Trade policy in a growth model with technology gap dynamics and simulations for South Africa

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Abstract

We extend an open economy Ramsey model to include the technology gap to the world technology frontier. The setting is a middle income country with productivity growth driven by technology adoption and foreign capital goods stimulating spillover and catching up. The interaction of technology adoption and capital accumulation generates prolonged transition growth and strengthens the growth effect of increased openness. Model simulations reproduce the changing openness in South Africa 1960–2005. International sanctions and protectionism are represented by a calibrated tariff equivalent, and the counterfactual elimination of the tariff equivalent shows large potential for GDP growth. According to our preferred parameterization increased trade share by 10% points raises GDP level over time by about 12%. Separating the effects of openness between investment and productivity we find that almost 60% of the increase in GDP is due to increased productivity, partly because of interaction with higher investment.

1. Introduction

Technology adoption is an important channel of productivity growth and is influenced by trade openness. Technological innovation is concentrated to the few most advanced economies, and most semi-industrialized middle income countries take benefit of innovations by foreign technology adoption. Keller (2004) and Saggi (2002) give an overview of the understanding of international technology spillover. Lucas (2009) understands the world growth pattern as a result of cross-country flows of production-related knowledge from the successful economies to the less successful ones. Caselli and Coleman (2006) show how adoption can be understood related to the gap to the world technology frontier.

We extend the standard open economy Ramsey model to take into account how foreign trade affects technology adoption and technology gap. Turnovsky (2009) gives an overview of small open economy models in this tradition. The technology gap dynamics typically is formulated as long run equilibrium gap with productivity growth equal to the world frontier. The technology gap affects the transitional growth towards a long run growth. The role of the technology gap has been given a first analysis in such models by Duczynski (2003), and we extend this by assuming that foreign trade affects the speed of technology adoption. This is handled in a model similar to Sen and Turnovsky (1989) separating between...
dynamics have been developed by Ngai (2004) for different country groups and Japan, Coleman (2005) for Japan, Duarte have been used in the Parente and Prescott (1994) tradition including barriers to capital accumulation (see for instance, represents an alternative to econometric analyses of the trade–growth relationship. Calibrated general equilibrium models process. The model is simulated based on the growth experience of South Africa. Our method of quantification also mechanism introduced here in interaction with investment is the dynamics of technology gap and adoption.

The quantitative effect is consistent with econometric studies. The cross-country analysis of Frankel and Romer (1999) for economic growth. Trade liberalization stimulates transition growth because of cheaper investment goods, more technology adoption, and productivity induced capital accumulation. Permanent anticipated trade liberalization leads to lower long run cost of foreign investment and higher long run productivity level. The steady state path has higher income and capital per worker. The higher productivity growth adds to the transition growth directly and indirectly by raising the capital accumulation. This is in accordance with the argument of Hulten (2001) that productivity improvements contribute to higher capital accumulation and must be seen as interdependent growth mechanisms.

The growth effects of tariff liberalization are channeled through investments and the choice between domestic and foreign capital goods. Tariffs influence relative prices including the cost of investment and lead to substitution effects towards foreign goods. The share of foreign capital goods encourages technology adoption and productivity growth given the gap to the technology frontier. Productivity and investment effects interact and strengthen the overall consequences for economic growth. Trade liberalization stimulates transition growth because of cheaper investment goods, more technology adoption, and productivity induced capital accumulation. Permanent anticipated trade liberalization leads to lower long run cost of foreign investment and higher long run productivity level. The steady state path has higher income and capital per worker. The core capital accumulation channel is already studied by Sen and Turnovsky (1989). The new mechanism introduced here in interaction with investment is the dynamics of technology gap and adoption.

We offer an attempt at quantifying the importance of investment and productivity and their interaction in the growth process. The model is simulated based on the growth experience of South Africa. Our method of quantification also represents an alternative to econometric analyses of the trade–growth relationship. Calibrated general equilibrium models have been used in the Parente and Prescott (1994) tradition including barriers to capital accumulation (see for instance, Chari et al., 1996; Restuccia, 2004). Broader applied growth models dealing with economic growth and productivity dynamics have been developed by Ngai (2004) for different country groups and Japan, Coleman (2005) for Japan, Duarte and Restuccia (2007) for Portugal, and Diao et al. (2005, 2006) for Thailand. The growth model of South Africa by Rattsø and Stokke (2007) has been the starting point for the analysis developed below. Ferreira and Trejo (2006) have made simulations of open economy dynamics by combining the Heckscher–Ohlin trade framework with a standard neoclassical model. Quantification of the model illustrates how protectionism may explain cross-country income and productivity differences. Similar results are found by Waugh (2010). While these analyses focus on the productivity effect from comparative advantage, we concentrate on the adoption of foreign technology.

South Africa represents an interesting case study of openness with changing relationship to the world market related to sanctions and trade policy reform. We calibrate a reference path that reproduces the broad economic development in South Africa during 1960–2005. Due to international sanctions against the Apartheid regime and a complex system of import quotas the degree of protectionism cannot be measured directly. Based on the model we offer an openness index by calibrating export and import taxes that reproduces the actual trade and growth path during the past decades. The effects of openness are analyzed by gradual elimination of the rise in the tariff equivalent. It should be noticed that the tariff equivalent captures both trade reform and changes in external trade conditions (sanctions). This counterfactual experiment raises the trade share by about 25% points and leads to an increase in the 2005 end of period GDP by 29%. The robustness of the result is investigated and the GDP-effect is in the range of 25–32% within standard parameterization. The quantitative effect is consistent with econometric studies. The cross-country analysis of Frankel and Romer (1999) finds that an increase in the trade share of 1% point raises the income level by 2%. By comparison a 1% point higher trade share leads to 1.2% higher GDP in our model.

A more open economy implies higher degree of technological catch-up, and given the productivity mechanism assumed the 2005 productivity level relative to the world technology frontier increases from 42% to 48%. Separating the effects of openness between investment and productivity we find that about 60% of the increase in GDP is due to increased productivity (including the induced capital accumulation effect). International technology spillovers feeding productivity are important to raise investment and growth. By decomposing the growth channels we find that the openness effect on long-run GDP is divided between about 30% for both the direct and the indirect productivity channel and 40% for the direct investment channel. Robustness tests show how the quantitative results depend on parameter values, in particular trade and productivity elasticities. The broad conclusion holds over a wide range of parameter values. The results add to and are
broadly consistent with the existing empirical studies of the importance of trade openness for productivity growth in South Africa. Aghion et al. (2008a) investigate the economic mechanisms involved in the relationship between trade and growth based on nominal tariffs, effective protection rates, and export taxes. Harding and Rattsø (2010) address the endogeneity problem of trade policy and use other regions’ tariff development as part of the WTO process as instruments for the tariff reductions since 1988. They find that tariffs have been important for labor productivity and confirm the importance of the world technology frontier.

The extended Ramsey model with endogenous technology gap dynamics is developed in Section 2 with analysis of the consequences of trade policy. Section 3 calibrates a reference growth path and an openness index that reproduces the trade and growth observed in South Africa during 1960–2005. Furthermore, we simulate the growth effects of trade barriers, and clarify the quantitative importance of the productivity and investment channels. The robustness of the results is checked using alternative parameter values. Concluding remarks are offered in Section 4.

