Efficiency Variation Among the Norwegian High Schools: Consequences of Equalization Policy

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Abstract — The performance of the regulated school system in Norway is investigated by analyzing the relationship between resource use and student achievement in 34 Norwegian high schools. The marginal school effect on student achievement is estimated, and the output of the schools is described by the number of graduates and the school effect. Using this separation between quantity and quality, a reference frontier representing best practice among the schools is established by Data Envelopment Analysis, and the technical efficiencies are measured. The schools have very different student achievements, but the variation is not related to differences in resource use. The school system is oriented towards the equalization of student results, but the schools show systematic differences in the handling of high and low achievers.

1. INTRODUCTION

The Norwegian school system is highly centralized both in terms of educational content and financing. Equal opportunities and standardization of the teaching are key characteristics. The present study addresses the resource use of Norwegian high schools. It is of interest to investigate if the schools under heavy regulation produce the equalization they are supposed to, and if the student performance is linked to the resource use in the schools.

The point of departure of this study of the efficiency variation between schools is the education production function approach. The high school is treated as a production unit transforming resources to student achievements. Four important characteristics of the transformation are captured. First, students are like raw materials entering the production to be processed, and the production is measured by the value added. The variation in the quality of the input and the output is represented by the evaluation of the students made through the examination marks. Second, the school production is measured both by the quantity (the number of graduates) and the quality (the average value added per graduate). Third, the student performance in each school is related to the quantity of teacher input. Fourth, the results of the high and low achievers from the junior high school are investigated separately. Different schools may follow different strategies regarding elitism and equalization.

The study describes the efficiency variation between the high schools and investigates the determinants of the performance. A reference frontier representing the best practice observed is established. A flexible methodology is chosen that allows for multiple outputs. The frontier is estimated non-parametrically (Data Envelopment Analysis, DEA) by enveloping the data. The production output is measured by the value added as recommended by Hanushek and Taylor (1990), and decomposes the value added for different student categories (low achievers and high achievers).

2. THE INSTITUTIONAL BACKGROUND

American studies dominate the literature on
school efficiency. This is certainly true for the use of education production functions since Coleman et al. (1966). Hanushek (1986) offers an authoritative overview of the methodology and the results obtained. This study represents an opportunity of comparing the results of the American studies to a different institutional setting. The public high-school system in the welfare state of Norway.

The decision making process determining the resource use in Norwegian schools represents complicated interactions between the national government, the county government and each school. The system is described in Granheim et al. (1990). The high schools (also called higher secondary schools), engaging mainly youth between 16 and 19 years of age, are the responsibility of the county governments, 19 in all. They run about 300 high schools for a total population of about 4.2 million. Since Norway has a unitary system, the national government controls the school activities by law and guidelines. The counties, and consequently the schools, are basically funded by national grants with no local discretion. The Norwegian school system is highly centralized both in terms of educational content and economic controls.

The counties are responsible for operating the schools and the construction of new schools. A very decentralized pattern of schools has developed with many small schools of less than 100 students in general subjects. The decentralized structure reflects the settlement pattern of the country with the population scattered in fjords and mountain valleys. The grant system basically has covered the costs involved, and the counties have had few incentives to exploit economies of scale.

The overall objective of the educational policy has been to provide equal opportunities for everyone independent of the social background and domicile. A decentralized pattern of school locations has been the main instrument to achieve this. In addition, the traditional gymnasium or grammar school has been merged with the more practically oriented schools at the upper secondary level. Most of the new schools are comprehensive high schools, integrating general and vocational subjects. This integration is made to reduce the differences in social status between schools of theoretical and occupational orientations. The teacher unions have fully supported the equalization strategy.

The Norwegian society is characterized by low mobility. The choice of school is basically determined by the location of residence. In fact, the applicants are geographically assigned in such a way as to avoid that some schools are underenrolled. Only students living in large towns (with several schools) are allowed to choose what high-school to attend. Self selection obviously is less important in this system.

Variation in school efficiency and student performance has not been an issue in the public debate. The regulated system is assumed to produce the same student results everywhere. There has been no evaluation of the school results. The initiation of this study was very controversial, and the presentation of the results has provoked strong responses from teacher unions and politicians who defend the existing system. The smaller schools located away from urban centers and the integration of academic and vocational subjects have been challenged, as will become clear below. The two aspects are essential in the regional policy and the equalization policy in the educational system. Also the educational production function approach has been seen to threaten the broader goals of the schools.

