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Trade policy in a growth model with technology gap dynamics and simulations for South Africa

Appendix with full model presentation and documentation of the calibration of the model to South Africa

Date: August 24, 2011

Content:

APPENDIX A: FULL MODEL DOCUMENTATION

Production technology, productivity dynamics and the firm's investment decisions

Gross output (X_t) is defined as a Cobb-Douglas function of effective labor (A_tL_t), foreign capital ($K_{F,t}$) and domestic capital ($K_{D,t}$):

$$X_t = (A_t L_t)^{1-\alpha-\beta} K_{F,t}^{\alpha} K_{D,t}^{\beta}$$

Intermediate goods (N_t) are employed according to the fixed coefficient (IO):

$$N_t = IO \cdot X_t$$

The productivity growth rate ($\hat{A}_t$) is specified as:

$$\hat{A}_t = \lambda_0 \left(K_{F,t} / K_t \right)^{\theta} \left(1 - \frac{A_t}{A_{F,t}} \right)$$

where $K_t = K_{D,t} + K_{F,t}$ is the total capital stock. The domestic and the frontier level of productivity are given by A_t and $A_{F,t}$ respectively, and $A_t / A_{F,t}$ is relative productivity. The elasticity of productivity growth with respect to the foreign capital share is given by θ and λ_0 is a positive parameter.

The representative firm makes its investment decisions according to intertemporal profit maximization, subject to the accumulation of the capital stocks over time:

$$\underset{L, I_D, K_D, I_F, K_F}{Max} \int_0^{\infty} e^{-rt} \left(P_{V,t} X_t - w_t L_t - I_{F,t}(P_{F,t} + \phi_{F,t}) - I_{D,t}(P_{D,t} + \phi_{D,t}) \right) dt$$

$$s.t. \qquad \dot{K}_{j,t} = I_{j,t} - \delta_j K_{j,t} \qquad\qquad j = F, D$$

where r is the world market interest rate, w_t is the wage rate, $I_{F,t}$ and $I_{D,t}$ are investments in foreign and domestic capital goods, $\phi_{F,t}$ and $\phi_{D,t}$ are unit investment adjustment costs, δ_F and δ_D

are the rates of depreciation on the foreign and domestic capital stocks, and $P_{D,t}$ is the price of domestic goods. The value added price ($P_{V,t}$) and the price of foreign goods ($P_{F,t}$) are defined as:

$$P_{V,t} = P_{X,t}(1-ta) - P_{N,t}IO$$

$$P_{F,t} = P_{WM,t}(1+tm_t)$$

where $P_{X,t}$ is the producer price, ta is the sales tax rate, $P_{N,t}$ is the composite price of intermediate goods, $P_{WM,t}$ is the exogenous world market price of import goods and tm_t is import tariffs.

Gross domestic product follows as:

$$GDP_t = P_{V,t}X_t$$

Unit adjustment costs depend positively on the size of investment relative to the capital stock:

$$\phi_{j,t} = P_{D,t} \cdot \frac{b_j}{2} \cdot \frac{I_{j,t}}{K_{j,t}} \qquad j = F, D$$

where b_F and b_D are positive parameters.

The first order conditions from the profit maximization follow as:

$$(1-\alpha-\beta)P_{V,t}X_t = w_t L_t$$

$$\alpha P_{V,t}X_t = Rk_{F,t}K_{F,t}$$

$$\beta P_{V,t}X_t = Rk_{D,t}K_{D,t}$$

$$q_{j,t} = P_{j,t} + P_{D.t} \cdot b_j \cdot \frac{I_{j,t}}{K_{j,t}} \qquad j = F, D$$

$$r \cdot q_{j,t} = Rk_{j,t} + P_{D,t} \cdot \frac{b_j}{2} \cdot \left(\frac{I_{j,t}}{K_{j,t}}\right)^2 - \delta_j \cdot q_{j,t} + \dot{q}_{j,t} \qquad j = F, D$$

where $Rk_{j,t}$ and $q_{j,t}$ are the capital rental rate and the shadow price of capital, respectively, for capital type j ($j = F, D$).

