De Janvry and Sadoulet (1983) present a model of unequalizing growth and the closure rule of the model is discussed in this note. It is shown that income distribution in the model is determined by the market clearing mechanism in the wage goods market and how the condition for social articulation is related to the excess demand function on the wage goods market.

1. Introduction

De Janvry and Sadoulet (1983), DJS, give a stimulating contribution to the discussion on income distribution and growth. This note offers a discussion and a clarification of the adjustment mechanisms implied in their model of unequalizing growth.

2. The model

To get a better understanding of the model, we turn it into a sequential dynamic framework. The idea is that we should be able to understand the story told by the model as a sequence of adjustments. The model then comes out with a nice recursive structure as shown below (with the symbols defined in the appendix):

\[ I_{i,t} = k_i I_{i,t-1}, \quad i=1,2, \]  
\[ K_{i,t} = K_{i,t-1} + I_{i,t-1} - \delta_i K_{i,t-1}, \quad i=1,2, \]  
\[ X_{i,t} = K_{i,t}, \quad i=1,2, \]  
\[ M_{i,t} = \beta_i X_{i,t}, \quad i=1,2, \]  
\[ L_{i,t} = \alpha_i X_{i,t}, \quad i=1,2, \]  

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The 14 independent equations of the model determine the time paths of the endogenous variables $I_{i,t}, K_{i,t}, X_{i,t}, M_{i,t}, L_{i,t}, P_{i,t} (i=1,2)$ and $I_t$. One degree of freedom is left to determine the relative wages $WM_t/WL_t$, that DJS call income distribution. The interpretation of the adjustment of income distribution is a key issue, and we think that it should include a story as to how wages are set. We will concentrate here on an alternative where the development of the wage level of the workers, $WL_t$, is set exogenously and the growth of the wage level of the managers, $WM_t$, is determined by the model. A closure with the opposite assumption is possible and will be discussed below.

The model can be solved according to the recursive structure when we start out with an investment level inherited from the previous period, $I_{t-1}$:

(i) Eqs. (1) and (2) allocate the investment goods to the two sectors and determine the new capital stocks $K_{1,t}$ and $K_{2,t}$.

(ii) The production outputs of the two sectors, $X_{1,t}$ and $X_{2,t}$, are determined by (3) when the capital stocks are known.

(iii) The inputs of the two variable production factors, workers $L_{i,t}$ and managers $M_{i,t} (i=1,2)$, are determined by fixed coefficients given the production outputs by (4) and (5).

(iv) The prices can be calculated from (6) as a function of the endogenous variable $WM_t$, the wage level of managers.

(v) Using the price equations, we get a market balancing condition for the wage goods sector 1 with only one endogenous variable $WM_t$. The excess demand equation for the wage goods sector looks like this:

$$ED_{1,t} = aWL_t^b(\alpha_1 X_{1,t} + \alpha_2 X_{2,t}) + aWM_t^b(\beta_1 X_{1,t} + \beta_2 X_{2,t}) - (\alpha_1 WL_t + \beta_1 WM_t)(1 + r_t)X_{1,t} = 0.$$  

(9)

This allows us to write the price equation in compact form with $\alpha_i^*$ and $\beta_i^* (i=1,2)$ dependent on the coefficients of the system,

$$\alpha_1^* = \frac{a_1 + \gamma_1 \delta_1 (1 + r_2)}{1 - \gamma_2 \delta_2 (1 + r_2)} \alpha_2,$$

$$\alpha_2^* = \frac{a_2}{1 - \gamma_2 \delta_2 (1 + r_2)}.$$
\[ \beta_1^* = \beta_1 + \frac{\gamma_1 \delta_1 (1 + r_2)}{1 - \gamma_2 \delta_2 (1 + r_2)} \beta_2, \quad \beta_2^* = \frac{\beta_2}{1 - \gamma_2 \delta_2 (1 + r_2)}. \]

It is important to notice that we can solve for the wage rate \( WM_t \) that clears the market. Consequently, the wage level of the managers and thereby the income distribution as defined by DJS are determined on the wage goods market. This conclusion about income distribution will go through whatever closure is chosen regarding \( WM_t \) and \( WL_t \). In this situation, it is not remarkable that income distribution is determined independently of the level of investment in the same period.

(vi) Since the neoclassical rule of savings-determined investment applies, we can solve for the investment level as a residual on the luxury goods market. For future reference, we write out the excess demand equation:

\[
ED_{2,t} = \left((1-s_L)WL_t - aWL_t^b\right)\left(\alpha_1 X_{1,t} + \alpha_2 X_{2,t}\right) + \left((1-s_M)WM_t - aWM_t^b\right)\left(\beta_1 X_{1,t} + \beta_2 X_{2,t}\right) + (\alpha_2^* WL_t + \beta_2^* WM_t)(1 + r_2)(I_t - X_{2,t}) = 0.
\]

3. Economic adjustments implied by the model

The model can be boiled down to two independent equations with two endogenous variables, \( WM_t \) and \( I_t \), in each period. We choose to deal with the two market balances, but the overall savings-investment balance can also be applied since Walras’ Law holds. The two excess demand equations (9) and (10) are driven to zero by adjustments in the wage rate of the managers and the investment level. The model in compact form may be written as

\[
ED_{1,t} = F_1\{WM_t\} = 0, \tag{11}
\]
\[
ED_{2,t} = F_2\{I_t; WM_t\} = 0. \tag{12}
\]

And the adjustment rule is

\[
\frac{\partial WM_t}{\partial t} = G_1\{ED_{1,t}\}, \tag{13}
\]
\[
\frac{\partial I_t}{\partial t} = G_2\{ED_{2,t}\}. \tag{14}
\]

The adjustment mechanism becomes clear when we differentiate the two excess demand equations with respect to \( WM_t \) and \( I_t \). Two effects of the
wage rate $WM_t$ on the excess demand for wage goods can then be identified. First, a rise in $WM_t$ creates demand by managers that increases the excess demand. This we will call the direct demand effect. Second, a rise in $WM_t$ increases the price level and reduces the excess demand for wage goods by reducing the real wage of the workers. This we will call the price effect. It is not a priori clear which of the effects will dominate, and this is the essential condition for social articulation. A closure with $WM_t$, exogenous and $WL_t$ endogenous will give a similar situation.

