

Model appendix:

A GROWTH MODEL FOR SOUTH AFRICA

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a) 1998 Social Accounting Matrix

The original SAM is developed by Thurlow and van Seventer (2002)¹ and includes 43 sectors, 14 household types and three labor categories. We aggregate this micro-SAM into a two-sector framework with a representative household and two labor types (skilled and unskilled). The model includes a traded and a nontraded sector. Tradables is dominated by industrial sectors, but also includes agriculture and mining. Nontradables is basically services. All savings are done by the representative household, which also pays interest on the foreign debt. Income from sales taxes and import tariffs are transferred to the household sector lump sum. Except for import tariffs, we ignore transfers between the rest of the world and domestic agents. Capital and wage income going abroad are included as income to the household. We do not adjust total export and import, and the current account therefore differs from the original SAM. The adjustments give negative foreign savings (trade surplus). In dynamic models the SAM is assumed to represent long-run balanced growth, and a trade surplus is consistent with growing foreign debt (as opposed to growing assets in the case of long-run trade deficit). The nontraded sector has very low values of exports and imports, which we adjust to zero, so the sector is completely nontraded.

Appendix Table 1: 1998 SAM SOUTH AFRICA (Measured in Millions of Rand)

	ACT M	ACT S	COMD M	COMD S	Lu	Ls	CAP
ACT_M			604 038				
ACT_S				723 490			
COMD_M	321 693	102 935					
COMD_S	233 933	149 791					
Lu	66 838	67 939					
Ls	54 357	182 629					
CAP	116 319	210 266					
HH					134 777	236 986	326 585
Mtax			6 642				
Atax	1 087	9 930					
SAV-INV							
RoW			181 632				
TOTAL	794 227	723 490	792 312	723 490	134 777	236 986	326 585

Note: The table continues on the next page.

¹ Thurlow, J. and D. E. van Seventer (2002), A standard computable general equilibrium model for South Africa, TMD Discussion paper no. 100, IFPRI.

Appendix Table 1 continues:

	HH	Mtax	Atax	SAV-INV	RoW	TOTAL
ACT_M					190 189	794 227
ACT_S						723 490
COMD_M	272 971			94 713		792 312
COMD_S	320 431			19 335		723 490
Lu						134 777
Ls						236 986
CAP						326 585
HH		6 642	11 017		-8 557	707 450
Mtax						6 642
Atax						11 017
SAV-INV	114 048					114 048
RoW						181 632
TOTAL	707 450	6 642	11 017	114 048	181 632	

Note: ACT_M = Traded activity, ACT_S = Nontraded activity, COMD_M = Traded commodity, COMD_S = Nontraded commodity, Lu = Unskilled labor, Ls = Skilled labor, CAP = Capital, HH = Household, Mtax = Import tariffs, Atax = Sales taxes, SAV-INV = Savings/Investments, RoW = Rest of world.

b) The mathematical documentation of the model

The following equations are the detailed description of the model. The numerical model is solved by the General Algebraic Modeling System (GAMS).

Production

Sectoral value added (X) is a Cobb-Douglas function of unskilled labor (Lu), skilled labor (Ls) and capital (K):

$$X_{i,t} = A_{i,t}^{\alpha_{1,i} + \alpha_{2,i}} Lu_{i,t}^{\alpha_{1,i}} Ls_{i,t}^{\alpha_{2,i}} K_{i,t}^{1 - \alpha_{1,i} - \alpha_{2,i}} \quad i = M, S$$

where subscripts M and S represents traded and nontraded sector, respectively, and t is the time period. Labor and capital are mobile between domestic sectors and are allocated based on marginal productivities.