The firm's export decision

We model imperfect substitution between sales to the domestic market and the world market through a constant elasticity of transformation (CET) function. Aggregate output follows from the production function, while the composition of exports (E_t) and domestic sales (D_t) is derived from maximizing current sales income subject to the CET function:

$$Max \quad P_{X,t}X_t = P_{D,t} \cdot D_t + P_{WE,t}(1-te_t) \cdot E_t$$

$$s.t. \quad X_t = a_X[m_X \cdot E_t^{\frac{1+\sigma_X}{\sigma_X}} + (1-m_X)D_t^{\frac{1+\sigma_X}{\sigma_X}}]^{\frac{\sigma_X}{1+\sigma_X}}$$

where σ_X is the constant elasticity of substitution between domestic and foreign markets, a_X is a shift parameter and m_X is the share parameter for exports. The producer price ($P_{X,t}$) is a composite of the exogenous world market price of export goods ($P_{WE,t}$) adjusted by export taxes (te_t) and the endogenous domestic price ($P_{D,t}$). The first order conditions give the following supply functions for exports and domestic sales:

$$\frac{E_t}{X_t} = a_X^{-(1+\sigma_X)} \left(m_X \frac{P_{X,t}}{P_{WE,t}(1-te_t)} \right)^{-\sigma_X}$$

$$\frac{D_t}{X_t} = a_X^{-(1+\sigma_X)} \left((1-m_X) \frac{P_{X,t}}{P_{D,t}} \right)^{-\sigma_X}$$

An increase in the world market price of export goods leads to a shift from domestic sales to more exports. Similar, higher domestic price stimulates domestic sales.

The household's consumption/savings decision

The representative household receives income (Y_t) through the primary factors, while interest payments on its foreign debt are subtracted. There is no independent government sector, and public tax revenues (sales and trade taxes) are transferred to the household in the form of a lump sum:

$$Y_t = w_t L_t + Rk_{F,t} K_{F,t} + Rk_{D,t} K_{D,t} + ta \cdot P_{X,t} X_t + tm_t P_{WM,t} M_t + te_t P_{WE,t} E_t + FS_t$$

where M_t is total imports and FS_t is foreign savings/trade deficit.

The household is forward-looking and maximizes an intertemporal utility function taking into account the lifetime budget constraint:

$$Max \int_0^\infty U(C_t) e^{-\rho t} dt$$

$$s.t. \int_0^\infty P_{C,t} C_t e^{-rt} dt = \int_0^\infty (Y_t - S_t) e^{-rt} dt$$

Assuming intertemporal elasticity of substitution equal to unity, the iso-elastic utility function is defined as $U(C_t) = \ln C_t$, where C_t is consumption in period t. S_t is private savings, $P_{C,t}$ is the endogenous price of consumption goods, and ρ is the positive rate of time preference. The utility maximization gives the Euler equation for optimal allocation of consumption over time:

$$\frac{\dot{C}_t}{C_t} = r - \rho - \frac{\dot{P}_{C,t}}{P_{C,t}}$$

Consumption growth depends on the interest rate, the time preference rate, and the price path.

Imports decisions

We model imperfect substitution between domestic and foreign consumption and intermediate goods through constant elasticity of substitution functions (Armington functions). Total consumption demand follows from the Euler equation, while the allocation between consumption imports (C_F) and domestic consumption goods (C_D) is derived from minimizing current expenditure subject to the Armington function:

$$Min \;\; P_{F,t} \cdot C_{F,t} + P_{D,t} \cdot C_{D,t}$$

$$s.t. \;\; C_t = a_C [m_C \cdot C_{F,t}^{\frac{\sigma_C - 1}{\sigma_C}} + (1-m_C) C_{D,t}^{\frac{\sigma_C - 1}{\sigma_C}}]^{\frac{\sigma_C}{\sigma_C - 1}}$$

where σ_C is the constant elasticity of substitution between domestic and foreign consumption goods, a_C is a shift parameter and m_C is the share parameter for the foreign consumption good. The price level facing domestic consumers ($P_{C,t}$) is a composite of the exogenous world market price of import goods adjusted by import tariffs ($P_{F,t}$) and the endogenous domestic price ($P_{D,t}$). The first order conditions give the following demand functions for foreign and domestic consumption goods:

$$\frac{C_{F,t}}{C_t} = a_C^{\sigma_C - 1}\left(m_C \frac{P_{C,t}}{P_{F,t}}\right)^{\sigma_C}$$

$$\frac{C_{D,t}}{C_t} = a_C^{\sigma_C - 1}\left((1-m_C)\frac{P_{C,t}}{P_{D,t}}\right)^{\sigma_C}$$

