

Model appendix:

## **RESOURCE BOOM, PRODUCTIVITY GROWTH AND REAL EXCHANGE RATE DYNAMICS**

### **- A dynamic general equilibrium analysis of South Africa**

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#### **a) 1998 Social Accounting Matrix**

The original SAM is developed by Thurlow and van Seventer (2002)<sup>1</sup> and includes 43 sectors, 14 household types and three labor categories. We aggregate this micro-SAM into a three-sector framework with a representative household and one labor type. The model includes a tradable industrial sector, nontradable services and mining.

<b>Tradables</b>	<b>Nontradables</b>
Agriculture, forestry and fishing	Electricity, gas and steam
Food	Water supply
Textiles	Wholesale and retail trade
Wearing apparel	Catering and accommodation services
Leather and leather products	Building construction & civil engineering
Footwear	Communication
Wood and wood products	Business services
Plastic products	Medical, dental, veterinary & other services
Glass and glass products	Other products
Basic iron and steel	Government services
Metal products excluding machinery	<b>Mining</b>
Furniture	Coal mining
Beverages & Tobacco	Gold and uranium ore mining
Paper and paper products	Other mining
Printing, publishing and recorded media	
Coke and refined petroleum products	
Basic chemicals	
Other chemicals and man-made fibers	
Rubber products	
Non-metallic minerals	
Basic non-ferrous metals	
Machinery and equipment	
Electrical machinery and apparatus	
Television, radio and communication equipment	
Professional and scientific equipment	
Motor vehicles, parts and accessories	
Other transport equipment	
Other manufacturing	
Transport and storage	
Finance and insurance	

<sup>1</sup> Thurlow, J. and D. E. van Seventer (2002), A standard computable general equilibrium model for South Africa, TMD Discussion paper no. 100, IFPRI.

In the model analysis the mining sector is treated as an enclave with no links to the rest of the economy. The SAM is adjusted accordingly. First, mining is assumed to be a pure exporting sector. Second, to capture the fiscal response to resource booms the income from the mining sector is transferred to a public sector that allocates the windfall to consumption of tradable and nontradable goods. All savings are done by the representative household, which also pays interest on the foreign debt. Income from sales taxes and import tariffs are transferred to the household sector lump sum. Except for import tariffs, we ignore transfers between the rest of the world and domestic agents. Capital and wage income going abroad are included as income to the household. We do not adjust total export and import, and the current account therefore differs from the original SAM. The adjustments give negative foreign savings (trade surplus). In dynamic models the SAM is assumed to represent long-run balanced growth, and a trade surplus is consistent with growing foreign debt (as opposed to growing assets in the case of long-run trade deficit). Trade in services is adjusted to zero, so the service sector is completely nontraded.

**Appendix Table 1: 1998 SAM SOUTH AFRICA** (Measured in Millions of rand)

	ACT_R	ACT_M	ACT_S	COMD_M	COMD_S	LAB	CAP
ACT_R							
ACT_M				665 313			
ACT_S					652 598		
COMD_M	26 685	338 748	131 540				
COMD_S	13 668	188 118	109 592				
LAB	22 622	124 158	224 982				
CAP	20 111	128 523	177 951				
HH						349 140	306 474
GOV						22 622	20 111
MTAX				6 640			
ATAX	713	1 772	8 533				
SAV-INV							
RoW				181 632			
<b>TOTAL</b>	83 799	781 319	652 598	853 585	652 598	371 762	326 585

Appendix Table 1 continues:

	HH	GOV	MTAX	ATAX	SAV-INV	ROW	TOTAL
ACT_R						83 799	83 799
ACT_M						116 006	781 319
ACT_S							652 598
COMD_M	293 598	23 591			39 423		853 585
COMD_S	246 740	19 855			74 625		652 598
LAB							371 762
CAP							326 585
HH			6 640	11 018		-18 173	654 386
GOV							43 446
MTAX							6 640
ATAX							11 018
SAV-INV	114 048						114 048
RoW							181 632
<b>TOTAL</b>	654 386	43 446	6 640	11 018	114 048	181 632	

*Note:* ACT\_R = Mining activity, ACT\_M = Industrial activity, ACT\_S = Service activity, COMD\_M = Industrial commodity, COMD\_S = Service commodity, LAB = Labor, CAP = Capital, HH = Household, GOV = Public sector, MTAX = Import tariffs, ATAX = Sales taxes, SAV-INV = Savings/Investments, RoW = Rest of world.