When the direct demand effect dominates, the following condition will hold:

$$ab\left(\frac{WM_t}{WL_t}\right)^{b-1} \left(\beta_1 + \beta_2 \frac{X_{2,t}}{X_{1,t}}\right) - \beta_1^* WL_t^{1-b}(1+r_1) \geq 0.$$  \hspace{1cm} (15)

The price effect will dominate if the expression at the left-hand side of (15) is negative.

On the luxury goods market as shown by (10), the wage rate $WM_t$ also has the same two types of effects. An increase in $WM_t$ both creates demand and increases the price level, and the dominating effect will determine the adjustments taken by the investment level.

It is easy to show that the short-run adjustment mechanism of the model is stable when the price effect dominates. However, the direct demand effect leads to a short-run unstable system with normal assumptions about the adjustment rule (13).

4. The condition for social articulation

DJS use the model to discuss the relationship between the relative wages $WM_t/WL_t$ and the relative outputs $X_{2,t}/X_{1,t}$. The relationship can be derived from the excess demand equation for the wage goods sector, eq. (9). When we divide through by $1/WL_t^b$ and $1/X_{1,t}$, and then differentiate with respect to $WM_t/WL_t$ and $X_{2,t}/X_{1,t}$, we get:

$$\frac{\partial(WM_t/WL_t)}{\partial(X_{2,t}/X_{1,t})} = \frac{-a(x_2 + \beta_2(WM_t/WL_t)^b)}{ab(WM_t/WL_t)^{b-1}(\beta_1 + \beta_2(X_{2,t}/X_{1,t})) - \beta_1^* WL_t^{1-b}(1+r_1)}.$$  \hspace{1cm} (16)

This expression is simpler than that reached by DJS, since we take advantage of imposing a sequential dynamic structure. They discuss the condition for social disarticulation, that is:

$$\frac{\partial(WM_t/WL_t)}{\partial(X_{2,t}/X_{1,t})} \geq 0.$$  \hspace{1cm} (17)
The numerator is always negative. The denominator is equal to our eq. (15), and the sign is dependent on whether the direct demand effect or the price effect is dominating on the wage goods market. The conclusion is:

If the direct demand effect is dominating we have a situation of social articulation.

If the price effect is dominating, we have a situation of social disarticulation.

In general, both the demand elasticities, the production structure, the price system, the wage levels and the income distribution will determine which effect will be dominating. In accordance with DJS, we find that a higher level of managers' wages will contribute to a situation of social disarticulation, since it will increase the importance of the price effect. Also a large wage goods sector and a high markup rate in the wage goods sector will tend to give a dominating price effect. On the other hand, an increase in the income elasticity of the wage goods will increase the importance of the direct demand effect and thereby social articulation.

The interpretation of this condition is best understood by discussing the adjustment mechanism on the wage goods market. We assume here that the luxury sector is the key growth sector, and we study a shift in the production structure towards the luxury sector. An increase in $X_{2,t}/X_{1,t}$ will create an excess demand on the wage goods market compared to the initial equilibrium. In the case of a dominating direct demand effect, the wage rate of managers, $WM_p$, has to go down to clear the wage goods market by reducing the demand from managers. Income distribution will be more equal and may be called social articulation. If the price effect is dominating, the wage rate $WM_t$ has to go up to increase the price level so that the demand from workers is contracted by a reduction in the real wage. The income distribution will be more unequal and we can name this social disarticulation.

The dynamics of the DJS story are obviously related to the changing production structure over time, represented by $X_{2,t}/X_{1,t}$, in condition (15).

5. Concluding remarks

The question to be raised is whether the adjustment mechanism of the model gives a realistic story. It is implicitly assumed that the income distribution across sectors of the economy is determined to clear one product market, the wage goods market. Further research seems to be needed to develop other closure rules for models of social articulation. In addition, future model work must allow for a more flexible response both in demand and production over time.
Appendix

Definition of symbols, all definitions refer to period $t$.

$x_{i,t}$ output of sector $i$ ($i=1,2$),
$L_{i,t}$ employment of workers in sector $i$ ($i=1,2$),
$M_{i,t}$ employment of managers in sector $i$ ($i=1,2$),
$K_{i,t}$ physical capital stock in sector $i$ ($i=1,2$),
$P_{i,t}$ production price of sector $i$ ($i=1,2$),
$r_{i,t}$ markup rate of sector $i$ ($i=1,2$),
$I_{i,t}$ investment in sector $i$ ($i=1,2$),
$ED_{i,t}$ excess demand in sector $i$ ($i=1,2$),
$WL_t$ workers wage rate,
$WM_t$ managers wage rate,
$I_t$ total investments,
$s_L$ propensity to save of workers,
$s_M$ propensity to save of managers,
$\alpha_i$ workers labor coefficient in sector $i$ ($i=1,2$),
$\beta_i$ managers labor coefficient in sector $i$ ($i=1,2$),
$\gamma_i$ capital coefficient in sector $i$ ($i=1,2$),
$a$ consumption function parameter,
$b$ income elasticity of consumption of good 1,
$k_i$ share of total investments allocated to sector $i$ ($i=1,2$),
$\delta_i$ depreciation rate in sector $i$ ($i=1,2$).

Reference