Similar, intermediate goods (N_t) are employed according to a fixed input-output coefficient, while the composition of intermediate imports (N_F) and domestic intermediate goods (N_D) follows from minimizing current expenditure subject to the Armington function for intermediate goods:

$$Min \;\; P_{F,t} \cdot N_{F,t} + P_{D,t} \cdot N_{D,t}$$

$$s.t. \;\; N_t = a_N[m_N \cdot N_{F,t}^{\frac{\sigma_N - 1}{\sigma_N}} + (1-m_N) N_{D,t}^{\frac{\sigma_N - 1}{\sigma_N}}]^{\frac{\sigma_N}{\sigma_N - 1}}$$

Total intermediate demand is given by N_t, σ_N is the constant elasticity of substitution between domestic and foreign intermediate goods, a_N is a shift parameter and m_N is the share parameter for the foreign intermediate good. The first order conditions give the following demand functions for foreign and domestic intermediate goods:

$$\frac{N_{F,t}}{N_t} = a_N^{\sigma_N - 1}\left(m_N \frac{P_{N,t}}{P_{F,t}}\right)^{\sigma_N}$$

$$\frac{N_{D,t}}{N_t} = a_N^{\sigma_N - 1}\left((1-m_N)\frac{P_{N,t}}{P_{D,t}}\right)^{\sigma_N}$$

Total imports (M_t) include the demand for foreign capital goods and total domestic demand (D_t) includes domestic investment demand:

$$M_t = C_{F,t} + N_{F,t} + I_{F,t}$$

$$D_t = C_{D,t} + N_{D,t} + TI_{D,t}$$

where $TI_{D,t} = I_{D,t} + \frac{1}{P_{D,t}}(\phi_{D,t} + \phi_{F,t})$ is domestic investment demand including adjustment costs.

Foreign borrowing and foreign debt

Foreign debt (FD_t) is accumulated over time from trade deficits (FS_t) and interest payments on outstanding debt:

$$FS_t = P_{WM,t} M_t - P_{WE,t} E_t$$

$$\dot{FD}_t = r \cdot FD_t + FS_t$$

APPENDIX B: 1998 SOCIAL ACCOUNTING MATRIX FOR SOUTH AFRICA

The original SAM is developed by Thurlow and van Seventer (2002) and includes 43 sectors, 14 household types and three labor categories. We aggregate this micro-SAM into a one-sector framework with a representative household and one labor type. All savings are done by the representative household, which also pays interest on the foreign debt. Income from sales taxes and import tariffs are transferred to the household sector lump sum. Except for import tariffs, we ignore transfers between the rest of the world and domestic agents. Capital and wage income going abroad are included as income to the household. We do not adjust total export and import, and the current account therefore differs from the original SAM. The adjustments give negative foreign savings (trade surplus). In dynamic models the SAM is assumed to represent long-run balanced growth, and a trade surplus is consistent with growing foreign debt (as opposed to growing assets in the case of long-run trade deficit).

Appendix Table 1: 1998 SAM SOUTH AFRICA (Measured in Millions of Rand)

	ACT	COMD	LAB	CAP	HH	Mtax	Atax	S-I	RoW	TOTAL
ACT		1 327 552							190 164	1 517 716
COMD	808 351				593 395			114 048		1 515 794
LAB	371 763									371 763
CAP	326 585									326 585
HH			371 763	326 585		6 642	11 017		-8 564	707 443
Mtax		6 642								6 642
Atax	11 017									11 017
S-I					114 048					114 048
RoW		181 600								181 600
TOTAL	1 517 716	1 515 794	371 763	326 585	707 443	6 642	11 017	114 048	181 600	

Note: ACT = Activity, COMD = Commodity, LAB = Labor, CAP = Capital, HH = Household, Mtax = Import tariffs, Atax = Sales taxes, S-I = Savings/Investments, RoW = Rest of world.