## b) The mathematical documentation of the model

The following equations are the detailed description of the model. Throughout the presentation subscript  $i$  refers to all sectors (industry, services and mining), while subscript  $j$  refers to industry and services only. The numerical model is solved by the General Algebraic Modeling System (GAMS).

### Production

Sectoral value added ( $X$ ) is a Cobb-Douglas function of capital ( $K$ ) and labor ( $L$ ):

$$X_{i,t} = A_{i,t}^{\alpha_i} L_{i,t}^{\alpha_i} K_{i,t}^{1-\alpha_i} \quad i = M, R, S$$

where subscripts  $M$ ,  $R$  and  $S$  represents industry, mining and services, respectively, and  $t$  is the time period. Labor and capital are mobile between industry and services and are allocated based on marginal productivities. The resource sector is modeled as an enclave with no direct links to the rest of the economy, and employs sector specific factors of production with supply growing exogenously at the long-run rate.

First order conditions for industry and services are:

$$\begin{aligned} \alpha_j PV_{j,t} X_{j,t} &= W_{1,t} \cdot L_{j,t} \\ (1 - \alpha_j) PV_{j,t} X_{j,t} &= Rk_{1,t} \cdot K_{j,t} \quad j = M, S \end{aligned}$$

First order conditions for mining are:

$$\begin{aligned} \alpha_R PV_{R,t} X_{R,t} &= W_{2,t} \cdot L_{R,t} \\ (1 - \alpha_R) PV_{R,t} X_{R,t} &= Rk_{2,t} \cdot K_{R,t} \end{aligned}$$

where  $L_R$  and  $K_R$  are the exogenous sector-specific supply of labor and capital used in the mining sector, while  $w_2$  and  $Rk_2$  are the endogenous factor returns.  $L_R$  and  $K_R$  are assumed to grow exogenously at the labor growth rate and the long run aggregate growth rate, respectively.

Value-added price for each sector:

$$PV_{i,t} = PX_{i,t} (1 - tax_i) - \sum_j P_{j,t} IO_{ji}$$

Intermediate goods are employed according to the fixed coefficient:

$$INT_{j,t} = \sum_i IO_{ji} \cdot X_{i,t}$$

GDP at factor price:

$$GDP_t = \sum_i PV_{i,t} \cdot X_{i,t}$$

### Investment decision

The investment decision is made according to intertemporal profit maximization, subject to the accumulation of the capital stock over time:

$$Max_{I,K} \sum_{t=1}^T (1+r)^{-t} [Rk_{1,t} \cdot K_t - W_{1,t} \cdot L_t - PI_t \cdot I_t - ADJ_t]$$

$$s.t. K_{t+1} = K_t \cdot (1 - \delta) + I_t$$

where

$$I_t = AK \cdot \prod_j IVD_{j,t}^{iels_j}$$

The adjustment costs in real terms consume the nontradable good and are specified as:

$$ADJ_t = a \cdot P_{S,t} \cdot \frac{I_t^2}{K_t}$$

where  $a$  is constant,  $P_S$  is the nontradable price (which equals  $PX_S$  since services are not traded internationally),  $I_t$  investment in real term, and  $K_t$  is the capital stock outside the mining sector.

Differentiating the intertemporal profit function of the investor with respect to  $I_t$  gives:

$$q_t = PI_t + 2 \cdot P_{S,t} \cdot a \cdot \frac{I_t}{K_t}$$

where  $PI$  is unit cost of investment net adjustment costs. This relationship says that the investor equilibrates the marginal cost of investment, which is given on the right hand side, and the shadow price of capital,  $q$ . Differentiating the same function with respect to  $K_t$  gives us the well-known no-arbitrage condition:

$$rq_{t-1} = Rk_{1,t} + P_{S,t} \cdot a \cdot \left( \frac{I_t}{K_t} \right)^2 - \delta \cdot q_t + \dot{q}$$

which states that marginal return to capital has to equal the interest payments on a perfectly substitutable asset of size  $q_{t-1}$ . The first term on right hand side,  $Rk_{1,t}$ , is the capital rental rate, while the second term is the derivative of capital in the adjustment cost function. The marginal return to capital also has to be adjusted by the depreciation rate,  $\delta$ , and capital gain or loss,  $\dot{q}$ .