2. Growth model with technology gap and technology adoption

The dynamic consequences of trade policy have been analyzed in various open economy Ramsey models. Trade policy has been analyzed as a price effect of changing tariffs. Sen and Turnovsky (1989) assume separate domestic and foreign goods and the change of tariffs induces substitution in consumption. Their main contribution is to clarify the different effects of anticipated versus unanticipated and permanent versus temporary reform. Osang and Turnovsky (2000) study a small open economy and separate between consumption and investment tariffs. Their main finding is that investment tariff has more adverse effect on growth than consumption tariff. Osang and Pereira (1996) analyze trade policy in a small open economy with real and human capital accumulation. The basic dynamics of a two-capital model with exogenous productivity has been solved analytically by Duczynski (2002), also separating between physical and human capital.

We extend the basic framework to include endogenous productivity dynamics related to the world technology frontier and the composition of the capital stock. We separate between foreign and domestic capital based on foreign and domestic investment goods. The starting point is a model concentrating on capital accumulation and exogenous productivity growth and assuming installation cost of investment. The productivity dynamics added is explained in Section 2.1. Productivity growth is determined by technology adoption and depends on the investment decision of the firm, in particular the share of foreign capital goods in the total capital stock. The analysis addresses the dynamic consequences of changing trade barriers in the form of import tariffs.

2.1. Production technology, productivity dynamics and the firm’s investment decisions

Gross output \((X_t)\) is defined as a Cobb–Douglas function of effective labor \((A_t L_t)\), foreign capital \((K_{F,t})\) and domestic capital \((K_{D,t})\):

\[
X_t = (A_t L_t)^{1-\beta} K_{F,t}^\beta K_{D,t}^{1-\beta}
\]

(1)

The labor force \((L_t)\) grows exogenously at rate \(n\). Productivity growth \((\dot{A}_t)\) is driven by technology adoption and is related to the gap to the world technology frontier. In the simplest form, the role of the technology gap can be introduced as

\[
\dot{A}_t = \lambda \left(1 - \frac{A_t}{A_{F,t}}\right)
\]

(2)

The domestic and the frontier level of productivity are given by \(A_t\) and \(A_{F,t}\), respectively, and \(A_t/A_{F,t}\) is relative productivity. The technology gap is measured as the productivity distance to the world technology frontier. The linear (rather than exponential) relationship between productivity growth and relative productivity implies logistic technology diffusion, as suggested by Benhabib and Spiegel (2005). The growth rate of productivity is determined by the gap and the speed of adjustment parameter \(\lambda\). The frontier productivity level is assumed to grow at an exogenous rate.

We expand this model by relating technology adoption to the investment decision of the firm, in particular to the composition of the capital stock. The speed of adjustment parameter \(\lambda\) is assumed to be a function of the share of foreign capital goods in the total capital stock, \(K_{F,t}/K_t\), where \(K_t = K_{D,t} + K_{F,t}\) is the total capital stock. The formulation is consistent with Findlay (1978) emphasizing the importance of technological contagion to benefit from the technology gap. He relates technology transfer to the degree of foreign direct investment in the domestic economy. Similarly, Dawid et al. (2010) relate absorptive capacity and technology transfer to the presence of foreign capital in the domestic economy. The main difference in our suggested specification is that the absorptive capacity is affected by the investment decision of the domestic firm, and is not exogenously given from abroad. The productivity growth rate is specified as

\[
\dot{A}_t = \lambda(K_{F,t}/K_t) \left(1 - \frac{A_t}{A_{F,t}}\right)
\]

(3)

where \(\lambda(K_{F,t}/K_t) = \lambda_0(K_{F,t}/K_t)^\theta\). The elasticity of productivity growth with respect to the foreign capital share is given by \(\theta\), while \(\lambda_0\) is a positive parameter.
The representative firm makes its investment decisions according to intertemporal profit maximization, subject to the accumulation of the capital stocks over time:

\[
\text{Max}_{\lambda, \phi_{D}, \phi_{F}, \delta_{F}, \delta_{D}} \int_{0}^{\infty} e^{-rt}(P_{V,T}X_{t} - w_{I}L_{t} - I_{F,t}(P_{F,t} + \phi_{F,t} - I_{D,t}(P_{D,t} + \phi_{D,t})))dt
\]

(4)

\[
\text{s.t. } \dot{K}_{j,t} = I_{j,t} - \delta_{j}K_{j,t}, \quad j = F, D
\]

(5)

The technology adoption function enters the optimization problem of the firm via gross output \((X_{t})\) and it follows that the investment decision separating between foreign and domestic capital goods takes into account the productivity effect of the investment. The exogenous world market interest rate is given by \(r\), \(w_{I}\) is the wage rate, \(I_{F,t}\) and \(I_{D,t}\) are investments in foreign and domestic capital goods, \(\phi_{F,t}\) and \(\phi_{D,t}\) are unit investment adjustment costs, \(\delta_{F}\) and \(\delta_{D}\) are the rates of depreciation on the foreign and domestic capital stocks, and \(P_{D,t}\) is the price of domestic goods. The value added price \((P_{V,t})\) is defined as \(P_{V,t} = P_{F,t}(1 - ta) - P_{N,t} IO\), where \(P_{F,t}\) is the producer price, \(ta\) is the sales tax rate, \(P_{N,t}\) is the composite price of intermediate goods, and \(IO\) is the fixed input–output coefficient. Gross domestic product \((GDP_{t})\) is thus given as \(GDP_{t} = P_{V,t}X_{t}\). The price of foreign goods \((P_{F,t})\) equals the exogenous world market price of import goods \((P_{WM,t})\) adjusted by import tariffs \((tm_{t})\):

\[
P_{F,t} = P_{WM,t}(1 + tm_{t})
\]

(6)

which captures the basic price effect of trade policy.

Following Hayashi (1982) and Abel and Blanchard (1983), unit adjustment costs depend positively on the size of investment relative to the capital stock:

\[
\phi_{j,t} = P_{D,t} \frac{b_{j} I_{j,t}}{K_{j,t}}, \quad j = F, D
\]

(7)

where \(b_{F}\) and \(b_{D}\) are positive parameters. The specification implies that total adjustment costs are an increasing and convex function of the investment level. To avoid that trade policy affects adjustment costs we assume that the costs only consume domestic investment goods.

The first order conditions from the profit maximization follow as

\[
(1 - \alpha - \beta)P_{V,T}X_{t} = w_{I}L_{t}
\]

(8)

\[
q_{j,t} = P_{j,t} + P_{D,t} \frac{b_{j} I_{j,t}}{K_{j,t}}, \quad j = F, D
\]

(9)

\[
rq_{j,t} = Rk_{j,t} + P_{D,t} \frac{b_{j} I_{j,t}}{K_{j,t}} - \delta_{j}q_{j,t} + \dot{q}_{j,t}, \quad j = F, D
\]

(10)

The equality between the wage rate and the marginal product of labor is given in Eq. (8). Eq. (9) says that for each type of capital the investor equilibrates the marginal cost of investment, which is given on the right hand side, and the shadow price of capital \((q_{j,t} \text{ and } \dot{q}_{j,t})\). The shadow price of foreign and domestic capital exceeds the respective price of investment goods due to investment adjustment costs. Eq. (10) is the no-arbitrage condition and states that the marginal return to each type of capital must equal the interest payments on a perfectly substitutable asset with a value equal to the respective shadow price. \(Rk_{j,t}\) and \(Rk_{D,t}\) are the capital rental rates (which equal the marginal products of foreign and domestic capital), while the second term on the right hand side is the partial derivative of total adjustment costs with respect to capital. The marginal return to each capital stock must be adjusted by the depreciation rate and by the capital gain or loss \((q_{j,F,t} \text{ and } \dot{q}_{j,D,t})\).