3. THE APPROACH

The American approach motivating this study has sought to understand the use of resources at the school level as the basis of reform. The studies oriented towards economic factors have compared schools as production units. Some of this literature has applied the methodology of traditional productivity analysis, frontier production functions and the identification of the productivity of each unit. The seminal article by Farrell (1957) used a non-parametric envelopment of data assuming constant returns to scale. Boles (1966), followed by Charnes, Cooper and Rhodes (1978), improved the Farrell approach by focusing on one facet of the frontier at the time instead of calculating the whole unit isoquant. Bessent et al. (1980, 1982), Charnes, Cooper and Rhodes (1981), Sengupta and Sfeir (1986), Jesson et al. (1987), Färe et al. (1988, 1989) and Ray (1991) have made use of this way of specifying the education production function.

The method identifies the best practice technology observed in the specific sample of schools. The efficiency of each school is measured relative to the best practice reference frontier. Two measures of
4. THE EDUCATION PRODUCTION FUNCTION

A simple starting point to describe the production process in schools is the relationship between the labor input of teachers and the number of students passing the final exams. The efficiency of the teacher input and the effect of scale (school size) can be investigated. The students can be said to satisfy certain minimum requirements, but no quality differences are included.

The production of knowledge is more completely described by introducing some measurement of student achievement. It is assumed that examination marks represent an acceptable measure of quality differences. The weighted average of the marks of the main subjects taught measures the level of knowledge of a student when leaving high school. The gross output of a school may be characterized by the sum of the weighted averages of all the graduates. Such an aggregation assumes that an ordinal scale can be used as interval. The distance between two marks following each other on the scale is the same everywhere.

The initial level of knowledge when the students enter the high school must be taken into account to measure the contribution of each school to the accumulation of knowledge. The junior high schools use a different measurement scale from the high schools, that is also used as interval. Hanushek and Taylor (1990) suggest a method of estimating the marginal effect of each school on student achievement taking into account data for each student. The level of knowledge of student $i$ when finishing high school, $A_{i,T}$, reflects the level of knowledge she brings with her from junior high school, $A_{i,T-1}$, and the value added at the high school — consisting of a school specific component for school $s$, $Q_s$, and a stochastic component $\epsilon_i$. The model estimated is:

$$A_{i,T} = \delta A_{i,T-1} + \sum_{s=1}^{S} H_s Q_s + \epsilon_i$$

where

- $A_{i,T}$ — the achievement of student $i$ at high school
- $A_{i,T-1}$ — the achievement of student $i$ at junior high school
- $H_s$ — school dummy variable, $H_s = 1$ if student is at school $s$, $s = 1, \ldots, S$
- $\epsilon_i$ — stochastic error term satisfying standard assumptions
δ, Qₙ — parameters to be estimated.

The formulation captures the previous history of the students by the variable Aᵢₜ₋₁. Unlike in the Hanushek–Taylor analysis, family background characteristics are not included due to lack of data. The estimated value added is biased upwards if family background variables are positively correlated with the value added. Hanushek and Taylor (1990) investigate the consequences of leaving out socioeconomic characteristics of students. They find that the resulting increase in the marginal bias of the school quality estimates is small. Serious mis-specification problems associated with missing data are not expected. The results of the estimation are reported in section 7.

The approach offers two output measures of school s, the number of graduates at the school, Xₙ, and the average value added estimated as Qₙ. The total value added of each school can be described by Qₙ*Xₙ. This output measure represents a value judgement of quantity versus quality, since a specific tradeoff between quantity and quality is assumed.

The schools produce multiple outputs also by treating students with different capabilities differently. The schools are assumed to have access to two types of students, low achievers and high achievers. This classification of the students is based on the scores from the junior high school. The allocation of resources to the two types of students may vary between the schools, and their performance may be influenced by differences between the schools regarding the organizing of the teaching (the quantity of home work, the frequency of tests, etc.). Final examination marks which favor the high achievers may result from the priority assigned to the elite. On the other hand, the priority for low achievers may generate equality. A third possibility is the existence of jointness in production so that both types of students benefit from the school priorities.