Investment demand:

$$IVD_{j,t} = \frac{iels_j \cdot PI_t \cdot I_t}{P_{j,t}}$$

Total investment demand for services includes the adjustment cost, while necessary investments to augment the mining capital stock according to the long run growth rate are included in total investment demand for industrial goods:

$$TIVD_{S,t} = IVD_{S,t} + a \cdot \frac{I_t^2}{K_t}$$

$$TIVD_{M,t} = IVD_{M,t} + I_{2,t}$$

### Factor market equilibrium

Labor and capital are mobile between industry and services, and the wage rate and the marginal product of capital are endogenously determined from the market clearing conditions:

$$L_t = \sum_j L_{j,t}$$

$$K_t = \sum_j K_{j,t}$$

### Endogenous productivity

In industry and services labor augmenting technical progress ( $A$ ) is endogenously determined from sectoral learning by doing (LBD) and technology adoption<sup>2</sup>:

$$\frac{\dot{A}_{j,t}}{A_{j,t}} = \lambda_{1,j} \left( \frac{L_{j,t}}{L_t} \right)^{\theta_{1,j}} + \lambda_{2,j} \left( \frac{TRADE_t}{GDP_t} \right)^{\theta_{2,j}} \left( 1 - \frac{A_{j,t}}{A_{j,t}^*} \right)$$

$A_{j,t}$  and  $A_{j,t}^*$  represent the domestic and frontier level of productivity, and  $A_{j,t}/A_{j,t}^*$  is the technology gap.  $L_{j,t}/L_t$  is the sectoral labor share and  $TRADE_t/GDP_t$  aggregate trade as share of GDP (not including mining exports):

$$TRADE_t = PWM \cdot M_t + PWE_M \cdot E_{M,t}$$

### The household's decision

The representative household maximizes an intertemporal utility function over time taking into account the current budget constraint for each period:

$$\text{Max } U_1 = \sum_{t=1}^T (1 + \rho)^{-t} \ln(Q_t) + \ln(Q_T) \frac{(1 + \rho)^{1-T}}{\rho}$$

$$\text{s.t. } PQ_t \cdot Q_t = Y_t - SAV_t$$

$U_1$  is the value of the intertemporal utility evaluated at time period 1's price, and aggregate consumption,  $Q_t$ , for each time period is defined as:

$$Q_t = cs \cdot \prod_j (C_{j,t} - \bar{C}_j)^{\alpha c_j}$$

where  $C_{j,t}$  is consumption for each good,  $\bar{C}_j$  is the minimum consumption and is constant over time.  $Y_t$  is the household income for each period and is defined as:

$$Y_t = W_{1,t} L_t + Rk_{1,t} K_t + FSAV_t + \sum_i tax_i \cdot PX_{i,t} \cdot X_{i,t} + mtr \cdot PWM \cdot M_t$$

The Euler equation for the consumer problem is:

$$\frac{Q_{t+1}}{Q_t} \frac{PQ_{t+1}}{PQ_t} = \frac{1+r}{1+\rho}$$

Consumer's demand for each commodity:

$$P_{j,t} \cdot (C_{j,t} - \bar{C}_j) = \alpha c_j \cdot PQ_t \cdot Q_t$$

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<sup>2</sup> Labor augmenting technical progress in the resource sector grows exogenously at the long-run rate.

### Resource wealth management (fiscal response to boom)

To capture the fiscal response to higher resource income we treat the mining sector as public with the income transferred to a myopic government keeping a balanced budget. The public resource income ( $Y_g$ ) comes from the factors of production in the resource sector, and is spent on a mix of industrial goods (both foreign and domestic) and services according to constant shares ( $\alpha g_i$ ):

$$Y_{g,t} = w_2 L_R + Rk_2 K_R$$

$$P_{j,t} \cdot G_{j,t} = \alpha g_j \cdot Y_{g,t} \quad \sum_j \alpha g_j = 1$$

$G_M$  and  $G_S$  are government consumption of industrial goods and services, respectively, with  $P_M$  and  $P_S$  as corresponding prices.

### Exports and Imports in the tradable industrial sector

We assume imperfect substitution between domestic and foreign industrial goods. Imports and domestic demand are endogenously determined through an Armington composite system. The demand functions are derived from minimizing current expenditure, subject to the Armington function:

$$\text{Min } PM \cdot M_t + PD_t \cdot D_t$$

$$\text{s.t. } CC_t = aa[ma \cdot M_t^{-\text{exa}} + (1-ma)D_t^{-\text{exa}}]^{-1/\text{exa}}$$

where  $PM = PWM(1+mtr)$  is the price of foreign goods.