2.2. The firm’s export decision

We model imperfect substitution between sales to the domestic market and the world market through a constant elasticity of transformation (CET) function. Aggregate output follows from the production function in Eq. (1), while the composition of exports \((E_{t})\) and domestic sales \((D_{t})\) is derived from maximizing current sales income subject to the CET function:

\[
\text{Max } P_{D,t}D_{t} + P_{WE,t}(1 - te_{t})E_{t}
\]

(11)

\[
\text{s.t. } X_{t} = a_{x}[m_{x}E_{t}^{(1+\sigma_{x})/\alpha_{x}} + (1-m_{x})D_{t}^{(1+\sigma_{x})/\alpha_{x}}]^{\sigma_{x}/(1+\sigma_{x})}
\]

(12)

where \(\sigma_{x}\) is the constant elasticity of substitution between domestic and foreign markets, \(a_{x}\) is a shift parameter and \(m_{x}\) is the share parameter for exports. The producer price \((P_{x,t})\) is a composite of the exogenous world market price of export goods \((P_{WE,t})\) adjusted by export taxes \((te_{t})\) and the endogenous domestic price \((P_{D,t})\).
2.3. The household’s consumption/savings decision

The representative household receives income through the primary factors, while interest payments on its foreign debt are subtracted. There is no independent government sector, and public tax revenues (sales and trade taxes) are transferred to the household in the form of a lump sum. The household is forward-looking and maximizes an intertemporal utility function taking into account the lifetime budget constraint:

\[
\text{Max } \int_0^\infty U(C_t)e^{-rt}dt
\]

s.t. \( \int_0^\infty P_{C,t}C_t e^{-rt}dt = \int_0^\infty (Y_t - S_t)e^{-rt}dt \)

(13) (14)

Assuming intertemporal elasticity of substitution equal to unity, the iso-elastic utility function is defined as \( U(C_t) = \ln C_t \), where \( C_t \) is consumption in period \( t \). \( Y_t \) is household income, \( S_t \) is private savings, \( P_{C,t} \) is the endogenous price of consumption goods, and \( \rho \) is the positive rate of time preference. The utility maximization gives the Euler equation for optimal allocation of consumption over time:

\[
\frac{\dot{C}_t}{C_t} = r - \rho \frac{P_{C,t}}{P_{C,t}}
\]

(15)

Consumption growth depends on the interest rate, the time preference rate, and the price path.

2.4. Imports decisions

We model imperfect substitution between domestic and foreign consumption and intermediate goods through constant elasticity of substitution functions (Armington functions). The model consequently operates with two composite goods (a consumption good and an intermediate good), and imports and domestic demand are endogenously determined.

Total consumption demand follows from the Euler equation, while the allocation between consumption imports \( (C_{D,t}) \) and domestic consumption goods \( (C_{D,t}) \) is derived from minimizing current expenditure subject to the Armington function for consumption goods:

\[
\text{Min } P_{F,t}C_{F,t} + P_{D,t}C_{D,t}
\]

s.t. \( C_t = a_C[m_C e^{-\sigma_C \sigma_C^{-1}}/\sigma_C + (1 - m_C)e^{-\sigma_C \sigma_C^{-1}}/\sigma_C]^{1/\sigma_C} \)

where \( \sigma_C \) is the constant elasticity of substitution between domestic and foreign consumption goods, \( a_C \) is a shift parameter and \( m_C \) is the share parameter for the foreign consumption good. The price level facing domestic consumers \( (P_{C,t}) \) is a composite of the exogenous world market price of import goods adjusted by import tariffs \( (P_{F,t}) \) and the endogenous domestic price \( (P_{D,t}) \).

Similar, intermediate goods are employed according to a fixed input–output coefficient, while the composition of intermediate imports \( (N_{F,t}) \) and domestic intermediate goods \( (N_{D,t}) \) follows from minimizing current expenditure subject to the Armington function for intermediate goods:

\[
\text{Min } P_{F,t}N_{F,t} + P_{D,t}N_{D,t}
\]

s.t. \( N_t = a_N[m_N N_{F,t}^{(\sigma_N n - 1)/\sigma_N} + (1 - m_N)N_{D,t}^{(\sigma_N n - 1)/\sigma_N}]^{1/\sigma_N} \)

(16) (17) (18) (19)

Total intermediate demand is given by \( N_t \). \( \sigma_N \) is the constant elasticity of substitution between domestic and foreign intermediate goods, \( a_N \) is a shift parameter and \( m_N \) is the share parameter for the foreign intermediate good.

Total imports include the demand for foreign capital goods and total domestic demand includes domestic investment demand.

2.5. Model solution and transition dynamics

In the long run equilibrium foreign and domestic capital per effective worker and the shadow price of both types of capital are constant. Productivity growth equals the world frontier rate, and the technology gap is constant. Output and capital stocks grow at the exogenous long run rate equal to \( g + n \), where \( g \) is the long-run rate of labor augmenting technical progress and \( n \) is the labor supply growth rate. The model includes a long-run restriction on foreign debt.

Based on the capital accumulation constraints in Eq. (5) the dynamics of foreign and domestic capital per effective worker \( (k_{F,t} \text{ and } k_{D,t} \text{, respectively}) \) is given as

\[
\dot{k}_{j,t} = i_j - (\delta_j + \dot{n} + \dot{\lambda}_{k})k_{j,t}, \quad j = F, D
\]

(20)
From Eq. (9) foreign and domestic investment per effective worker \((i_{F,t} \text{ and } i_{D,t}, \text{ respectively})\) equals:

\[
i_{j,t} = \frac{q_{j,t} - P_{j,t}}{P_{D,t}b_j} k_{j,t}, \quad j = F, D
\]  

(21)

Combining Eqs. (20) and (21) gives us the dynamics of foreign and domestic capital per effective worker as a function of the respective capital shadow price and the productivity growth rate:

\[
k_{j,t} = \left( \frac{q_{j,t} - P_{j,t}}{P_{D,t}b_j} - (\delta_j + n + \hat{\Lambda}_t) \right) k_{j,t}, \quad j = F, D
\]  

(22)

For each type of capital, the long-run stability condition of \(\dot{k}_{j,t} = 0\) implies a horizontal curve in the \(q_j - k_j\) diagram, which is constant as long as commodity prices and productivity growth are constant:

\[
q_{j,t} = P_{j,t} + P_{D,t}b_j(\delta_j + n + \hat{\Lambda}_t), \quad j = F, D
\]  

(23)