The production process is formulated as a two-output system consuming a common resource input, as formalized by Chizmar and Zak (1983). The analysis of joint production is similar to the study of public goods. Public goods are defined by non-rivalry in consumption, that is, the access to the good is not reduced when new consumers are added. Teaching in the classroom is similar to a collective good. The learning of each student is not necessarily reduced when new students enter the class. As suggested above, teaching also may include characteristics of private goods. But since the allocation of resources to different students in the classroom cannot be observed (except at high costs), modelling the consumption of a common resource input is considered the only approach available.

5. MODELLING SCHOOL PRODUCTION

We begin with a simple benchmark model relating the number of graduates of the high school, Xₙ, and the number of teacher manyears, Lₙ. The strength of this formulation is that it is a technical relationship with no indexing problems of measurement. The differences between the schools are the result of school size or technical efficiency:

Model 1: Xₙ = Γ₁(Lₙ)

The model serves as a basis of comparison when student achievements are taken into account. Model 2 is an extension of 1 by including the average value added, Qₙ, as a separate output. The formulation is general since no a priori assumption is made regarding the tradeoff between the number of graduates and the average value added:

Model 2: (Xₙ, Qₙ) = F₂(Lₙ)

The total value added of each school can intuitively be described by the product Qₙ*Xₙ. As mentioned in section 4, this output measure assumes a value judgement between the number of graduates and the average value added. Two schools have the same total value added if 10% lower average value added in one of them is compensated by 10% more graduates. Model 3 analyzes the total value added measured in this way, and the results are compared with the more general model 2 to evaluate the tradeoff between quality and quantity:

Model 3: Qₙ*Xₙ = F₃(Lₙ)

Based on the models 1–3, investigating the basics of technical efficiency, an extension is offered to analyze the priorities of the schools regarding student with different backgrounds. A hypothesis regarding the choice of teaching strategy between elitism and equality is tested. Formally this is handled as multiple output production by describing the total value added separately for low achievers.
Efficiency variation among high schools

Q_{s}^{L}X_{s}^{L}, and high achievers, Q_{s}^{H}X_{s}^{H}, in the junior high school:

Model 4: (Q_{s}^{L}X_{s}^{L}, Q_{s}^{H}X_{s}^{H}) = F_{s}(L_{s})

The model is consistent with an assumption of joint production. Compared to model 3, some of the efficiency differences are associated with different teaching strategies.\textsuperscript{5}

6. EFFICIENCY MEASUREMENT

The technical efficiency of a school reflects the potential for increasing the school output without increasing the use of resources (output efficiency) or the potential for reducing the use of resources without reducing the school output (input efficiency). The analysis makes use of the output efficiency definition. The calculation of the efficiency measures for each school makes use of Data Envelopment Analysis (DEA). The method also allows the measurement of scale efficiency, the distance to optimal scale.

Technical and scale efficiency are shown in Figure 1, describing the relationship between one input and one output. The observed schools A, B and C represent the best practice and they define the reference frontier. Production at the frontier implies technical efficiency, but not necessarily scale efficiency. School A is technically efficient, but scale inefficient. School K does not satisfy any of the efficiency requirements. The efficiency measures under variable returns to scale introduced by Forsund and Hjalmarsson (1979) define the technical efficiency of school K as \( E_{2} = NK/NV \). Technical efficiency under constant returns to scale is \( E_{3} = NK/NM \) and scale efficiency is \( E_{5} = E_{5}/E_{2} = NL/NM \). The intuition of the technical efficiency measure \( E_{2} \) can be easily understood: It indicates the possible increase in output when the best practice technology is applied given the resource input.

The point L serves as an efficient reference for school K. L is a linear combination of the efficient schools. The weights of the efficient schools in the linear combination are determined as a maximizing problem: The ratio NL/NK (the inverse of the technical efficiency measure \( E_{2} \)) is maximized for every school, given three conditions:

\[
\text{Max } 1/E_{2s},
\]

\[
\left(\frac{1}{E_{2s}}\right) Q_{ks} \leq \sum_{j=1}^{S} \gamma_{kj}Q_{kj}, \quad k = 1, \ldots, K
\]

\[
L_{ks} \geq \sum_{j=1}^{S} \gamma_{kj}L_{kj}, \quad l = 1, \ldots, P
\]

\[
\sum_{j=1}^{S} \gamma_{kj} = 1
\]

\[
\lambda_{j} \geq 0, \quad j = 1, \ldots, S
\]

where

\( Q_{ks} \) — value added achievement for group k in school s

\( L_{1s} \) — input type 1 in school s

\( \lambda_{ij} \) — reference weights for school s

S — number of schools

K — number of achievement groups

P — number of inputs.

