in our example. Chart 5 illustrates the effect on optimal output and spending for three different values of γ. The higher is γ, the faster is optimal output growth (shown in the upper graphs) and the larger is the share of the foreign exchange gift that should be allocated to the first generations (shown in the lower graphs). We notice that although the optimal path for R falls for a sufficiently high γ, optimal aggregate consumption would increase over time in our model for all $\gamma < 1$.

** Chart 5 about here **

The effect on the spending path from a higher interest rate is analogous in our model to that in non-growing or exogenous growth economies. In all cases optimal saving increases and so the R path becomes steeper. However, in our endogenous growth framework, this would also imply that output growth increases initially and then becomes lower as M approaches. Likewise, an increase in the rate of time preference lowers optimal saving in all models considered, implying that it would be optimal to distribute more of the foreign exchange gift to the first generations. As a consequence optimal output growth would decrease initially and increase in later periods in the LBD model.

5 Discussion and extensions

5.1 Descriptive implications

Although our theory is normative, it may also contain some positive implications that is difficult to study in the existing theories of the Dutch disease. These theories assume an exogenous flow of resource income in each period and assume that all of the income is used in the same period. Hence, public saving out of the resource income is assumed to be equal to zero. Thus in these theories the more natural resources, the worse the growth outcome. This is clearly in contrast to reality. While resource rich Venezuela and Zambia have done poorly, resource rich Botswana and Norway have done well. The most interesting aspect of the resource curse is not to explain why resource income may lower growth - but why some countries have escaped the resource curse while others have not. Although normative, our model may help explain such diverging experiences in resource abundant countries. In contrast to the previous Dutch disease literature, policy in our model is endogenous; public saving is determined to satisfy the optimal response by the public sector. A prediction of our model is thus that countries who manage their resource wealth more in accordance with the optimality criteria will fare better than those who do not. In particular, our theory suggests the importance of savings out of the resource income - with Dutch disease productivity dynamics the optimal saving needs to be adjusted up as the effective interest rate is higher than the market interest rate.

According to Abidin (2001) resource abundant developing countries that have escaped the resource curse comprises Botswana, Chile, Malaysia, Oman and Thailand. According to Mehlum et al. (2002) additional resource rich countries that have escaped the curse are Australia, Canada, Ireland, New Zealand, Norway and United States. On the other hand Algeria, Congo, Ecuador, Mexico, Nigeria, Saudi Arabia, Sierra Leone, Trinidad & Tobago, Venezuela and Zambia are often mentioned as examples of cursed countries (see e.g. Gelb (1988), Karl (1997), Auty (2001), Gylfason (2001), Robinson et al. (2002), Olsson and Congdon Fors (2004), and Papyrakis and Gerlagh (2004)). In light of our model it should be of some interest to study if there are systematic differences in savings among the resource abundant winners and loosers. However, a potential problem is that savings from the national accounts can be very misleading for resource abundant countries. The reason for this is that savings as defined in the national accounts do not take into account that when non-renewable natural resources are extracted and sold this is really a reduction in a countrys' wealth, and not income in the traditional sense. We thus need to construct resource wealth adjusted savings data that takes this into account. We start off with national savings as a share of GNI from World Bank (2003) (we use GNI rather than GDP to account for the fact that some resource abundant countries have transformed part of their resource wealth into foreign assets that yield net factor income from abroad). We then subtract energy depletion, mineral depletion and net forest depletion (all as a share of GNI and from the same data source) to arrive at what we term the natural resource wealth adjusted savings rate. We then calculate the average of this savings rate over the last 25 years for each country. The results are shown in Table 1, where the above mentioned countries are grouped into countries that are claimed to have escaped the resource curse and countries that have not.

** Table 1 about here **

Table 1 reveals at least two interesting features. First, there is large variation in resource wealth adjusted savings rates between the countries.

The assumption about savings in the traditional Dutch disease literature is not supported by data. Second, although there are exceptions[12], there is a tendency that those countries that have escaped the resource curse have higher resource wealth adjusted savings rates than those who have not. For instance, ten out of the eleven countries in the first group have positive resource wealth adjusted savings rates, while seven out of the nine countries in the second group have negative resource wealth adjusted savings rates. We hasten to add, however, that this simple calculation should not be viewed as a test of our theory. Among obvious problems such as few countries one may also question to what extend causality runs from savings to economic performance, and to what extend it is the other way around. In light of our theory model, however, it is nevertheless interesting to note that countries in the two groups differ exactly in the dimension that the traditional Dutch disease literature assumes away.

5.2 Uncertainty

Although we have extended existing theories to deal with normative questions, we have maintained the simplifying assumption of full certainty. In reality, managers of resource wealth face substantial uncertainty about the level of resource reserves and the future price of the resource in question (see Katz and Bartsch, 2003 for a discussion of practical challenges in resource wealth management). As a first pass, introducing uncertainty would probably lower the consumption propensity out of resource wealth further, as it would give the planner an additional precautionary motive for saving.[13] However, once uncertainty is introduced it would also be natural to allow the planner to save in risky high-return assets in addition to the riskless bonds we include in our model. Rate-of-return risk would interact with the savings decision of the planner, but the theory is more ambiguous here as such risk would entail counteracting income and substitution effects on the savings decision.