From the no-arbitrage conditions in Eq. (10) and the expressions for investment per effective worker in Eq. (21), the dynamics of the foreign and domestic capital shadow price follows as

\[
\dot{q}_{j,t} = (r + \delta_j)q_{j,t} - Rk_{j,t} - \frac{(q_{j,t} - P_{j,t})^2}{2b_j P_{D,t}}, \quad j = F, D
\]  

(24)

The long-run stability condition of \(\dot{q}_{j,t} = 0\) implies the following relationships between each capital shadow price and the two types of capital per effective worker:

\[
(q_{F,t} - P_{F,t})^2 - 2b_F P_{D,t} (r + \delta_F) q_{F,t} + 2b_F P_{D,t} P_{V,t} \alpha k_{F,t}^{-\alpha - 1} k_{D,t} = 0
\]  

(25)

\[
(q_{D,t} - P_{D,t})^2 - 2b_D P_{D,t} (r + \delta_D) q_{D,t} + 2b_D P_{D,t} P_{V,t} \beta k_{F,t}^{\beta - 1} k_{D,t} = 0
\]  

(26)

where the foreign and domestic capital rental rate \((Rk_{F,t} \text{ and } Rk_{D,t}\)) are substituted with the respective marginal product of capital. Holding commodity prices and domestic capital per effective worker constant, Eq. (25) gives a decreasing curve in the \(q_F - k_F\) diagram. Similar, for fixed values of \(P_{D,t}, P_{V,t} \text{ and } k_{F,t}\), Eq. (26) represents a decreasing curve in the \(q_D - k_D\) diagram (documented in Appendix A).

With exogenous productivity growth equal to \(g\), Eqs. (22) and (24) form a two-dimensional system of differential equations in capital per effective worker and the shadow price of capital (Tobin’s \(q\)) for each type of capital. The phase diagram for foreign capital is drawn in Fig. 1. The model has a long run equilibrium foreign capital per effective worker equal to \(k_F^*\). Given an initial low capital per effective worker \(k_{F,0}\), and with a shadow price of capital \(q_{F,0}\) above total investment cost, the economy follows the saddle path (drawn with arrows) towards the equilibrium \(k_F^*\). The transition growth path is driven by capital accumulation. High investment initially declines towards the equilibrium. The equilibrium capital per effective worker implies that the capital stock grows at the rate \(g + n\), the productivity growth plus the population growth. Investment covers this capital accumulation in addition to replacement investment. The dynamics of domestic capital is similar.

![Fig. 1. Standard open economy Ramsey model: Dynamics of foreign capital per effective worker and foreign capital shadow price, and transitional and long-run effects of technology gap dynamics.](image-url)
As seen from Eqs. (24)–(26), the dynamics of each capital shadow price depends on both foreign and domestic capital per effective worker, which implies that the phase diagrams for foreign and domestic capital affect each other. Increasing domestic capital per effective worker during transition generates an outward shift to a steeper \( q_f = 0 \) curve, and similar, higher foreign capital per effective worker gives an outward shift to a steeper \( q_D = 0 \) curve. But the shifts become gradually smaller, and eventually fade out. The positive feedback effect from domestic capital is included in the \( q_f = 0 \) curve in Fig. 1. In the long run equilibrium both foreign and domestic capital per effective worker are constant.\(^1\)

The basic dynamics of the technology gap formulation similar to (2) has been solved analytically by Duczynski (2003) in a Ramsey growth model with one capital good. The solution is made simple by the dynamic adjustment rule for productivity that is independent of the rest of the model. Compared to the Duczynski model with technology gap we have added the endogenous effect of the foreign capital share for the ability to take advantage of the world frontier. The extension is important for the effect of trade policy (as described in Section 2.6), but does not change the transition growth path much in the case of stable capital shares over time. The productivity growth is primarily determined by the technology gap, and productivity growth and investment are declining from an initial gap towards the long run equilibrium. The convergence is faster than in the standard open economy model with exogenous productivity because of the interaction between high and declining productivity growth and investment.

In our setup, Eqs. (3), (22) and (24) form a three-dimensional system of differential equations for each type of capital. The growth process depends on initial conditions, in particular the gap to the world frontier. We illustrate how the technology gap dynamics affects foreign capital per effective worker and its shadow price in the phase diagram of Fig. 1. The implications for domestic capital are similar. Since long run productivity growth equals the exogenous productivity growth rate (and the world frontier rate), the equilibrium \( k_F^e \) is not affected. The initial gap generates a high productivity transition growth that declines towards the long run rate. The consequences for the transition growth can be described by an upward shift in the \( k_F = 0 \) curve resulting from a positive shift in the growth rate of productivity given the starting point \( k_{F0} \) (see Eq. (23)). The size of the shift in the growth rate of \( A \) depends on the size of the gap, as seen from Eq. (3). The dynamics of the productivity growth is imposed on the diagram, and the declining productivity growth towards the frontier rate generates gradual shifts down in the \( k_F = 0 \) curve back to the original curve with productivity growth \( g \). It follows that the economy will move along a shifting saddle path towards the old equilibrium. The speed of convergence is determined by the interaction of productivity and investment. While the long run capital per effective worker is the same, the income and capital per worker are higher since the level of productivity is higher. Compared to the standard transition growth higher productivity growth stimulates the investment profitability.

2.6. The effects of lower import tariffs

Trade liberalization is analyzed as an anticipated permanent reform that applies to both consumption, intermediate, and investment goods. Lower import tariffs decrease the price of foreign goods \( P_{fi} \), and important for the growth effect, the cost of foreign investment goods is reduced. As seen from Section 2.5, lower investment costs stimulate investment activity. We add a possible channel of effects of tariff reform through productivity growth. A more open economy changes the composition of the capital stock in favor of foreign capital and stimulates technology adoption.

The full effect of trade liberalization is a combination of investment and productivity responses and they strengthen each other.\(^2\) The reduction in the price of foreign investment goods generates a downward shift in the \( k_F = 0 \) curve and an outward shift to a steeper \( q_f = 0 \) curve (the calculations are given in Appendix A). The shifts, shown as dotted lines in Fig. 2, determine the new and lower long run cost of foreign investment \( q_f^e \) and the higher long run equilibrium of foreign capital per effective worker \( k_F^e \). The increase in foreign capital per effective worker stimulates the profitability of domestic capital, and gives an outward shift to a steeper \( q_D = 0 \) curve. The higher level of domestic capital per effective worker has positive feedback effects on foreign capital (already included in the outward shift in the \( q_f = 0 \) curve in Fig. 2), but the positive interaction gradually diminishes and we reach a long-run equilibrium with constant foreign and domestic capital per effective worker at new higher levels. Trade liberalization has positive long run level effects, but also changes the composition of the aggregate capital stock in favor of foreign capital, which stimulates productivity growth.

The implication for productivity growth can be described based on Eq. (3). Trade liberalization increases the foreign capital share, and productivity growth is higher given the technology gap. The new equilibrium gap implies that the long run distance to the frontier is smaller, although the long run growth rate remains unchanged. Trade liberalization generates higher transitional growth towards a reduced gap to the world frontier. The dynamics is consistent with the common understanding that differences in income and productivity levels are permanent, while differences in growth rates are transitory (Acemoglu and Ventura, 2002).