The first condition says that the inverse of \( E_{2} \) multiplied by the observed output must be less or equal to a weighted average of the outputs of the other schools. Observed output is adjusted upwards to be equal to or less than the output of the reference point at the best practice frontier. Second, the observed use of resources must be greater than or equal to a weighted average of the inputs of the other schools. The two restrictions require the output of the reference unit to be at least as large as the observed unit, given the same use of resources or less.

![Figure 1. Technical efficiency and scale efficiency.](image-url)
The third restriction of the maximization problem sets the sum of weights equal to 1. In this way, variable returns to scale are allowed. The intuition is that the units included in the reference point must be similar to the observed school. Färe et al. (1989) apply this specification. When the third restriction is not assumed, the reference frontier reflects constant returns to scale, as in Bessent et al. (1982).

7. THE 34 HIGH SCHOOLS

The data cover all high schools in two counties, Sør-Trøndelag and Nordland, offering training in general subjects. The study is limited to the training in general as opposed to vocational subjects at the high school level. The size of the schools vary from 12 to 187 in terms of the number of graduates. The distribution of school sizes is shown in Table 1. The total number of general subject students per school is approximately the number of graduates multiplied by 3 (years).

The Sør-Trøndelag county includes the 5 largest and the 4 smallest schools, reflecting the regional center Trondheim (150,000 inhabitants) and the sparsely populated areas in the mountains and at the coast. The Nordland county (a total of 250,000 inhabitants) has most of the population concentrated to townships of 5000–15,000 inhabitants, and most of the high schools are medium-sized.

The schools may be separated in two groups according to age. The largest schools specializing in general subjects are old and include traditional gymnasia or grammar schools. The schools established during the last two decades concentrate on vocational subjects, and general subjects are only a small part of their activities.

The level of knowledge at the final high school exams is measured by the marks given in the four central subjects: Norwegian language and literature, mathematics, social studies and English language and literature. The student achievements at each school are documented in Table 2. The average marks given at each school vary from 2.39 to 3.60, averaging 3.05 (standard deviation is 0.24). The scale is 0–6 and only one digit marks are given. Six is the best result, while 0 and 1 represent failure to pass the exam. The differences in results between the schools must be considered serious.

The level of knowledge at the end of junior high school is documented by a score. The marks given to the exams of the junior high school are based on a different system of evaluation. The average points of the students of each high school achieved at the junior high school vary from the lowest 40.1 to the best 44.8, averaging 42.6 (standard deviation of 0.96).

The average value added per graduate is shown in Table 2 and the estimation is documented in Appendix Table 1, based on the model outlined in Section 4. The estimated coefficients of each school come out with a negative sign and are all highly significant. They can be interpreted as the marginal reduction of the average value added at each school given the effect of junior high school results on average. Grosskopf et al. (1991) transform similar negative estimates to positive values by adding a fixed positive value for all units. To have a more direct interpretation related to the average marks of the high school, we have changed the signs of the coefficients and turned them around their average.

Both methods keep the absolute differences between the schools as estimated. We have tested whether the school effects are different from one another, and a hypothesis that all coefficients are equal is rejected (F-statistics $F^* = 3.30$, $F_{0.05} = 1.46$ with $v_1 = 32$ and $v_2 = 2578$ degrees of freedom).