Uncertainty in the present setup would also mean that the planner must make a portfolio choice in addition to the savings decision. The small open economy framework point to an internationally well-diversified portfolio of financial assets as the optimal saving vehicle. However, since resource income

[12] Oman beeing the most obviuos one. Although Oman has experienced positive per capita growth over the last decades one may question the claim of Abidin (2001) that it has escaped the resource curse. One may argue that the measured income growth is to a large extend the reflection of a massive rundown of natural resource wealth, and that it is not sustainable. However, since we group countries according to what earlier studies have argued, Oman belongs in the group of countries that are claimed to have escaped the resource curse.

[13] It has been known since Sandmo (1970) that endowment uncertainty triggers precautionary savings as long as the utility function of decision maker has a positive third derivative. The utilty function of the social planner in this paper has this property.

would be a dominant revenue source for many resource abundant countries, so-called hedging portfolios could be highly relevant for resource wealth managers. This would imply that the investment portfolio would include positions that, as far as possible, offsets the income risk associated with the resource revenue. In general, this would imply that the manager "overweights" in assets which returns have a low (preferably negative) correlation with the resource revenue, and invests less than a neutral position (perhaps even sell short) assets that are highly correlated with resource income. It is beyond the scope of this paper to give a full-blown analysis of savings and portfolio choice in the presence of the Dutch disease, so we leave that as a natural topic for future research.

6 Conclusions

The growth literature on the Dutch disease has provided important contributions towards understanding why resource abundance may reduce growth. In addition the literature has raised new questions that need to be analyzed in a normative setting. If the use of resource wealth hurts productivity growth, an important question is how such wealth should then be managed.

In this paper we have studied this question by extending the growth literature on the Dutch disease from a positive to a normative setting. This extension may also help explain positive features such as the diverging experiences between resource abundant countries. Adopting the same assumptions of productivity growth that were used in the earlier growth literature on the Dutch disease, we have derived the implications for optimal saving of resource wealth and the corresponding optimal growth of consumption and output. LBD implies that the optimal share of national wealth consumed in each period needs to be adjusted downwards. However, some Dutch disease is always optimal in the sense that a positive fraction of the resource wealth should be consumed in each period. We have seen that the optimal consumption decision differs from models of both zero and exogenous growth. The spending path of the resource wealth may be increasing or decreasing over time. The less important traded goods are as a proportion of consumption, the more likely it is that the optimal spending path of the resource wealth is increasing over time.

7 References

Abidin, M.H. (2001) "Competitive Industrialization with Natural Resource Abundance: Malaysia." in R.M. Auty (2001).

Auty R.M. (1999) "The transition from rent-driven growth to skill-driven growth: recent experience of five mineral economies." In: Maier, J., Chambers, B., Farooq, A. (eds.) *Development Policies in Natural Resource Economies*, Edward Elgar, Cheltenham.

Auty R.M. (2001) *Resource Abundance and Economic Development*, Oxford University Press, Oxford.

Baland, J.-M. and P. Francois. (2000) "Rent-seeking and resource booms." *Journal of Development Economics* 61: 527-542.

Busby, G., J. Isham, L. Pritchett and M. Woolcock (2002) "The varieties of rentier experience: how natural resource export structures affect the political economy of economic growth." Mimeo, The World Bank.

Corden, W.M. (1984) "Booming sector and Dutch disease economics: survey and consolidation." *Oxford Economic Papers* 36: 359-380.

Corden, W.M. and J.P. Neary (1982) "Booming sector and de-industrialisation in a small open economy." *Economic Journal* 92: 825-848.

Damania, R. and E. Bulte (2003) "Resources for sale: Corruption, democracy and the natural resource curse." Mimeo, University of Adelaide.

Diamond, P. (1965) "National debt in a neoclassical growth model." *American Economic Review* 55: 1126-1150.

Gelb, A. (1988) *Windfall Gains: Blessing or Curse?*, Oxford University Press, Oxford.

Gylfason, T., T.T. Herbertsson and G. Zoega (1999) "A mixed blessing: Natural resources and economic growth." *Macroeconomic Dynamics* 3: 204-225.

Gylfason, T. (2001) "Natural resources, education, and economic development." *European Economic Review* 45: 847-859.

Hannesson, R. (2001) *Investing for Sustainability: The Managment of Mineral Wealth*, Kluwer Academic Publishers, Boston.

Hartwick, J. (1977). "Intergenerational equity and the investing of rents from exhaustible resources." *American Economic Review* 67: 972-991.

Karl, T.L. (1997) *The Paradox of Plenty: Oil Booms and Petro States*, University of California Press, Berkeley.