The first order conditions:

$$\frac{M_t}{CC_t} = aa^{\frac{-\text{exa}}{\text{exa}+1}} \cdot \left( ma \frac{P_{M,t}}{PM} \right)^{\frac{1}{\text{exa}+1}}$$

$$\frac{D_t}{CC_t} = aa^{\frac{-\text{exa}}{\text{exa}+1}} \cdot \left( (1-ma) \cdot \frac{P_{M,t}}{PD_t} \right)^{\frac{1}{\text{exa}+1}}$$

where  $\text{exa} = \frac{1}{\sigma_m} - 1$ .

Sales to export market versus domestic market are endogenously determined through a CET function, and domestic and export goods are imperfect substitutes. The supply functions are derived from maximizing current sales income, subject to the CET function:

$$\text{Max } PD_t \cdot D_t + PWE_M \cdot E_{M,t}$$

$$\text{s.t. } X_{M,t} = ac[mc \cdot E_{M,t}^{\text{exc}} + (1-mc)D_t^{\text{exc}}]^{1/\text{exc}}$$

The first order conditions:

$$\frac{D_t}{X_{M,t}} = ac^{\frac{exc}{1-exc}} \cdot \left( (1-mc) \cdot \frac{PX_{M,t}}{PD_t} \right)^{\frac{1}{1-exc}}$$

$$\frac{E_{M,t}}{X_{M,t}} = ac^{\frac{exc}{1-exc}} \cdot \left( mc \cdot \frac{PX_{M,t}}{PWE_M} \right)^{\frac{1}{1-exc}}$$

where  $exc = \frac{1}{\sigma_e} + 1$ .

### Commodity market equilibrium

$$\begin{aligned} CC_t &= INT_{M,t} + C_{M,t} + TIVD_{M,t} + G_M && \text{for industrial goods} \\ X_{S,t} &= INT_{S,t} + C_{S,t} + TIVD_{S,t} + G_{S,t} && \text{for services} \\ X_{R,t} &= E_{R,t} && \text{for mining products} \end{aligned}$$

### Foreign borrowing and foreign debt

$$FSAV_t = PWM \cdot M_t - PWE_M \cdot E_{M,t} - PWE_R \cdot E_{R,t}$$

Since mining is a pure exporting sector  $PX_R = PWE_R$ .

$$DEBT_{t+1} = DEBT_t \cdot (1+r) + FSAV_t$$

Foreign debt is accumulated over time from trade deficits and interest payments on outstanding debt.

### Terminal conditions (steady state constraints)

The terminal conditions are imposed in the model, such that when the time is beyond T, which is the last period in the model, all endogenous variables have to approach approximately to their steady state situation.

$$FSAV_T = (g + n - r) \cdot DEBT_T$$

$$I_T = (\delta + g + n) \cdot K_T$$

$$Rk_{1,T} + a \cdot P_{S,T} \cdot \left( \frac{I_T}{K_T} \right)^2 = (r + \delta) \cdot q_T$$

These conditions state that foreign debt and capital stock grow at a constant rate given by  $g + n$ , and that marginal return to capital becomes constant.

## Notation

### Parameters

$\alpha_i$	share parameter for labor in value added function sector $i$
$IO_{ji}$	input-output coefficient for commodity $j$ used in sector $i$
$exa$	exponent in Armington functions
$\sigma_m$	elasticity of substitution between imported and domestic industrial goods
$ma$	distribution parameter in Armington function for industrial goods
$aa$	shift parameter in Armington function for industrial goods
$exc$	exponent in CET functions
$\sigma_e$	elasticity of substitution between domestic markets and export markets for industrial goods
$mc$	distribution parameter in CET function for sale of industrial goods
$ac$	shift parameter in CET function for sale of industrial goods
$cs$	shift parameter in total consumption function
$\alpha c_j$	share of household consumption for commodity $j$
$\alpha g_j$	share of government consumption for commodity $j$
$AK$	shift parameter in total investment function
$iels_j$	share of investment demand for commodity $j$
$a$	coefficient in adjustment cost function
$\rho$	rate of consumer's time preference
$\delta$	capital depreciation rate
$\theta_{1,j}$	parameter in the productivity growth function
$\theta_{2,j}$	parameter in the productivity growth function
$\lambda_{1,j}$	parameter in the productivity growth function
$\lambda_{2,j}$	parameter in the productivity growth function