The same procedure has been applied when the value added for low and high achievers are estimated separately, as reported in the Tables. A Chow test was conducted to test for whether or not pooling high and low achievers in the value added equations is appropriate. A null hypothesis that there is no difference in the coefficients (i.e. school effects) obtained from the two samples is rejected. ($F^* = 2.13$, $F_{0.05} = 1.40$ with $v_1 = 35$ and $v_2 = 2543$ degrees of freedom.) Similar to the overall estimation, we also reject a hypothesis that the coefficients in the equations that estimate school effects separately for high and low achievers are equal. (High achievers: $F^* = 2.94$, $F_{0.05} = 1.46$ with $v_1 = 32$ and $v_2 = 1327$ degrees of freedom.)
Efficiency variation among high schools

Table 2. School description

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<th>Marks junior school</th>
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<th>Value added high sch.</th>
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<th>Teachers per 100 students</th>
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Low achievers: $F^* = 2.22$, $F_{0.05} = 1.46$ with $v_1 = 32$ and $v_2 = 1216$ degrees of freedom.

Observing the average marks of high schools with their junior high school starting point in Table 2, one can conclude that the marks achieved at the junior high school do not predict the performance at the high school level very well. It is of interest to notice the different ranking of schools by marks at final exams and the school effect. The rank correlation is 0.65. The raw material must be taken into account when the efficiency of the schools is evaluated, as suggested by Hanushek and Taylor (1990).

The teacher input associated with the graduates is measured by the number of manyears in the general subjects section of each high school multiplied by the share of total graduates (about 1/4). In Table 2, the number of manyears per 100 students is presented, since it is easier to interpret. The variation of the teacher input per graduate basically reflects the size of the schools. The small schools have the smallest classes due to the limited population basis of recruitment of students in the periphery.

8. RESULTS: TECHNICAL EFFICIENCY AND QUANTITY VERSUS QUALITY

We concentrate on the results regarding technical efficiency and alternative output measures. School size (measured by teacher manyears) is shown to be
an important variable. More research is needed to determine the scale efficiency of the schools properly. Scale effects in the school production are related to both class size and school size. Consequently, data at the class level are necessary to determine scale efficiency.

The benchmark model 1 directs attention to the relationship between the teacher input and the number of students. All models assume variable returns to scale. The results are reported in Figure 2 and Appendix Table 2, together with the documentation of a robustness test, jackknifing. Jackknifing is an iterative technique which yields a distribution of efficiency measures for each observation by dropping one school at a time and recalculating the efficiency scores in order to account for outliers. In the table, average efficiency values and standard deviation are based on 34 estimations of the model, each with 33 schools included. The standard deviations are small, and overall the results seem robust with respect to outliers defining the efficiency frontier.

According to Figure 2 and Appendix Table 2, 5 schools are efficient and located at the best practice frontier. School size and efficiency are clearly positively correlated. Schools with efficiency (E₂) less than 0.75 are small, implying less than 10 teacher manyears and 120 students (40 graduates). They can increase the number of graduates by more than 25% given their teacher input if they operate according to best practice. Schools with technical efficiency in the range 0.95–1.00 are medium sized or large (more than 20 teacher manyears and 300 students). Class size is a background variable. Less teacher input per student is needed in large classes, and small schools have smaller classes. Differences among the large schools are expected to reflect the variety of specialization that they offer. Many varieties imply smaller classes at least for some subjects.

When a quality description of the production of knowledge is included, the schools can be characterized by two product measures. In model 2, no a priori assumption is made regarding the choice between quantity and quality, the number of graduates and the average value added. The analysis shows the priorities of the schools regarding the two products, and marginal rates of transformation can be derived. Appendix Table 3 reveals the ranking of technical efficiency when the value added is taken into account. The conventional wisdom says that large teacher input per graduate produces high average value added. But the results do not confirm such a pattern. Many small schools with large teacher input per graduate keep a low rank or even have their position worsened compared to model 1, because they have low average value added. (Schools 10, 17, 18, 21, 27, 32 and 33 serve as examples.) These small schools have about the same teacher input per graduate as their efficient reference units, but they have fewer graduates and lower average value added.

Many of the medium sized and large schools with low teacher input per graduate keep or improve their high ranking relative to model 1, because they have high average value added. (Schools 3 and 16 are the best examples.) The rank correlation between models 1 and 2 is 0.88, confirming that the inclusion of student achievement does not change the ranking of efficiency much. However, some exceptions from the general pattern can be identified. Some schools with large teacher input per graduate have high average value added (such as schools 4, 8 and 9), and they improve their ranking in model 2 compared to model 1.