Katz, M. and U. Bartsch (2003) "Transforming the oil 'curse' into a blessing." *IMF Survey* 32 (12): 194-196.

Krugman, P. (1987) "The narrow moving band, the Dutch disease, and the competitive consequences of Mrs. Thatcher: notes on trade in the presence of dynamic scale economies." *Journal of Development Economics* 37: 41-55.

Lane, P.R. and A. Tornell (1996) “Power, growth and the voracity effect.” *Journal of Economic Growth* 1: 213-241.

Matsuyama, K. (1992) “Agricultural productivity, comparative advantage and economic growth.” *Journal of Economic Theory* 58: 317-334.

Mehlum, H., K.O. Moene and R. Torvik (2002) “Institutions and the resource curse.” Mimeo, Department of Economics, University of Oslo.

Neary, J.P. and D.D. Purvis (1983) “Real adjustment and exchange rate dynamics.” In: Frenkel, J.A. (ed.) *Exchange Rates and International Macroeconomics*, University of Chicago Press, Chicago.

Neary, J.P. and S. van Wijnbergen (1986) “Natural resources and the macroeconomy: a theoretical framework.” In: Neary, J.P., van Wijnbergen, S. (eds.) *Natural Resources and the Macroeconomy*, Blackwell, Cambridge.

Obstfeld, M. and K. Rogoff (1996) *Foundations of International Macroeconomics*, MIT Press, Cambridge, MA.

Olsson, O. and H. Congdon Fors (2004) ”Congo: The prize of predation.” *Journal of Peace Research* 41: 321-336.

Papyrakis, E. and R. Gerlagh (2004) ”The resource curse hypothesis and its transmission channels.” *Journal of Comparative Economics* 32: 181-193.

Robinson, J.A., R. Torvik and T. Verdier (2002) ”Political foundations of the resource curse.” CEPR Discussion Paper No. 3422.

Rodriguez, F. and J. Sachs (1998) ”Why do resource-abundant economies grow more slowly.” *Journal of Economic Growth* 4: 277-303.

Ross, M.L. (1999) ”The political economy of the resource curse.” *World Politics* 51: 297-322.

Ross, M.L. (2001) *Timber Booms and Institutional Breakdown in Southeast Asia*, Cambridge University Press, New York.

Røed Larsen, E. (2003) ”Are rich countries immune to the resource curse? Evidence from Norway’s managment of its oil riches.” Discussion paper no. 362, Statistics Norway.

Sachs, J.D. and A.M. Warner (1995) “Natural resource abundance and economic growth.” NBER Working Paper No. 5398.

Sachs, J.D. and A.M. Warner (1997) “Sources of slow growth in African economies.” *Journal of African Economies* 6: 335-376.

Sachs, J.D. and A.M. Warner (2001) “The curse of natural resources.” *European Economic Review* 45: 827-838.

Sandmo, A. (1970) ”The effect of uncertainty on saving decisions.” *Review of Economic Studies* 37: 353-360.

Solow, R. (1974) ”Intergenerational equity and exhaustible resources.” *Review of Economic Studies* 41: 29-45.

Solow, R. (1986) ”On the intergenerational allocation of natural resources.” *Scandinavian Journal of Economics* 88: 143-154.

Stijns, J.P. (2002) “Natural resource abundance and economic growth revisited.” Mimeo, Department of Economics, UC Berkeley.

Tornell, A. and P.R. Lane (1999) "The voracity effect." *American Economic Review* 89: 22-46.

Torvik, R. (2001) "Learning by doing and the Dutch disease." *European Economic Review* 45: 285-306.

Torvik, R. (2002) "Natural resources, rent seeking and welfare." *Journal of Development Economics* 67: 455-470.

van Wijnbergen, S. (1984) "The 'Dutch disease': a disease after all?" *Economic Journal* 94: 41-55.

World Bank (2003) "World Development Indicators." Online version.

A Proof of proposition 1

For the proposed value function J_t, the Bellman optimality equation is:

$$\begin{aligned}
& \Phi_t + \Theta_t \log NW_t \\
= & \max_{R_t} \left[\log(R_t + H_t) + \frac{1}{1+\delta}(\Phi_{t+1} + \Theta_{t+1} \log NW_{t+1}) \right], \quad \text{(A.1)}
\end{aligned}$$

subject to (12). The first-order condition can be written as:

$$\begin{aligned}
C_t^{-1} &= \frac{\Theta_{t+1}}{1+\delta} \left[1 + r + \alpha(1-\gamma) \sum_{s=t+1}^{M} \left(\frac{1+\alpha\gamma}{1+r} \right)^{s-t+1} \right] NW_{t+1}^{-1} \\
&= \frac{1+r}{1+\delta} \Theta_{t+1} \left[1 + \frac{\alpha(1-\gamma)}{r-\alpha\gamma} \left(1 - \left(\frac{1+\alpha\gamma}{1+r} \right)^{M-t} \right) \right] NW_{t+1}^{-1}.
\end{aligned}$$