The temporary higher productivity growth rate gives temporary upward shifts in the \( k_j = 0 \) curves \((j=F, D)\) that gradually shifts back down as productivity growth returns to the frontier rate (similar to that described in Fig. 1). This

\(^1\) The steady state solutions are given in Appendix A.

\(^2\) For now, we ignore general equilibrium price effects. For instance, lower import tariffs affect the domestic price through the Armington composite system, with further consequences for the long run values of the capital rental rates, the capital shadow prices and the foreign and domestic capital per effective worker. These effects are taken into account in the numerical simulations in Section 3.
implies that the new long run levels of foreign and domestic capital per effective worker (the former illustrated as $k_F^{**}$ in Fig. 2) are not affected by the productivity dynamics, but since the level of productivity is higher so are the levels of income per worker and foreign and domestic capital per worker. The endogenous interaction between productivity and investment generates prolonged transition growth towards the higher long run levels of foreign and domestic capital per effective worker. But the road to get to the new equilibrium is complicated and has not been fully worked out analytically. As for the foreign capital dynamics in Fig. 2, the new saddle path (not drawn) may be above or below the saddle path without trade liberalization (to $k_F^*$). Trade liberalization and increased trade openness in this situation prolongs transition growth because of more technology adoption, cheaper foreign investment goods and productivity induced capital accumulation.

In this section we have considered the effects of lower import tariffs. Trade liberalization can also follow from lower export taxes. The implications for the transition dynamics and long run equilibrium are straightforward given the analysis of import tariffs above. The cost of investment goods is not affected by lower export goods, and thus the producer price and the value added price ($P_{VL}$) increases. This generates outward shifts to steeper $q_j = 0$ curves, and the long run equilibrium has higher foreign and domestic capital per effective worker. If lower export taxes affect the share of foreign capital in the total capital stock, we get additional productivity effects that prolong the transition growth and lead to higher income and capital per worker in the long run equilibrium. But these effects are likely to be small.

We offer an attempt at quantifying the importance of investment and productivity and their interaction in the growth process. In the next section we apply the model to the case of South Africa to identify the relative importance of these channels. The model reproduction of South African growth during the past decades is of transitional character, and thus endogenous.

3. Quantification of trade openness effects in South Africa

The recent economic history of South Africa allows an investigation and quantification of the dynamic adjustments and we have calibrated the model to reproduce the economic development in South Africa during 1960–2005. The economic growth was promising post WWII and the country was named ‘the Japan of Africa’. This growth period has been understood as catching up growth based on openness and industrial diversification, but ended in the 1970s and was turned into a long period of stagnation. *Pritchett (2000)* describes South Africa as a ’mountain’, where per capita growth above 1.5% per year is turned into negative numbers. Economic growth has been on the policy agenda in South Africa with the government’s Accelerated and Shared Growth Initiative (ASGI-SA). The policy program primarily discusses domestic binding constraints on growth. The government has invited a group of experts to do a growth diagnostic, and input to this process has been produced by *Aghion et al. (2008b)*, *Edwards and Lawrence (2008)*, *Hausmann and Klinger (2008)*, and *Rodrik (2008)*, among others. The background growth diagnostic approach is outlined by *Hausmann, Rodrik and Velasco (2008)*. Our analysis concentrates on the links to the rest of the world as constraints for growth in a general equilibrium model.

The starting point for the quantitative evaluation of the adjustment mechanisms following change in trade openness is a reference growth path and a measure of trade restrictions. The model is calibrated to reproduce the broad economic
development in South Africa during 1960–2005. The effects of trade openness can be studied in a counterfactual analysis of reduced tariff equivalent and thereby offer a quantification of the growth effect of trade barriers. The robustness of the results is investigated by using alternative parameter values.

3.1. Calibrating the growth path and the tariff equivalent for South Africa

The parameters are set based on a 1998 Social Accounting Matrix, as well as available econometric estimates and stylized facts. The parameters are made consistent with long run equilibrium, where the growth rate is assumed to equal 2% (1.3% technological progress rate and 0.7% labor growth). Long run technical progress follows the growth rate of the world technology frontier. To reproduce actual GDP growth, the initial levels of productivity, foreign capital and domestic capital are scaled down compared to the steady state path. The scaling back serves as an exogenous shock that takes the economy outside the equilibrium long run path in 1960, and transitional economic growth is driven by endogenous adjustment back to equilibrium growth.

The key parameter determining the role of trade openness is the elasticity of productivity growth with respect to the share of foreign capital in the total capital stock [given by the parameter \( \theta \) in Eq. (3)]. This is set equal to 3 and implies that an increase in the foreign capital share of 1% point gives about 0.15% point higher productivity growth rate when starting from the assumed steady state rate. Caselli and Wilson (2004) and Xu and Wang (1999) show the importance of foreign capital goods, but do not offer a basis for quantification. Griffith et al. (2003) find that increased foreign presence within UK industries stimulates technological catch-up. They measure foreign presence by the share of employment in foreign owned plants, and show that 1% point increase in the employment share generates about 0.1% point higher productivity growth. Even though the development in the share of the labor force in foreign owned plants might differ from the development in the foreign capital share it gives an indication of a reasonable size of the effect. Keller and Yeaple (2009) perform a similar analysis using US data between 1987 and 1996, and find that 1% point increase in the share of employment in foreign affiliates gives 0.4–0.9% point higher productivity growth. Compared to this estimate, the elasticity of productivity growth with respect to the foreign capital share applied in our model can be seen as conservative.

The foreign trade conditions have been changing over time in South Africa, in particular related to the sanctions and protectionism from the mid-1970s to the early 1990s. The general equilibrium model allows for measurement of the various internal and external trade restrictions. We calibrate export and import taxes necessary to reproduce the observed export and import paths during 1960–2005. The development of terms of trade and real effective exchange rate are calibrated consistent with data to adjust for the impact of world price shocks on the trade level. Total trade taxes as share of trade represents our measure of openness, as illustrated in Fig. 3.

While the tariff equivalent decreases during the 1960s, the slow growth of exports and imports in the 1970s and 1980s requires a gradual increase of the tariff-equivalent with a peak in the late 1980s of about 55%. After 1990 the removal of sanctions together with gradual liberalization of the trade policy increased trade rapidly, reflected in the model by decreasing tariffs. The underlying paths of the export tax and the import tax are documented in Appendix B. Interestingly, the calibrated tariff paths are consistent with tariffs calculated from partial analyses of exports and imports with reasonable values of elasticities.

Existing measures of openness in South Africa are scarce. A recent contribution by Edwards and Lawrence (2008) offers data on tariffs and surcharges since 1960. Since our calibrated tariff equivalent takes external trade restrictions (sanctions) into account it lies at a higher level than the tariffs reported by Edwards and Lawrence. But the development path (illustrated in their Fig. 3) with liberalization in the 1960s, increasing protectionism since the mid–1970s, peak in 1990, and liberalization since 1990, is consistent with our calibrated measure of openness. Aron and Muellbauer (2002) develop an openness indicator for South Africa based on econometric estimation. Their model includes a measure of tariffs and surcharges, while the unobservable effect of sanctions and quotas are captured by a non-linear stochastic trend. The indicator illustrates the changing degree of openness during 1970–2000 with increasing protectionism in the 1970s, sanctions and protectionism in the 1980s and trade liberalization after 1990. Compared to the analysis by Aron and Muellbauer our openness indicator takes into account that both imports and exports are held back by sanctions, covers a longer time period, and gives a more intuitive measure of openness (export and import tax as share of total trade).