Model 2 allows the calculation of marginal rates of transformation between the two products, the number of graduates and the average value added. The calculations reflect the production technology given that the schools are technically efficient. Schools 3 and 16 represent one large and one medium sized efficient school. They have marginal rates of transformation of 0.045 and 0.262 respectively. The implied elasticities were calculated, and we notice that a 1% increase in the average value added requires 0.6% reduction in the number of
graduates in the large school and 0.3% reduction in the medium sized school.

The desired marginal rate of transformation between quantity and quality represents a value judgment. Some may argue in favor of many students with a given minimum value added, where value added in excess of the minimum is not valued much. On the other hand, an elitist orientation emphasizes the merits of having few students with the very best results. The valuation of quantity versus quality is important for school policy priorities. This study offers no normative conclusions on the issue. 

Model 3 represents a restrictive, but intuitive, measure of the total production of each school, the number of graduates multiplied by the average value added. The marginal rate of transformation is restricted to 1. Two schools are assumed to have the same total production if the one with half as many students as the other has double the average value added per graduate.

The balancing of quantity versus quality assumed in this formulation means that schools with marginal rates of transformation which vary greatly from 1 are evaluated as inefficient. Except for a few schools, the ranking of the schools is very similar between models 2 and 3, as shown by comparing Appendix Tables 3 and 4. The rank correlation is 0.90. Different priority of quantity versus quality is not sufficiently important to influence the efficiency evaluation of the schools. The fact that several model formulations are estimated with very similar ranking of the schools confirms the robustness of our results as shown by the jackknifing.

9. THE ROLE OF EQUALIZATION

The high priority of student equalization in Norway has led us to investigate the differences between high and low achievers in model 4. All the graduates are classified in two groups based on their scores at junior high school, above and below the average. The value added separated by high and low achievers, presented in Table 2, shows systematic differences between the two groups and between the schools.

The schools can be classified in four groups:

1. Schools producing above average value added for both high and low achievers
2. Elitist schools producing high value added for high achievers, but not for low achievers
3. Equality oriented schools producing high value added for low achievers, but not for high achievers
4. Schools generating below average value added for both groups.

Focusing on school size, we note that the small comprehensive high schools dominate groups 3 and 4. These schools have low overall value added per graduate, attributable largely to high achievers showing weak performance. Referring to Table 2, schools 23 and 25 are typical among the equality oriented and schools 33 and 34 are among the worst (group 4). Most of the largest schools are in the elitist group 2, with large overall value added per graduate (note schools 3, 4 and 7). Medium sized schools are the majority of the best, group 1 (school 16, for instance).

The efficiency results of the model investigating low and high achievers are reported in Appendix Table 5, together with a documentation of the jackknifing. The relationship between school size, efficiency and degree of equalization can be illustrated by representative examples from the DEA-analysis. An investigation of reference schools of different sizes shows that the best practice reflects either high value added overall (small and medium sized schools) or elitist orientation (large schools).

Consequently, the authorities can have some influence on the performance of different types of students by deciding the size of the schools. Deller and Rudnicki (1993) find that “more size efficient schools may be detrimental to the learning process”. According to our results, this is not true, since low achievers are losers in the large schools.

Our discussion of the DEA-results concentrates on the inefficient schools (in groups 3 and 4), to identify the problems of the equalization policy. A close examination of schools 23 (group 3) and 18 (group 4) offers an understanding of the factors involved.

School 23 is medium-sized and equality oriented with 24 high achievers and 23 low achievers graduating. The DEA-method compares the school to a reference unit, constructed by a combination (weights) of schools at the frontier. The weights enable us to identify the reference unit. The reference school is a combination of schools 1, 2 and 16 weighted by 0.12, 0.05 and 0.83, respectively. They are all shown in Table 3. The technical efficiency $E_z = 0.69$ implies that the production can be increased by 31% to the frontier without any increase in input.
The reference school is a medium sized school which produces high value added for both types of students, while school 23 is oriented towards equality and the value added for high achievers is low. There is a potential for efficiency improvement along two lines, quantity and quality. The number of students can be raised, from 23 to 32 low achievers and from 24 to 31 high achievers as shown in Table 3. Or the average value added of the students can be raised, from 3.14 to 3.35 for low achievers and from 4.39 to 4.82 for high achievers. The improvement of the average value added represents 1 standard deviation for the low achievers and 1.6 standard deviations for the high achievers.