### Exogenous variables

$PWM$	world import price for industrial goods
$PM$	price of foreign industrial goods (including import tariffs)
$PWE_M$	world export price for industrial goods
$PWE_R$	world export price for mining goods (= $PX_R$ )
$\bar{C}_j$	minimum consumption commodity $j$
$A_j^*$	productivity level at the frontier
$tax_i$	sales tax rate for commodity $i$
$mtr$	tariff rate for industrial goods
$r$	world market interest rate
$n$	exogenous growth rate labor supply
$g$	exogenous productivity growth
$L_t$	supply of labor outside the mining sector
$L_{R,t}$	supply of labor in the mining sector

$K_{R,t}$	supply of capital in the mining sector
$I_{2,t}$	investments to augment the mining capital stock according to long run growth rate

*Endogenous variables*

$X_{i,t}$	output of commodity $i$
$K_{j,t}$	capital demand industry and services
$L_{j,t}$	labor demand industry and services
$A_{j,t}$	labor augmenting technical progress sector $j$
$D_t$	industrial goods produced and consumed domestically
$M_t$	imports of industrial goods
$CC_t$	total absorption of industrial goods
$E_{M,t}$	exports of industrial goods
$E_{R,t}$	exports of mining products ( $= X_{R,t}$ )
$C_{j,t}$	household consumption demand for good $j$
$G_{j,t}$	public consumption demand for good $j$
$INT_{j,t}$	intermediate demand for good $j$
$IVD_{j,t}$	investment demand for good $j$
$TIVD_{j,t}$	total investment demand good $j$ (including adjustment cost and mining investments)
$I_t$	investment in quantity
$K_t$	capital stock outside the mining sector
$ADJ_t$	adjustment costs
$Q_t$	aggregate consumption
$Y_t$	household income
$Y_{g,t}$	public resource income
$SAV_t$	total savings
$GDP_t$	gross domestic product
$FSAV_t$	foreign savings
$DEBT_t$	foreign debt
$PV_{i,t}$	value added price for commodity $i$
$W_{1,t}$	wage rate labor in industry and services
$W_{2,t}$	wage rate labor in mining
$Rk_{1,t}$	rate of return to capital outside the mining sector
$Rk_{2,t}$	rate of return to capital in the mining sector
$PQ_t$	aggregate consumption price
$PX_{j,t}$	producer price for commodity $j$
$PD_t$	domestic price industrial goods

$P_{M,t}$	composite consumer price for industrial goods
$P_{S,t}$	consumer price for services
$PI_t$	unit cost of investment that builds up capital equipment
$q_t$	shadow price of capital
$TRADE_t$	total trade (excluding mining exports)

### c) Selected calibrated parameters and initial values of selected endogenous variables

Definition	Symbol in the model	Value
<b>Parameters</b>		
Share of labor in value added, industry	$\alpha_M$	0.51
Share of labor in value added, services	$\alpha_S$	0.44
Share of labor in value added, mining	$\alpha_R$	0.47
Parameter in technical progress function, industry	$\theta_{1,M}$	0.8
Parameter in technical progress function, industry	$\theta_{2,M}$	0.4
Parameter in technical progress function, services	$\theta_{1,S}$	0.8
Parameter in technical progress function, services	$\theta_{2,S}$	0.1
Share of household consumption to industrial goods	$\alpha c_M$	0.52
Share of household consumption to services	$\alpha c_S$	0.48
Share of public consumption to industrial goods	$\alpha g_M$	0.4
Share of public consumption to services	$\alpha g_S$	0.6
Distribution parameter CET function industrial sector	$mc$	0.64
Distribution parameter Armington function industrial good	$ma$	0.4
Elasticity in Armington function	$\sigma_m$	3
Elasticity in CET function	$\sigma_e$	3
Coefficient in adjustment cost function	$a$	5.02
Interest rate	$r$	0.125
Labor growth rate	$n$	0.02
Long run technical progress rate	$g$	0.01
Time preference rate	$\rho$	0.092
Depreciation rate	$\delta$	0.04
<b>Endogenous variables</b>		
Marginal product of capital in the mining sector	$Rk_2$	0.18
Marginal returns to capital outside the mining sector	$Rk_1 + a \cdot P_S \left(\frac{I}{K}\right)^2$	0.20
Marginal product of capital outside the mining sector	$Rk_1$	0.18
Derivative of adjustment cost w.r.t capital	$-a \cdot P_S \left(\frac{I}{K}\right)^2$	-0.02
Shadow price of capital	$q$	1.24
Adjustment cost per unit of investment	$a \cdot P_S \frac{I}{K}$	0.35