Inverting this expression, substituting for NW_{t+1} from (13), and simplifying gives:

$$\begin{aligned}
C_t &= \frac{(1+r)(1+\delta)}{(1+r)(1+\delta) + (1+r)\Theta_{t+1} \left[1 + \frac{\alpha(1-\gamma)}{r-\alpha\gamma} \left(1 - \left(\frac{1+\alpha\gamma}{1+r} \right)^{M-t} \right) \right]} NW_t \\
&\equiv h_t NW_t. \qquad \text{(A.2)}
\end{aligned}$$

Substituting for C in (A.1) gives:

$$\begin{aligned}
\Phi_t + \Theta_t \log NW_t &= \log(h_t NW_t) \\
&\quad + \frac{1}{1+\delta} \left\{ \Theta_{t+1} \log\left[(1+r)(1-h_t) NW_t \right] + \Phi_{t+1} \right\} \\
&= \left(1 + \frac{1}{1+\delta} \Theta_{t+1} \right) \log NW_t \\
&\quad + \log h_t + \frac{\Phi_{t+1}}{1+\delta} + \frac{\Theta_{t+1}}{1+\delta} \log\left((1+r)(1-h_t) \right).
\end{aligned}$$

Thus, the proposed value function is established for:

$$\Theta_t = 1 + (1+\delta)^{-1} \Theta_{t+1}, \qquad \text{(A.3)}$$

and:

$$\Phi_t = \log h_t + \frac{\Phi_{t+1}}{1+\delta} + \frac{\Theta_{t+1}}{1+\delta} \log\left((1+r)(1-h_t)\right).$$

(A.3) can be evaluated recursively by observing that $\Theta_M = 1$. Hence, $\Theta_{M-1} = 1 + \frac{1}{1+\delta}$, $\Theta_{M-2} = 1 + \frac{1}{1+\delta} + \left(\frac{1}{1+\delta}\right)^2$, etc. In general,

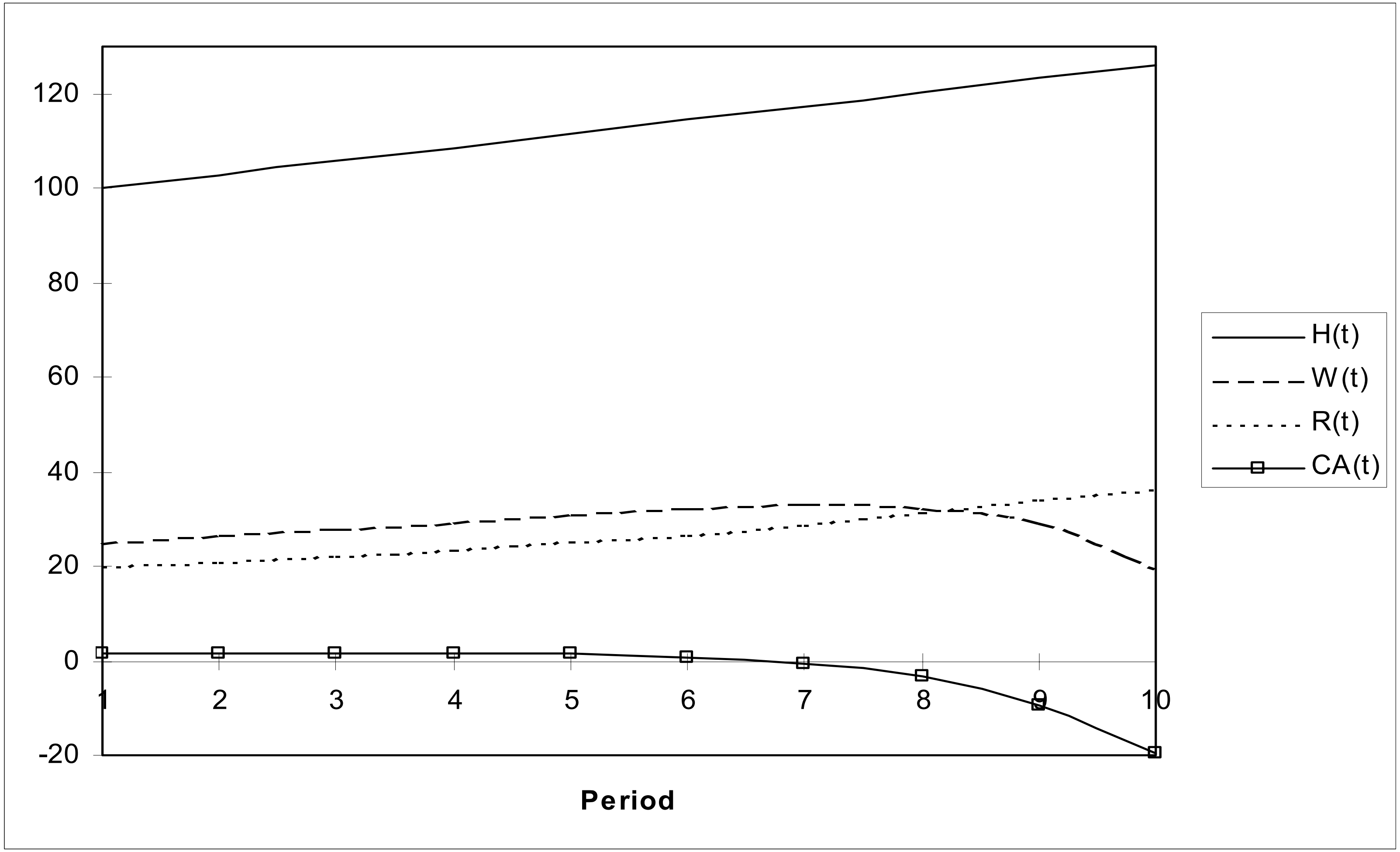
$$\begin{aligned} \Theta_t &= 1 + \frac{1}{1+\delta} + \left(\frac{1}{1+\delta}\right)^2 + \cdots + \left(\frac{1}{1+\delta}\right)^{M-t} \\ &= \frac{1+\delta}{\delta}\left(1 - \left(\frac{1}{1+\delta}\right)^{M-t+1}\right). \end{aligned} \tag{A.4}$$