First order conditions:

$$\alpha_{1,i} \cdot PV_{i,t} X_{i,t} = Wu_t \cdot Lu_{i,t}$$

$$\alpha_{2,i} \cdot PV_{i,t} X_{i,t} = Ws_t \cdot Ls_{i,t}$$

$$(1 - \alpha_{1,i} - \alpha_{2,i}) PV_{i,t} X_{i,t} = Rk_t \cdot K_{i,t}$$

Value-added price for each sector:

$$PV_{i,t} = PX_{i,t} (1 - ta_i) - \sum_j P_{j,t} IO_{ji}$$

Intermediate goods are employed according to the fixed coefficient:

$$INT_{i,t} = \sum_j IO_{ij} \cdot X_{j,t}$$

GDP at factor price:

$$GDP_t = \sum_i PV_{i,t} \cdot X_{i,t}$$

Factor market equilibrium

The wage rate and the marginal product of capital are endogenously determined from the market clearing conditions:

$$Lu_t = \sum_i Lu_{i,t}$$

$$Ls_t = \sum_i Ls_{i,t}$$

$$K_t = \sum_i K_{i,t}$$

Investment decision

The investment decision is made according to intertemporal profit maximization, subject to the accumulation of the capital stock over time:

$$\text{Max}_{I_t, K_t} \sum_{t=1}^{\infty} (1+r)^{-t} [Rk_t \cdot K_t - (PI_t \cdot I_t + ADJ_t)]$$

$$\text{s.t. } K_{t+1} = K_t \cdot (1 - \delta) + I_t$$

The adjustment costs in real terms consume the traded good and are specified as:

$$ADJ_t = a \cdot P_{M,t} \cdot \frac{I_t^2}{K_t}$$

where a is constant, P_M is the traded price, I_t is investment, and K_t is the aggregate capital stock.

Differentiating the intertemporal profit function of the investor with respect to I_t gives:

$$q_t = PI_t + 2 \cdot P_{M,t} \cdot a \cdot \frac{I_t}{K_t}$$

where PI is unit cost of investment net adjustment costs. This relationship says that the investor equilibrates the marginal cost of investment, which is given on the right hand side, and the shadow price of capital, q . Differentiating the same function with respect to K_t gives us the well-known no-arbitrage condition:

$$rq_{t-1} = Rk_t + P_{M,t} \cdot a \cdot \left(\frac{I_t}{K_t} \right)^2 - \delta \cdot q_t + \dot{q}$$

which states that marginal return to capital has to equal the interest payments on a perfectly substitutable asset of size q_{t-1} . The first term on right hand side, Rk_t , is the capital rental rate, while the second term is the derivative of capital in the adjustment cost function. The marginal return to capital also has to be adjusted by the depreciation rate, δ , and capital gain or loss, \dot{q} .

Aggregate investment is given as:

$$I_t = AK \cdot \prod_i IVD_{i,t}^{els_i}$$

which gives sectoral investment demand:

$$IVD_{i,t} = iels_i \cdot PI_t \cdot I_t / P_{i,t}$$

Total investment demand for traded goods includes the adjustment cost:

$$TIVD_{M,t} = IVD_{M,t} + a \cdot \frac{I_t^2}{K_t}$$

$$TIVD_{S,t} = IVD_{S,t}$$

Endogenous productivity

Labor augmenting technical progress ($A_{i,t}$) is endogenously determined from domestic and international technology spillovers:

$$\hat{A}_{M,t} = H_t^{\theta_1} + \lambda_M H_t^{\theta_2} T_t^{\theta_3} \left(1 - \frac{A_{M,t}}{A_t^*} \right)$$

$$\hat{A}_{S,t} = H_t^{\gamma_1} + \lambda_S H_t^{\gamma_2} N_t^{\gamma_3} \left(1 - \frac{A_{S,t}}{A_{M,t}} \right)$$