Fig. 4 shows how we track the actual growth rate as a steady decline in the model growth rate during 1961–1990, followed by more stable growth since 1990. The South African growth experience can be understood as neoclassical convergence, trade openness affecting technology transfer through the share of foreign capital in the total capital stock, and endogenous interaction between investment and technology adoption. While the initial high growth is driven by investment and profitability, sanctions and protectionism increase the cost of foreign capital and the associated change in the composition of the capital stock gives a drop in productivity growth, with further consequences for overall investment.

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3 The model is solved numerically in discrete time using the software GAMS.
4 Detailed documentation of the model, the calibration and the 1998 South African Social Accounting Matrix is given in Appendix B.
5 The assumption of 0.7% labor growth is consistent with data on average annual employment growth in South Africa during 1971–2005 (Quantec Research, 2007).
6 The calculation is based on foreign capital shares in the range 0.25–0.3, which is consistent with the values in the model simulations.
profitability. Post Apartheid, the elimination of sanctions and trade liberalization stimulate economic growth through less expensive foreign capital goods and more technology adoption.

3.2. Quantification of the investment and productivity responses to openness

As explained in Section 3.1, we have calibrated a tariff equivalent growing from the late 1960s and with a peak in the late 1980s to reproduce the actual trade and growth path. Eliminating the rise in the tariff-equivalent during the period of sanctions and protectionism, we can simulate the economic development in a more open economy. In the experiment, the tariff equivalent level decreases gradually, as illustrated in Fig. 3. The average tariff rate during 1960–2005 equals 16%, down from 38% along the reference path reproducing the actual growth in South Africa.

The new GDP growth path is shown in Fig. 5. Given the investment and productivity links to openness assumed, the analysis shows that South Africa could have avoided some of the decline in the growth rate. The sanctions and protectionism have contributed to more costly foreign investment goods and less technology adoption and consequently held back economic growth. In the counterfactual open economy scenario the average foreign capital share during 1980–2005 is 3% points higher than along the reference path reproducing the actual growth in South Africa.

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The新GDP增长路径图示于图5。考虑到投资和生产力与开放度之间的联系被假设,分析显示南非可以避免一些增长率的下降。制裁和保护主义导致了更昂贵的外国投资商品和更少的技术采纳,从而阻碍了经济增长。在开放经济的反事实情景下,平均外资比重在1980–2005年期间提高了3%。与沿参考路径模拟的实际增长在南非相比。

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GDP by 29%, implying that 1% point higher trade share leads to about 1.2% higher GDP level in our model. By comparison, the cross-country analysis of Frankel and Romer (1999) finds that an increase in the trade share by 1% point raises the income level by 2%. Romalis (2007) finds that 10% point increase in the trade share raises the GDP per capita growth rate by 0.2–0.5% point in a set of developing countries. The general equilibrium effects included here do not change the size of the effect. Our numbers imply that 10% point higher trade share translates into about 0.25% point higher GDP per capita growth rate during transition.

The model shows how the timing and expectation of trade policy can generate a complicated dynamic pattern of response. In our setting, future trade liberalization is expected and influences current investment and production decisions. Gradual trade liberalization gives an immediate drop in the trade share and in investment rates compared to the reference path. Higher expected productivity with a more open economy increases the expected profitability of future investments and current investments in foreign and domestic capital are postponed. The effect on initial foreign capital investment is strengthened since investors will take advantage of cheaper imported investment goods in the future. Over time the profitability of capital accumulation increases and the investment rates are higher in the open economy scenario. Gradual trade liberalization has a similar effect on foreign trade. The first decade the trade share drops by about 3% points compared to the reference path, mainly driven by lower export share. When cheaper foreign goods and lower export taxes are expected in the future, current trade is held back. Over time the trade share increases, and is about 25% points higher than along the calibrated South Africa path.

Our main interest is a clarification of the vehicles from openness to growth, the endogenous adjustment of productivity and investment. To separate different channels of effects we run counterfactual experiments with exogenous productivity growth and compare the quantitative effects of reduced tariffs to the results with endogenous productivity growth.

The effect of trade liberalization on capital investments works via several channels. First, lower import tariffs imply less expensive foreign capital goods, which stimulates foreign capital accumulation and affects the composition of the capital stock. Second, higher foreign capital share generates more technology adoption and consequently higher level of productivity, which in turn increases the overall investment profitability. Third, lower export taxes increases the producer price and stimulates investment activity through increased production capacity. Fourth, there are positive interactions between foreign and domestic capital investments. When the productivity effect is excluded, the increase in the 2005 domestic real investment level of 43% is reduced to 26%. This implies that the productivity channel accounts for about 40% of the trade liberalization effect on investment in domestic capital goods. This is the induced capital accumulation effect highlighted by Hulten (2001). About 30% of the response to trade liberalization on investment in foreign capital goods is due to higher productivity level. The productivity channel is relatively more important for domestic capital investments, since the price effect (the first channel above) only applies to foreign capital investments.

The effect of trade liberalization on long-run GDP works via three channels. First, lower import tariffs imply less expensive foreign capital goods and lower export taxes give higher production, both of which generate more capital accumulation (the direct investment effect). Second, increased foreign capital share stimulates adoption of foreign technology, and productivity growth increases (the direct productivity effect). Third, trade liberalization increases GDP growth indirectly through the endogenous interplay between productivity and investment profitability. The increase in the 2005 level of real GDP is down from 29% to 12% with exogenous productivity growth. This implies that almost 60% of the increase in real GDP comes from higher productivity, working either directly or indirectly in interaction with investment profitability. To separate the direct productivity effect from the induced capital accumulation effect, we calculate the long-run GDP effect of trade liberalization without the interaction effect.8 We find that the openness effect on growth is divided between about 30% for both the direct and the indirect productivity channel and 40% for the direct investment channel.

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8 This calculation is based on the foreign and domestic capital path from the exogenous productivity scenario and the productivity path implied by the foreign capital share from the exogenous productivity scenario.
3.3. Robustness tests

The quantitative results reported above obviously depend on parameter values, in particular trade and productivity elasticities. The trade elasticities represent substitution possibilities between domestic and foreign consumption and intermediate goods ($\sigma_C$ and $\sigma_N$, respectively) and between sales to domestic markets versus export markets ($\sigma_x$). In the base-run simulations we set $\sigma_C=\sigma_N=3$ and $\sigma_x=2$, which is consistent with national and international estimates (Hertel et al., 2007; Senhadji and Montenegro, 1999). As documented in Section 3.1, we assume an elasticity of productivity growth with respect to the foreign capital share of 3, in line with available econometric estimates. Below we investigate how the quantitative effects of trade barriers depend on these parameter values.