Applying in (A.2) gives:

$$h_t = \frac{(1+r)(1+\delta)}{(1+r)(1+\delta) + (1+r)(1+\delta)\left(\Theta_t - 1\right)\left[1 + \frac{\alpha(1-\gamma)}{r-\alpha\gamma}\left(1 - \left(\frac{1+\alpha\gamma}{1+r}\right)^{M-t}\right)\right]}.$$

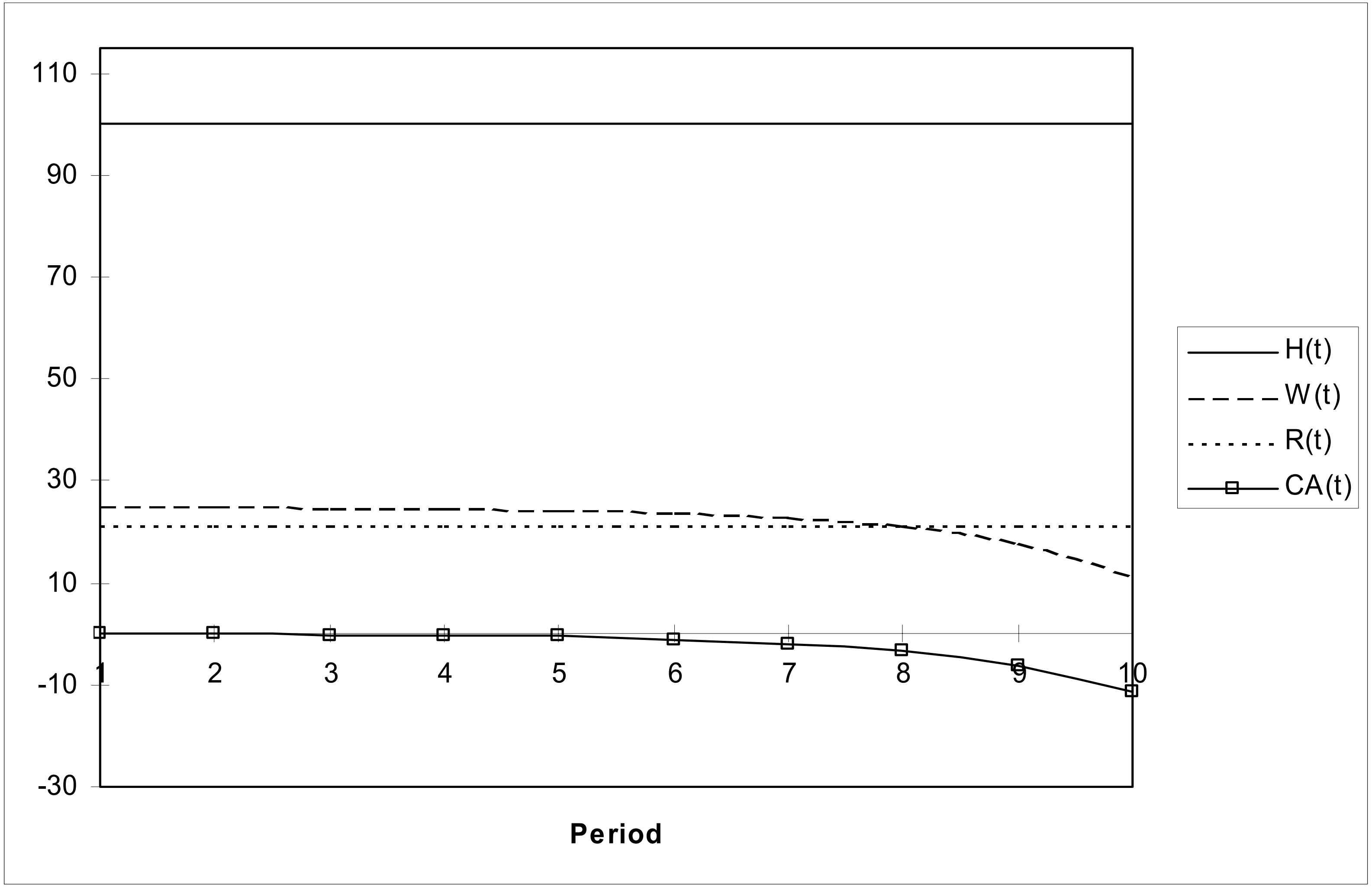
Inserting from (A.4) gives us equation (15), and completes the proof.■