A_t^* is the frontier level of productivity, and $A_{M,t}/A_t^*$ and $A_{S,t}/A_{M,t}$ represent the international and domestic technology gap, respectively. We measure human capital (H_t) by the share of skilled workers in the labor force. Trade barriers (T_t) are represented by total trade as share of GDP, and N_t represents the nontraded sector's intermediate supply to tradables as share of GDP:

$$H_t = \frac{Ls_t}{Lu_t + Ls_t}$$

$$T_t = \frac{IM_t + E_t}{GDP_t}$$

$$N_t = \frac{IO_{SM} X_{M,t}}{GDP_t}$$

The household's decision

The representative household maximizes an intertemporal utility function over time taking into account the current budget constraint for each period:

$$\text{Max} \sum_{t=1}^{\infty} (1 + \rho)^{-t} \ln(Q_t)$$

$$\text{s.t. } PQ_t \cdot Q_t = Y_t - SAV_t$$

Aggregate consumption, Q_t , for each time period is defined as:

$$Q_t = cs \cdot \prod_i (C_{i,t} - \bar{C}_i)^{\alpha c_i}$$

and consumer's demand for each commodity:

$$P_{i,t} \cdot (C_{i,t} - \bar{C}_i) = \alpha c_i \cdot PQ_t \cdot Q_t$$

where $C_{i,t}$ is consumption for each good, \bar{C}_i is the minimum consumption and is constant over time.

Y_t is the household income for each period and is defined as:

$$Y_t = Wu_t Lu_t + Ws_t Ls_t + Rk_t K_t + FSAV_t + \sum_i ta_i \cdot PX_{i,t} \cdot X_{i,t} + tm_t \cdot PWM_t \cdot IM_t + te_t \cdot PWE_t \cdot E_t$$

The Euler equation for the consumer problem is:

$$\frac{Q_{t+1}}{Q_t} \frac{PQ_{t+1}}{PQ_t} = \frac{1+r}{1+\rho}$$

Exports and Imports in the traded sector

We assume imperfect substitution between domestic and foreign traded goods. Imports and domestic demand are endogenously determined through an Armington composite system. The demand functions are derived from minimizing current expenditure, subject to the Armington function:

$$\begin{aligned} \text{Min } & PM_t \cdot IM_t + PD_t \cdot D_t \\ \text{s.t. } & CC_t = aa[ma \cdot IM_t^{-exa} + (1-ma)D_t^{-exa}]^{-1/exa} \end{aligned}$$

where $PM_t = PWM_t(1+tm_t)$ is the price of foreign goods.

The first order conditions:

$$\begin{aligned} \frac{IM_t}{CC_t} &= aa \frac{-exa}{exa+1} \cdot \left(ma \frac{P_{M,t}}{PM_t} \right)^{\frac{1}{exa+1}} \\ \frac{D_t}{CC_t} &= aa \frac{-exa}{exa+1} \cdot \left((1-ma) \cdot \frac{P_{M,t}}{PD_t} \right)^{\frac{1}{exa+1}} \end{aligned}$$

where $exa = \frac{1}{\sigma_m} - 1$.

Sales to export market versus domestic market are endogenously determined through a CET function, and domestic and export goods are imperfect substitutes. The supply functions are derived from maximizing current sales income, subject to the CET function:

$$\begin{aligned} \text{Max } & PD_t \cdot D_t + PE_t \cdot E_t \\ \text{s.t. } & X_{M,t} = ac[mc \cdot E_t^{exc} + (1-mc)D_t^{exc}]^{1/exc} \end{aligned}$$

where

$PE_t = PWE_t(1 - t_e)$ is the price of export goods.

The first order conditions:

$$\frac{D_t}{X_{M,t}} = ac^{\frac{exc}{1-exc}} \cdot \left((1 - mc) \cdot \frac{PX_{M,t}}{PD_t} \right)^{\frac{1}{1-exc}}$$

$$\frac{E_t}{X_{M,t}} = ac^{\frac{exc}{1-exc}} \cdot \left(mc \cdot \frac{PX_{M,t}}{PE_t} \right)^{\frac{1}{1-exc}}$$

where $exc = \frac{1}{\sigma_e} + 1$.