School 23 illustrates a hypothesis put forward by Brown and Saks (1987), that excess resources are allocated to low achievers. This school has teacher input per graduate well above the average, as shown in Table 2, and it is truly equality oriented. However, in this case, the extra teacher input is also associated with inefficiency. The strategy can be a costly way of reaching the low achievers.

School 18 produces low value added for both types of students and is part of group 4. It is representative of small schools with a high teacher-student ratio and many high achievers. The efficiency aspects are described in Table 4. The reference school is a combination of schools 2, 12 and 16 (with weights 0.41, 0.47 and 0.12 respectively). Both the reference school and school 18 have about 70% high achievers. The production of the reference school is typical of the best of the small, a high value added for low achievers (0.84 standard deviations above the mean) and a relatively low value added for high achievers (0.33 standard deviations above the mean).

School 18 has a potential for efficiency improvement of 30% \((E_2 = 0.7028)\), primarily related to the low achievers (2.5 standard deviations from the reference school). Likewise, many inefficient small schools with many high achievers are producing bad results for low achievers. Small schools with a large share of high achievers represent a kind of a mismatch. They have too many students that they cannot handle, and all students suffer.

Summers and Wolfe (1977) conclude that low achievers are doing better in small classes with a large share of high achievers. School 23, analyzed

### Table 3. School 23 and reference schools

<table>
<thead>
<tr>
<th>No.</th>
<th>Weight</th>
<th>(X_L^H)</th>
<th>(X_L^L)</th>
<th>(Q_H^H)</th>
<th>(Q_L^L)</th>
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<td>4.63</td>
<td>3.46</td>
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<tr>
<td>16</td>
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<td>3.27</td>
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<td>RS</td>
<td></td>
<td>31</td>
<td>37</td>
<td>4.87</td>
<td>3.35</td>
</tr>
</tbody>
</table>

\(E_{2,23} = 0.6902\)

\(X_L^H\) — number of high achieving students.
\(X_L^L\) — number of low achieving students.
\(Q_H^H\) — value added per high achieving student.
\(Q_L^L\) — value added per low achieving student.
RS — reference school constructed.
\(E_{2,23}\) — technical efficiency for school 23.

### Table 4. School 18 and reference schools

<table>
<thead>
<tr>
<th>No.</th>
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<th>(X_L^L)</th>
<th>(Q_H^H)</th>
<th>(Q_L^L)</th>
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<td>12</td>
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<td>137</td>
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<td>4.63</td>
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<tr>
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<tr>
<td>RS</td>
<td></td>
<td>90</td>
<td>37</td>
<td>4.65</td>
<td>3.31</td>
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</table>

\(E_{2,18} = 0.7028\)
Efficiency variation among high schools

above, fits this pattern, with a high ratio of teachers per graduate and orientation towards equality. However, school 18 is in conflict with the suggestion of Summers and Wolfe, since low achievers may do poorly when the school is not capable of stimulating the high achievers.

The two examples of inefficient schools enable us to identify winners and losers of the educational policy emphasizing decentralization (small schools) and high teacher input per student. All the schools in groups 3 and 4 are a result of this policy, since they have been established during the last two decades. The high achievers are definitely the losers. Low achievers have been winners in some of the schools with large teacher input. When many high achievers are recruited to these small schools, low achievers are also losers.

10. CONCLUDING REMARKS

Significant variation in the average value added per graduate is documented among the Norwegian high schools studied. The variation of student results is not positively correlated with the variation in resource use, and teacher manyears per 100 student varies from 6 to 13. It follows that the efficiency variation among the high schools is quite serious. The conclusion is consistent with a number of American education production function studies summarized by Hanushek (1986).

The efficiency variation is investigated by the estimation of a best practice production frontier consistent with production theory. The methodology chosen allows for multiple outputs and variable returns to scale. The school tradeoffs between quantity and quality are analyzed in terms of the number of students and the average value added in the measurement of production.

The regulated school system in Norway attempts at equalizing the level of knowledge of the students. The consequences of the policy are analyzed by separating between the high and low achievers following Brown and Saks (1975, 1980). The schools come out with very different performances in this respect: Large and medium sized schools produce successful results with the high achievers, while small comprehensive schools (combining general subjects with vocational subjects) are inefficient with this group. Some of the smaller schools do better with the low achievers, and some of the smaller and newer schools do worse with both the low and high achievers.