Chart 1: Optimal paths for output, spending of the foreign exchange gift, the current account, and beginning-of-period foreign assets.



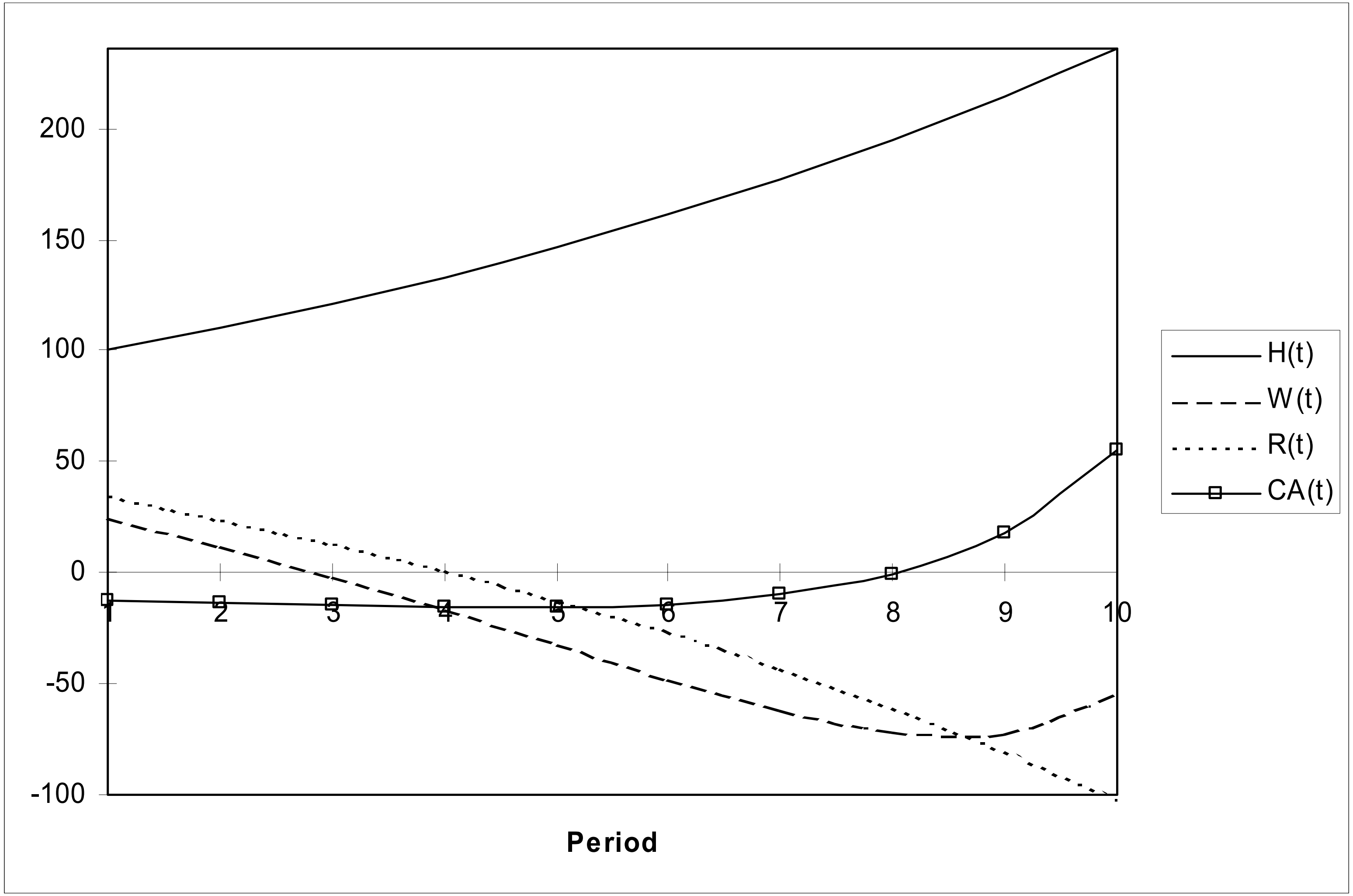
Note: Based on following parameter- and initial state variable values: $r = \delta = 85.4$ %, $\alpha = 0.1$, $\gamma = 0.4$, $H_1 = 100$, $W_1 = 25$.