Commodity market equilibrium

$$CC_t = INT_{M,t} + C_{M,t} + TIVD_{M,t} \quad \text{for traded goods}$$

$$X_{S,t} = INT_{S,t} + C_{S,t} + TIVD_{S,t} \quad \text{for nontraded goods}$$

Foreign borrowing and foreign debt

$$FSAV_t = PWM_t \cdot IM_t - PWE_t \cdot E_t$$

$$DEBT_{t+1} = DEBT_t \cdot (1 + r) + FSAV_t$$

Foreign debt is accumulated over time from trade deficits and interest payments on outstanding debt.

Terminal conditions (steady state constraints)

The terminal conditions are imposed in the model, such that when the time is beyond period T, which is the last period in the model, all endogenous variables have to approach approximately to their steady state situation.

$$FSAV_T = (g + n - r) \cdot DEBT_T$$

$$I_T = (\delta + g + n) \cdot K_T$$

$$Rk_T + a \cdot P_{M,T} \cdot \left(\frac{I_T}{K_T} \right)^2 = (r + \delta) \cdot q_T$$

These conditions state that foreign debt and capital stock grow at a constant rate given by $g + n$, and that marginal return to capital becomes constant.

Notation

Parameters

$\alpha_{1,i}$	share parameter for unskilled labor in value added function sector i
$\alpha_{2,i}$	share parameter for skilled labor in value added function sector i
IO_{ij}	input-output coefficient for commodity i used in sector j
exa	exponent in Armington functions
σ_m	elasticity of substitution between imported and domestic traded goods
ma	distribution parameter in Armington function for traded goods
aa	shift parameter in Armington function for traded goods
exc	exponent in CET functions
σ_e	elasticity of substitution between domestic markets and export markets for traded goods
mc	distribution parameter in CET function for sale of traded goods
ac	shift parameter in CET function for sale of traded goods
cs	shift parameter in total consumption function
ac_i	share of household consumption for commodity i
AK	shift parameter in total investment function
$iels_i$	share of investment demand for commodity i
a	coefficient in adjustment cost function
ρ	rate of consumer's time preference
δ	capital depreciation rate
θ_1	parameter in tradables productivity growth function
θ_2	parameter in tradables productivity growth function
θ_3	parameter in tradables productivity growth function
λ_M	parameter in tradables productivity growth function
γ_1	parameter in nontradables productivity growth function
γ_2	parameter in nontradables productivity growth function
γ_3	parameter in nontradables productivity growth function
λ_S	parameter in nontradables productivity growth function

Exogenous variables

PWM_t	world import price
PWE_t	world export price
PM_t	import price (including tariffs)
PE_t	export price (including taxes)
ta_i	sales tax rate
tm_t	import tariff rate
te_t	export tax rate
\bar{C}_i	minimum consumption commodity i

A_t^*	productivity level at the frontier
r	world market interest rate
n	exogenous growth rate labor supply
g	exogenous productivity growth
$g + n$	steady state growth rate
Lu_t	supply of unskilled labor
Ls_t	supply of skilled labor
H_t	human capital level

Endogenous variables

$X_{i,t}$	output of commodity i
$K_{i,t}$	capital demand sector i
$Lu_{i,t}$	unskilled labor demand sector i
$Ls_{i,t}$	skilled labor demand sector i
$A_{i,t}$	labor augmenting technical progress sector i
$\hat{A}_{i,t}$	productivity growth rate
D_t	traded goods produced and consumed domestically
IM_t	imports of traded goods
CC_t	total absorption of traded goods
E_t	exports of traded goods
$C_{i,t}$	household consumption demand for good i
$INT_{i,t}$	intermediate demand for good i
$IVD_{i,t}$	investment demand for good i
$TIVD_{i,t}$	total investment demand good i (including adjustment cost)
I_t	investment
K_t	capital stock
ADJ_t	adjustment costs
Q_t	aggregate consumption
Y_t	household income
SAV_t	total savings
GDP_t	gross domestic product
$FSAV_t$	foreign savings
$DEBT_t$	foreign debt
T_t	total trade as share of GDP
N_t	nontraded sector's intermediate supply to the traded sector
$PV_{i,t}$	value added price for commodity i
Wu_t	wage rate unskilled labor
Ws_t	wage rate skilled labor