In the model we separate between foreign and domestic capital goods, and the SAM is adjusted accordingly. We assume that 2/3 of the capital stock is domestic and 1/3 foreign. Total capital is typically about equally divided between buildings and other structures on the one hand, and equipment and machinery on the other hand. This stylized fact, together with the assumption that 2/3 of equipment and machinery are imported, gives a total foreign capital share of 1/3. Further, we assume that imported consumption goods account for 5% of total consumption demand, while the residual imports are intermediate goods. This implies that total imports are divided between 17% capital goods, 16% consumption goods and 67% intermediate goods. Data on the composition of merchandise imports in South Africa during 1980-92 offered by Hawkins (1997) is fairly consistent with our numbers.

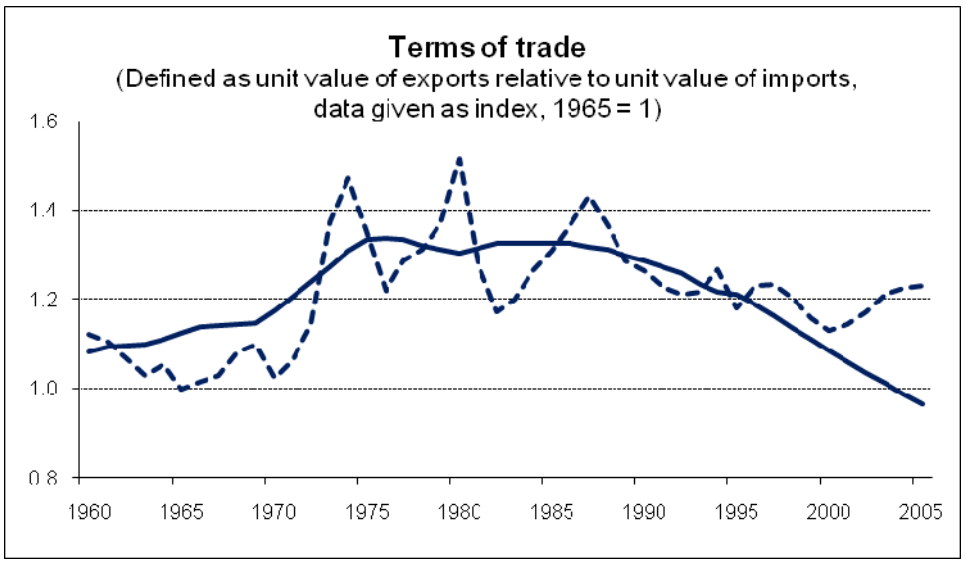
APPENDIX C: CALIBRATION OF PARAMETERS

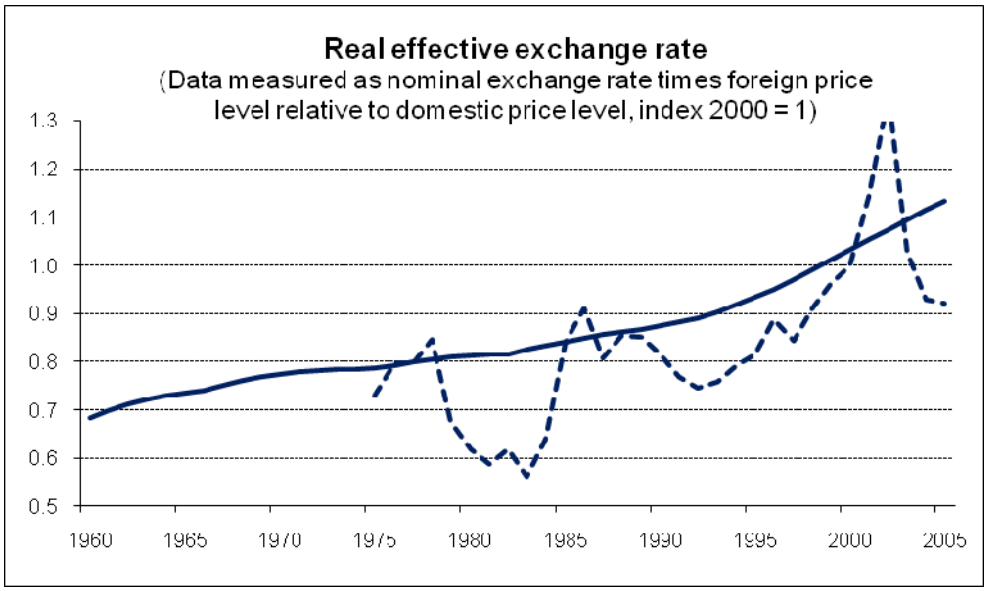
The model parameters are calibrated consistent with long-run equilibrium, where the growth rate is assumed to equal 2% (1.3% technological progress rate and 0.7% labor growth). The long-run growth path must be consistent with the macroeconomic equilibrium as represented by the Euler equation: $r = (1+\rho)(1+g+n)-1$, where $g+n$ is the exogenous long-run growth rate. With the time preference rate set equal to 9% the world market interest rate equals 11%. The parameters in the production, demand, and trade functions are calibrated based on the 1998 social accounting matrix (SAM) for South Africa. Most parameters and initial values of variables related to the investment dynamics follow from the long-run assumptions of the model. The capital depreciation rate is set to 3.7% for both domestic and foreign capital. The value of total investments (including adjustment costs) and capital incomes are given from the SAM. Since adjustment costs are assumed to consume domestic investment goods only, the foreign capital