A low elasticity of substitution implies that it is hard to substitute between domestic and foreign consumption/intermediate goods, as well as between domestic and foreign markets. Trade is therefore kept relatively high also along the reference path with an increasing tariff equivalent. The lower the elasticity of substitution, the smaller the quantitative effects of reducing trade barriers on the trade share. With trade elasticities equal to 1.5 the trade share is 18% points higher in the more open economy, compared to 25% points in the base run scenario. With high elasticity of substitution (equal to 4.5 for consumption and intermediate imports and 2.5 for exports) trade is reduced more when the tariff equivalent increases, and the trade share is 30% points higher in the open economy scenario. Since foreign and domestic investment goods are separate and not affected by the degree of substitution in the Armington functions, the quantitative effects on real investment, productivity catch-up and GDP do not differ much across different trade elasticities.

During international isolation the composition of the capital stock changes in favor of domestic capital and productivity growth is held back. The higher the elasticity of productivity growth with respect to the foreign capital share ($\theta$), the larger is the negative effect of isolation on productivity growth and the lower is the degree of catch-up. Hence, the quantitative effects of trade barriers increase with the elasticity of productivity growth with respect to the foreign capital share. With low ($\theta=2$) and high ($\theta=4$) elasticity the increase in the 2005 real GDP level due to a more open economy is 25% and 32%, respectively, compared to 29% in the base run scenario.

Independent of the values of trade and productivity elasticities the relationship between the trade share and GDP is quite robust. The GDP effect of an increase in the trade share of 1% point is in the range 0.9–1.7% (compared to 1.2% with the preferred values of elasticities). The decomposition of the effects of trade liberalization is fairly stable across different parametrizations. The importance of the productivity channel for the investment response to lower tariffs remains around 30% for investments in foreign capital goods and lies in the range 35–44% for domestic investments (compared to 40% with the preferred values of elasticities). Higher productivity (working either directly or indirectly as an induced capital accumulation effect) contributes to 50–62% of the increase in GDP. The endogenous interaction between productivity and investment profitability (the third channel of growth) accounts for about 30% of the long-run GDP effect of trade liberalization in all scenarios.

4. Concluding remarks

Trade openness influences investment and productivity and thereby the growth path of the economy. We study a model where trade barriers affect the technology adoption dependent on the gap to the world technology frontier. Technology gap productivity growth implies constant long run productivity growth equal to the frontier and is well suited to study transition dynamics. The standard open economy Ramsey model is extended to include the technology gap, and we allow technology adoption to be affected by trade openness. The inclusion of the technology gap implies a high and declining growth rate of productivity starting with a large gap to the world frontier. Compared to the standard transition path higher productivity growth stimulates investment and leads to higher long term equilibrium income and capital per worker. The higher productivity growth adds to the transition growth directly and indirectly by raising the capital accumulation.

The growth effects of tariff liberalization are channeled through investments and the choice between domestic and foreign capital goods. Tariffs influence relative prices including the cost of investment and lead to substitution effects towards foreign goods. The share of foreign capital goods encourages technology adoption and productivity growth given the gap to the technology frontier. Productivity and investment effects interact and strengthen the overall consequences for economic growth. Trade liberalization stimulates transition growth because of cheaper investment goods, more technology adoption, and productivity induced capital accumulation. Permanent anticipated trade liberalization leads to lower long run cost of foreign investment and higher long run productivity level. The steady state path has higher income and capital per worker.

We offer a quantification of the importance of investment and productivity and their interaction in the growth process. The model is implemented for South Africa to make lessons from the large shifts in openness observed. We produce a quantification of how investment and productivity interact and respond to change of openness based on calibration. Due to international sanctions against the Apartheid regime and a complex system of import quotas the degree of protectionism cannot be measured directly. Based on the model we offer an openness index by calibrating a tariff equivalent that reproduces the actual trade path during 1960–2005.

The growth model allows a counterfactual analysis of the role of international trade. The effects of openness are analyzed by gradual elimination of the rise in the tariff equivalent. This counterfactual experiment raises the trade share by about 25% points and leads to an increase in the 2005 end of period GDP by 29%. The robustness of the result is investigated and the GDP-effect is in the range of 25–32% within standard parameterization. The implied relationship...
between trade share and GDP is consistent with recent econometric studies. Given the productivity mechanism assumed, a more open economy reduces the cost of technology adoption and contributes to higher degree of technological catch-up. Separating the effects of openness between investment and productivity we find that about 60% of the increase in GDP is due to increased productivity (including the induced capital accumulation effect). By decomposing the growth channels we find that the openness effect on long-run GDP is divided between 40% directly via investment, 30% directly via productivity and 30% indirectly via the productivity effect on investment profitability. Robustness tests show how the quantitative results depend on parameter values, in particular trade and productivity elasticities. The broad conclusion holds over a wide range of parameter values.

The quantitative results of the analysis reflect the growth potential assuming well-functioning domestic markets taking advantage of international spillovers. South African growth under new rule has been reluctant and there is widespread suggestions without implications from Alice Amsden, Rob Davies, Xinshen Diao, Lawrence Edwards, Johannes Fedderke, International Finance of the Federal Reserve System in Washington, DC and at the University of Surrey, and comments and Angeles, the IT-FA conference in Las Vegas, the ETSG conference in Lausanne, and at staff seminars at the Division of International Finance of the Federal Reserve System in Washington, DC and at the University of Surrey, and comments and suggestions without implications from Alice Amsden, Rob Davies, Xinshen Diao, Lawrence Edwards, Johannes Fedderke, Stephen Gelb, David Greenaway, Torfinn Harding, Ravi Kanbur, Neil McCulloch, Oliver Morrissey, Sherman Robinson, Terry Roe, Francis Teal, Peter van der Windt, Dirk van Seventer, Adrian Wood, an associate editor and the editor. The project is financed by the Norwegian Research Council.

Appendix A. Documentation of phase diagrams in Figs. 1 and 2

For each type of capital the three-dimensional system of differential equations is given as

\[
\dot{k}_{jt} = \left( \frac{q_{jt} - P_{jt}}{P_{jt}b_j} - (\delta_j + n + \hat{A}_t) \right) k_{jt}, \quad j = F, D
\]

(A.1)

\[
\dot{q}_{jt} = (r + \delta_j)q_{jt} - R k_{jt} - \frac{(q_{jt} - P_{jt})^2}{2b_j P_{jt}}, \quad j = F, D
\]

(A.2)

\[
\dot{\hat{A}}_t = \lambda (K_{F,t}/K_t) \left( 1 - \frac{\hat{A}_t}{\hat{A}_{F,t}} \right)
\]

(A.3)

The long run stability conditions give the following curves in the \( q_j - k_j \) diagram:

\[
\dot{k}_j = 0 \Rightarrow q_{jt} = P_{jt} + P_{D,t} b_j (\delta_j + n + \hat{A}_t)
\]

(A.4)

\[
\dot{q}_F = 0 \Rightarrow (q_{F,t} - P_{F,t})^2 - 2b_F P_{D,t}(r + \delta_F)q_{F,t} + 2b_F P_{D,t} P_{V,t} 2k_{F,t}^{\beta_F - 1} k_{D,t}^{\beta_F} = 0
\]

(A.5)

\[
\dot{q}_D = 0 \Rightarrow (q_{D,t} - P_{D,t})^2 - 2b_D P_{D,t}(r + \delta_D)q_{D,t} + 2b_D P_{D,t} P_{V,t} \beta_F k_{F,t}^{\beta_F} k_{D,t}^{\beta_F - 1} = 0
\]