Chart 2: Optimal paths for output, spending of the foreign exchange gift, the current account, and beginning-of-period foreign assets in a non-growing economy.



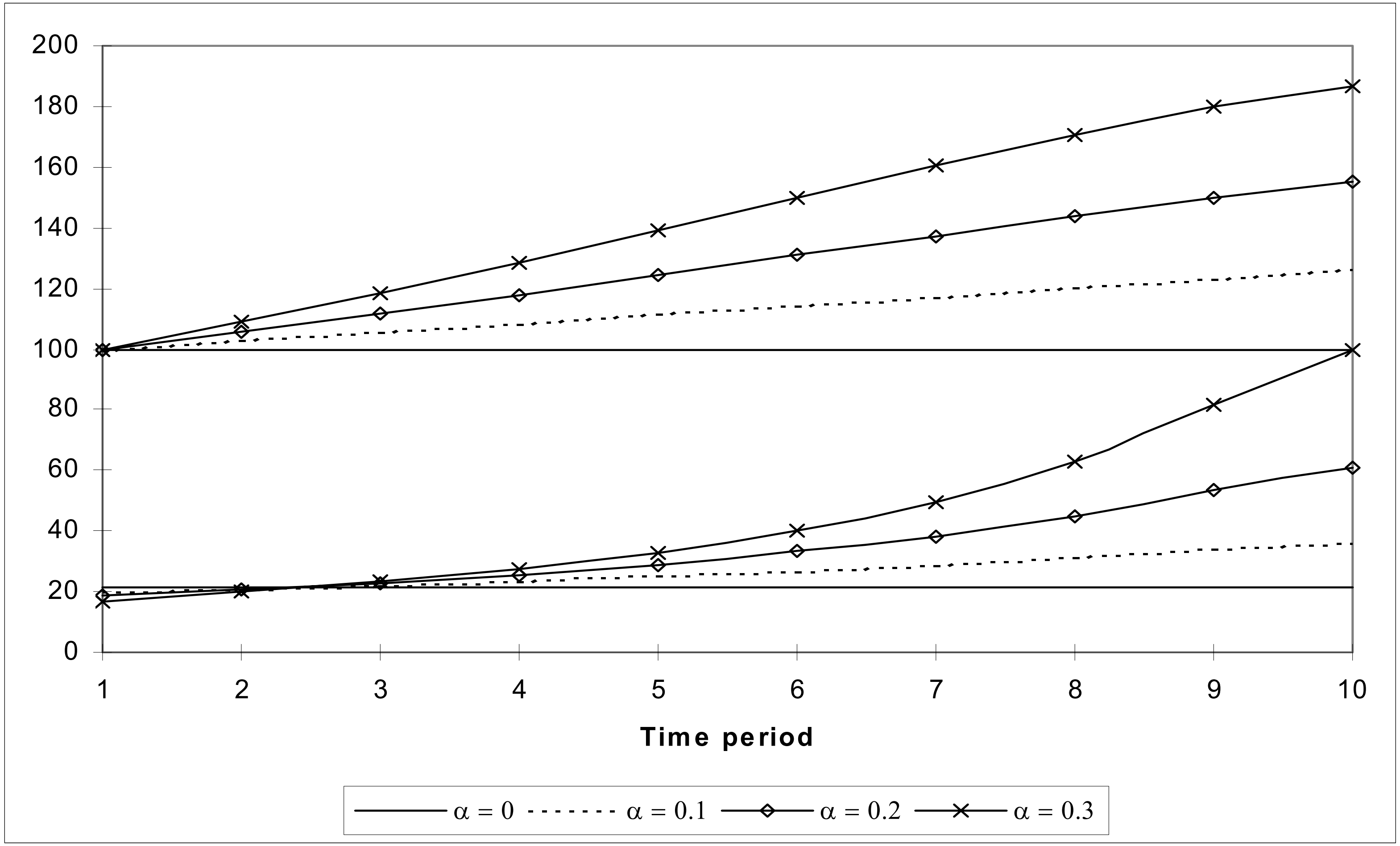
Note: Based on following parameter- and initial state variable values: $r = \delta = 85.4$ %, $\alpha = 0$, $\gamma = 0.4$, $H_1 = 100$, $W_1 = 25$.

Chart 3: Optimal paths for output, spending of the foreign exchange gift, the current account, and beginning-of-period foreign assets in an economy with exogenous growth.