stock follows from the long-run constraint on capital accumulation. The domestic capital stock is calibrated from the rate of depreciation, the long-run growth rate, the interest rate, the respective firm value, the value of total domestic investments, and the domestic capital income (an expression deduced from the long-run constraint on capital accumulation). Given the foreign and domestic levels of capital, the marginal product of each capital type is calculated from the capital income. The shadow price of each capital stock equals the respective firm value relative to the capital stock. The coefficient *b* in the adjustment cost functions is calibrated based on the no-arbitrage conditions. With the calibrated value of *b*, the initial value of adjustment costs follows. Domestic investment is calibrated as the value of total domestic investments minus adjustment costs. The initial level of foreign debt is set by the long-run constraint on debt accumulation, given data about trade deficit/surplus together with the long-run growth rate and the interest rate.

The elasticity of substitution in the Armington and CET functions are assumed to equal 3 and 2, respectively, in accordance with national and international estimates. Hertel et al. (2007) combine parameter estimation and general equilibrium modeling. Based on data from five Latin American countries, the US and New Zealand they estimate the elasticity of substitution among imports from different countries. The “rule of two” says that the elasticity of substitution across imports by sources is equal to twice the elasticity of substitution between domestic and foreign goods (empirical support is offered by Liu et al., 2004). Based on this hypothesis the average Armington elasticity across sectors equals 3.5. Available estimates of export elasticities are more limited. Senhadji and Montenegro (1999) estimate export elasticities for 53 developing and developed economies. The average elasticity across middle income countries is 1.7.