(A.6)

A.1. The steady state solutions

In the long run equilibrium foreign and domestic capital per effective worker, the shadow price of foreign and domestic capital and relative productivity are all constant. Productivity growth equals the long-run growth rate of the world technology frontier, \( g \), and the steady state value of relative productivity follows from Eq. (A.3):

\[
\left( \frac{A^*}{\hat{A}_t} \right)^* = 1 - \frac{g}{\lambda (K_F/K)}
\]

(A.7)

The steady state values of the capital shadow prices \( (q^*_j) \) follow directly from Eq. (A.4)\(^9\):

\[
q^*_j = P_j + P_{D,t} b_j (\delta_j + n + g), \quad j = F, D
\]

(A.8)

\(^9\) In the long-run steady state commodity prices and the value added price are all constant.
Combining Eqs. (A.2) and (A.8) gives the long run values of the foreign and domestic capital rental rates (\(Rk^*_F\)):

\[
Rk^*_F = (r - \hat{\delta}_j)P_F + P_Db_j(\hat{\delta}_j + n + g) \left( r + \hat{\delta}_j - \frac{\hat{\delta}_j + n + g}{2} \right) \quad j = F,D
\]  
(A.9)

Using that each capital rental rate equals the respective marginal product of capital, \(Rk_{jt} = PV_{t}^jX_t^j\partial_k^j\), the steady state values of foreign and domestic capital per effective worker follow as

\[
k^*_F = \left( \frac{PV_F^{1-\beta_1} \beta_1^\beta_1}{(Rk^*_F)^{1-\beta_1}(Rk^*_D)^{\beta_1}} \right)^{1/(1-\beta_1)}
\]  
(A.10)

\[
k^*_D = \left( \frac{PV_D^{1-\beta_2} \beta_2^\beta_2}{(Rk^*_F)^{1-\beta_2}(Rk^*_D)^{\beta_2}} \right)^{1/(1-\beta_2)}
\]  
(A.11)

Finally, the steady state value of output per effective worker equals:

\[
x^* = (k^*_F)^{\beta_1}(k^*_D)^{\beta_2} = \left( \frac{PV_F^{1-\beta_1} \beta_1^\beta_1}{(Rk^*_F)^{1-\beta_1}(Rk^*_D)^{\beta_1}} \right)^{1/(1-\beta_1)}
\]  
(A.12)

A.2. The slope of the \(q_j = 0\) curves

The \(q_j = 0\) curves (\(j = F, D\)) are downward sloping. This can be verified through implicit derivation of Eqs. (A.5) and (A.6) with respect to \(k_{F,t}\) and \(k_{D,t}\), respectively. Illustrated for the \(q_F = 0\) curve

\[
2(q_{F,t} - P_{F,t}) \frac{dq_{F,t}}{dk_{F,t}} - 2b_FPD_{D,t}(r + \hat{\delta}_F) \frac{dq_{F,t}}{dk_{F,t}} + (\alpha - 1)2b_FPD_{F,t}PV_{t}^Fk_F^{1-2}k_D = 0 \Rightarrow \frac{dq_{F,t}}{dk_{F,t}} = \frac{(1-\alpha)2b_FPD_{F,t}PV_{t}^F\beta k_F^{2-2}k_D^{\beta}}{q_{F,t} - P_{F,t} - PD_{D,t}b_j(\hat{\delta}_j + n + \hat{\Delta})} < 0
\]  
(A.13)

From Eq. (A.8) we know that in equilibrium \(q_{F,t} = P_{F,t} + PD_{D,t}b_j(\hat{\delta}_j + n + \hat{\Delta})\), and since \(r > n + g\) (follows from the Euler equation with positive rate of time preference) the denominator is negative, which gives the negative slope of the \(q_F = 0\) curve. Downward sloping \(q_D = 0\) curve follows from similar calculations.

A.3. Movements of \(k_{jt}\) and \(q_{jt}\) outside of equilibrium

Eq. (A.1) implies that \(\hat{k}_j > 0\) (\(j = F, D\)) when \(q_{jt} > P_{jt} + PD_{jt}b_j(\hat{\delta}_j + n + \hat{\Delta})\) and \(\hat{k}_j < 0\) when \(q_{jt} < P_{jt} + PD_{jt}b_j(\hat{\delta}_j + n + \hat{\Delta})\). By inserting for the capital rental rate in Eq. (A.2) it follows that \((\partial q_j/\partial k_j) > 0\) (\(j = F, D\)), implying that the shadow price of capital decreases to the left of the \(q_j = 0\) curve, while it increases to the right of the curve. These movements of \(k_{jt}\) and \(q_{jt}\) are illustrated by arrows in Figs. 1 and 2 of the paper.

A.4. Implications of lower \(P_{F,t}\) for the \(k_j = 0\) and the \(q_j = 0\) curves

As seen from Eq. (A.4), a reduction in the price of foreign investment goods generates a downward shift in the \(k_F = 0\) curve. The implications for the \(q_F = 0\) curve involve an outward shift to a steeper curve, as documented below. The \(k_D = 0\) and \(q_D = 0\) curves are not directly affected by the reduction in \(P_{F,t}\). From Eq. (A.13) it follows that \((\partial q_{F,t}/\partial k_{F,t}) > 0\).

Lower \(P_{F,t}\) leads to a reduction in the negative slope of the curve, in other words, the curve becomes steeper. To verify that the \(q = 0\) curves shift outward for lower import tariffs, we calculate the effect of lower \(P_{F,t}\) on the value of foreign capital per effective worker along the curve for a given value of \(q_{F,t}\). By setting \(q_{F,t} = 1\) in Eq. (A.5), the corresponding value of \(k_{F,t}\) is given by

\[
(1 - P_{F,t})^2 - 2b_FPD_{D,t}(r + \hat{\delta}_F) + 2b_FPD_{F,t}PV_{t}^Fk_F^{1-1}k_D = 0
\]

Solving for \(k_{F,t}\) gives

\[
k_{F,t} = \left( \frac{2b_FPD_{D,t}PV_{t}^Fk_D^{\beta}}{2b_FPD_{F,t}(r + \hat{\delta}_F) - (1 - P_{F,t})} \right)^{1/(1-\beta)}
\]

The value of foreign capital per effective worker is positive for all possible values of \(P_{F,t}\). The denominator can be written as \(2(P_{F,t} + PD_{D,t}(r + \hat{\delta}_F)) - P_{F,t}^2 - 1\), and since \(P_{F,t} < q_{F,t} = 1\) due to adjustment costs and \(P_{F,t} + PD_{D,t}(r + \hat{\delta}_F) = q_{F,t} = 1\) [same reasoning as for Eq. (A.13)], the denominator and \(k_{F,t}\) are always positive. It follows from the expression for \(k_{F,t}\), above that \((\partial k_{F,t}/\partial P_{F,t}) < 0\). This implies that for a given value of \(q_{F,t}\), lower \(P_{F,t}\) generates a higher value of \(k_{F,t}\), in other words, the \(q = 0\) curve shifts outwards.
Appendix B. Supplementary material

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.jedc.2012.02.005.

References