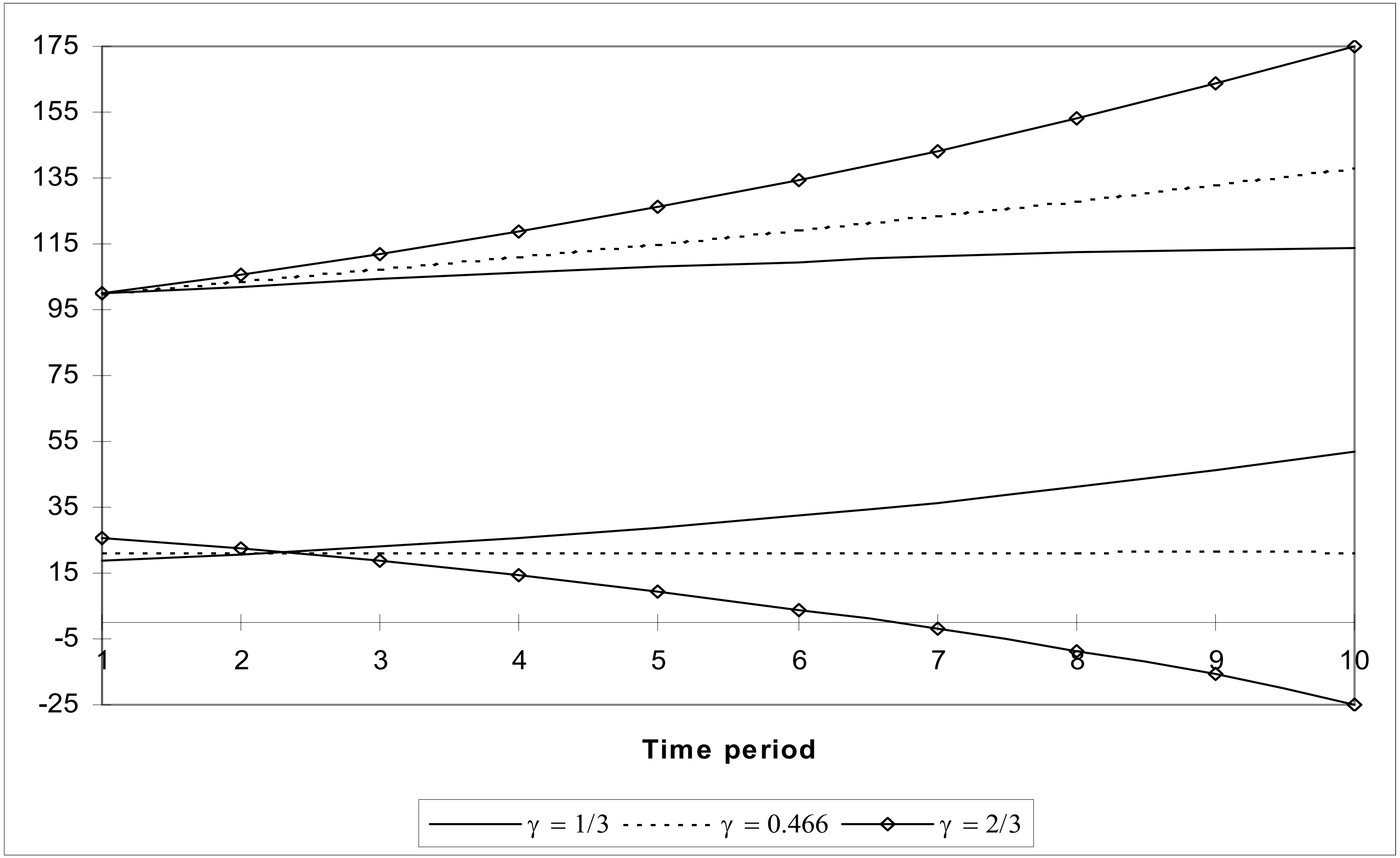
Note: Based on following parameter- and initial state variable values: $r = \delta = 85.4\ \%$, $\alpha = 0.1$, $\gamma = 1$, $H_1 = 100$, $W_1 = 25$.

Chart 4: Optimal paths for output (upper 4 graphs) and spending of the foreign exchange gift (lower 4 graphs) for different values of α.



Note: Except for α, all parameters and initial state variables have the same values as in Chart 1.

Chart 5: Optimal paths for output (upper 3 graphs) and spending of the foreign exchange gift (lower 3 graphs) for different values of γ.



Note: Except for γ, all parameters and initial state variables have the same values as in Chart 1.

Table 1: Resource Wealth Adjusted Saving Rates, 1972-2000.

Escapers		Non-escapers	
Australia	18.0 %	Algeria	6.11 %
Botswana	33.0 %	Congo	-11.9 %
Canada	15.7 %	Ecuador	n.a.
Chile	7.4 %	Mexico	10.8 %
Ireland	22.0 %	Nigeria	-22.0 %
Malaysia	19.9 %	Saudi Arabia	-21.5 %
New Zealand	18.4 %	Sierra Leone	-1.8 %
Norway	17.0 %	Trinidad & Tobago	-3.9 %
Oman	-26.6 %	Venezuela	-1.8 %
Thailand	20.0 %	Zambia	-5.8 %
USA	15.1 %		

Note: The table report the average value of national saving less energy depletion, mineral depletion and net forest depletion, as a share of gross national income. The averages are corrected for missing observations for some countries. Source: World Bank (2003).