APPENDIX D: CALIBRATION OF WORLD PRICES

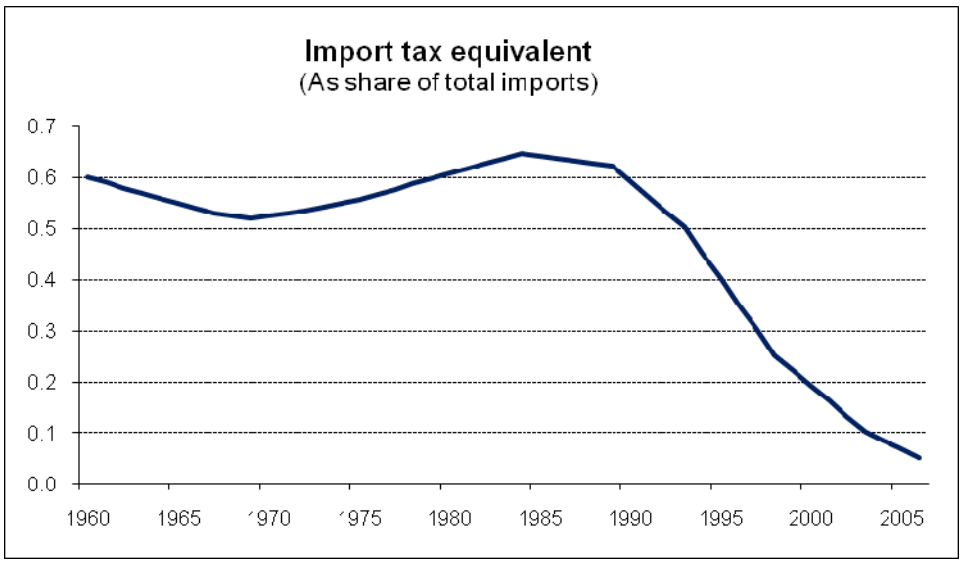
To adjust for the impact of world price shocks on the trade level the terms of trade and the real effective exchange rate are calibrated based on available data (IMF, 2006). In the model, the terms of trade is given by the ratio between the exogenous world market prices of exports and imports ($P_{WE,t}/P_{WM,t}$), while the real exchange rate is defined as the exogenous world market price of imports relative to the endogenous domestic price ($P_{WM,t}/P_{D,t}$).

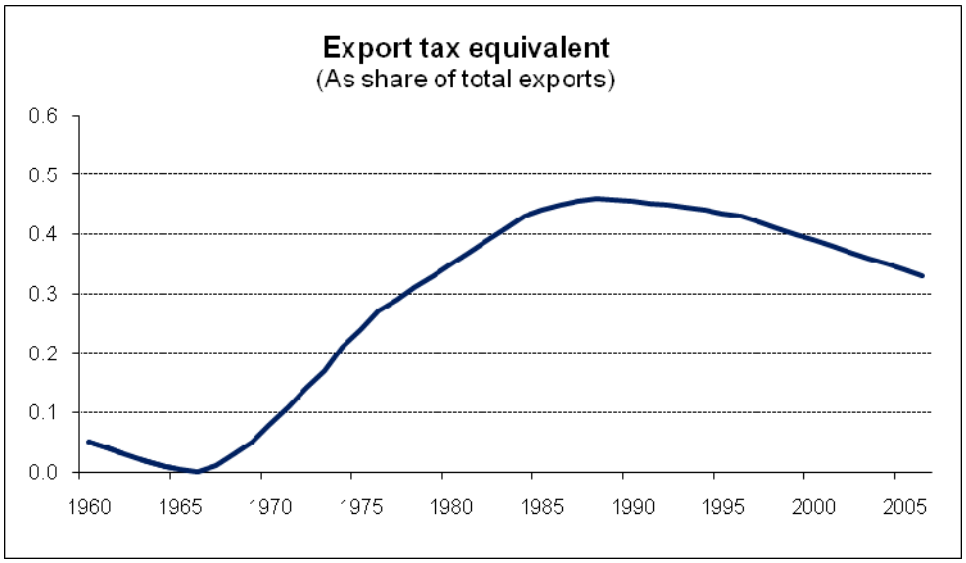




APPENDIX E: SOUTH AFRICAN OPENNESS INDICATOR

The calibrated tariff equivalent measure of openness (given in Figure 3 of the paper) is defined as export tax plus import tax as share of total trade. The underlying paths of import and export tax rates are illustrated below. The tariffs are calibrated to reproduce the observed export and import paths in South Africa during 1960-2005.





To reproduce the actual import path the import tax is initially high, decreases during the 1960s, increases to a peak of about 65% in the late 1980s, and then decreases rapidly to 7% in 2005. The export tax is low during the 1960s, but to capture the slowdown in exports in the 1970s and 80s it is necessary to gradually increase the tax to about 45%. The export tax is also declining post Apartheid, but remains above 30% even in 2005 indicating that domestic conditions are holding back South African exports. The calibrated tariff paths are consistent with tariffs calculated from

partial analyses of exports and imports with reasonable values of elasticities. The export function is assumed to depend on the world level of GDP and the real effective exchange rate, with elasticities set equal to 1. The import function is assumed to depend on the South African GDP level and the real effective exchange rate, with elasticities set equal to 1.2 and -1.5, respectively.

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