Rk_t	rate of return to capital
PQ_t	aggregate consumption price
$PX_{i,t}$	producer price for commodity i
PD_t	domestic price traded goods
$P_{M,t}$	composite consumer price for traded goods
$P_{S,t}$	consumer price for nontraded goods
PI_t	unit cost of investment that builds up capital equipment
q_t	shadow price of capital

c) Calibration

The model parameters are calibrated consistent with long run equilibrium, where the growth rate is assumed to equal 2% (1.3% technological progress rate and 0.7% labor growth), the structure of the economy is stable, and the trade surplus with interest payments balances the projected development of foreign debt. The long run growth path must be consistent with the macroeconomic equilibrium as represented by the Euler equation: $r = (1 + \rho)(1 + g + n) - 1$, where $g + n$ is the exogenous long-run growth rate. With the time preference rate set equal to 9% the world market interest rate equals 11%. The elasticity of substitution in the Armington and CET functions are assumed to equal 3 and 2, respectively, in accordance with national and international estimates. The parameters in the production, demand, and trade functions are calibrated based on the 1998 social accounting matrix (SAM) for South Africa. Most parameters and initial values of variables related to the investment dynamics follows from the long run assumptions of the model. The capital depreciation rate is set to 4%. Given marginal product of capital (which is assumed equal to 18%), the initial capital stock is calculated based on capital income. Investment is calibrated from the long-run constraint on capital accumulation, for given values of depreciation rate and long run growth rate. The shadow price of capital equals the firm value relative to the capital stock, and follows when we know the interest rate. The initial level of foreign debt is set by the long-run constraint on debt accumulation, given data about trade deficit/surplus together with the long-run growth rate and interest rate.

Appendix Table 2: Selected calibrated parameters and initial values of selected endogenous variables

Definition	Symbol in the model	Value
<i>Parameters</i>		
Share of unskilled labor in value added, tradables	$\alpha_{1,M}$	0.28
Share of skilled labor in value added, tradables	$\alpha_{2,M}$	0.23
Share of unskilled labor in value added, nontradables	$\alpha_{1,S}$	0.15
Share of skilled labor in value added, nontradables	$\alpha_{2,S}$	0.40
Parameter in technical progress function, tradables	θ_1	0.6
Parameter in technical progress function, tradables	θ_2	0.6
Parameter in technical progress function, tradables	θ_3	1.3
Parameter in technical progress function, nontradables	γ_1	0.3
Parameter in technical progress function, nontradables	γ_2	0.3
Parameter in technical progress function, nontradables	γ_3	1.2
Distribution parameter CET function traded sector	mc	0.64
Distribution parameter Armington function traded good	ma	0.4
Elasticity in Armington function	σ_m	3
Elasticity in CET function	σ_e	2
Interest rate	r	0.11
Labor growth rate	n	0.007
Long run technical progress rate	g	0.013
Time preference rate	ρ	0.09
Depreciation rate	δ	0.04
<i>Endogenous variables</i>		
Marginal returns to capital	$Rk + a \cdot P_M \left(\frac{I}{K} \right)^2$	0.19
Marginal product of capital	Rk	0.18
Derivative of adjustment cost w.r.t capital	$-a \cdot P_M \left(\frac{I}{K} \right)^2$	-0.01
Shadow price of capital	q	1.25
Adjustment cost per unit of investment	$a \cdot P_M \frac{I}{K}$	0.20