The large and medium-sized schools with successful results for high achievers are basically old, while the small schools were established during the last two decades. It follows that high achievers may have been the losers of the ambitious decentralization of the Norwegian high school system. Even worse, the record for the low achievers in the new schools is mixed, too. The best practice is characterized by old schools with a tradition for academic demands and discipline.

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NOTES

1. The exams are arranged at the national level. They are common for all schools and are marked in a common pool. The rates of pay and the working conditions of the teachers are regulated by national bargaining between the government and the trade unions. The pay scale is common for all schools, and involves detailed regulation by work experience and education.
2. A third disaggregation, output described by the student achievements in different subjects, is analyzed and will be reported in a separate paper.
3. The role of the composition of the teachers at each school with respect to their age, age homogeneity and education has been investigated. See footnote 7.
4. No student has exams in all the four subjects, but in two or three. The different subjects are weighted together to a common average mark where each subject has the same weight.
5. The score follows from the marking at each junior high school, and the scaling of the marks may vary between schools. However, this is the best available data describing the students entering high schools.
6. The transformation can be illustrated by an example: School 1 has an estimated coefficient of $-4.05$, the average school coefficient is $-4.66$, and our value added measure is $5.27 (4.66 + (4.66-4.05))$. In an earlier version of the paper, we have normalized the two scales (number of standard deviations
from the mean) and defined the value added of each school as the distance between the averages of the two scales. The ranking of the schools is very close to the ranking presented here using the estimated school effect.

7. Quality variables describing the teacher input have been added to model 2, and the average age of the teachers seems to be important. The efficiency of the small schools is improved when the age composition of the teachers is taken into account. The relative youth of teachers in the small schools explains some of the inefficiency.

8. The reference school for the large inefficient school 19 is composed of the efficient large schools 7, 12 and 29 (weights 0.58, 0.39 and 0.04). The reference school produces a value added of 3.10 for low achievers and 4.74 for high achievers, i.e. elite orientation. The medium sized and inefficient school 23 has a reference school producing a value added of 3.35 for low and 4.82 for high achievers, a good performance of both groups. (See Table 3 for a close examination of this school.) The reference school of the small and inefficient school 33 consists of schools 1, 2 and 16 (weights 0.27, 0.03 and 0.70), implying a value added of 3.42 and 4.86 for low and high achievers respectively, a good result for both groups.

9. Other schools in group 4 with high teacher input per graduate and a large share of high achievers include schools 10 (7.95 teacher manyears per 100 students, 78% high achievers), 17 (8.60, 59%), 21 (9.80, 58%) and 32 (9.88, 61%).

REFERENCES


### Appendix Table 1. Estimation of the value added equation (r-values in parentheses)

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Estimated coefficient all</th>
<th>Estimated coefficient high ach.</th>
<th>Estimated coefficient low ach.</th>
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<td>$\delta$</td>
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<td>0.178 (35.7)</td>
<td>0.142 (11.1)</td>
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<td>$Q_1$</td>
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<td>3.037 (5.8)</td>
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<td>$Q_2$</td>
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<td>-3.037 (5.8)</td>
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<td>$Q_3$</td>
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<td>-4.652 (9.3)</td>
<td>-3.409 (6.7)</td>
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<td>$Q_4$</td>
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<td>-4.039 (9.3)</td>
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<td>$Q_5$</td>
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<td>-2.980 (5.7)</td>
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<tr>
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<tr>
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<td>$Q_{32}$</td>
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</tbody>
</table>

$Q_{mcu} = -4.66$  $Q_{meu} = -4.45$  $Q_{meu} = -3.11$

$r^2_{adj} = 0.861$  $r^2_{adj} = 0.176$  $r^2_{adj} = 0.146$

$N = 2613$  $N = 1479$  $N = 1134$
### Appendix Table 2. The efficiency results of model 1

<table>
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<th>Sch. no.</th>
<th>Noniterated technical efficiency</th>
<th>Iterated Mean technical efficiency</th>
<th>S.D. of the mean</th>
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<tr>
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Efficiency variation among high schools

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