Interest Rate Decisions in an Asymmetric Monetary Union*

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Abstract

Decision rules matter for monetary policy in a currency union if the interest rate affects member states differently. We examine the consequences for inflation, output and interest rate fluctuations and the welfare loss of four alternative types of decision procedures. We show that the alternative decision rules have very dissimilar properties and that different rules favour different types of countries. In addition to asymmetric transmission mechanisms, we consider asymmetric shocks. We show that it is the combination of a country's interest rate elasticity and the covariance between the shocks to the country and the shocks to the union that determines which decision rule the country would favour.

Keywords: Monetary union, decision making, regional influences

JEL Classification: E52, E58, E61, F33

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1 Introduction

Interest rate decisions are usually made by a committee rather than by a single person. In a monetary union, the composition of the monetary policy board may reflect the union’s regional composition. For example, in the European Monetary Union (EMU), the Governing Council consists of the Governors of the National Banks of the EMU countries, and the President, the Vice President and the four Directors of the European Central Bank (ECB). The Federal Open Market Committee (FOMC) in the United States consists of seven Board members, the President of the Federal Reserve Bank of New York and four of the other eleven regional Reserve Bank presidents, who vote on a fixed rotation. Because of this regional heterogeneity, it is possible that committee members are more concerned about economic development in their respective home regions than in the union as a whole. Meade and Sheets (2001) provide empirical evidence for a bias towards the home region among the members of the FOMC. The ECB, however, is very explicit about neglecting regional developments, as illustrated by the following statement by President Duisenberg:1 “... our decisions today, again and as always, were based on a Euro area-wide analysis of economic and financial developments—and nothing else.” Nevertheless, commentators have argued that the fact that the majority of the Governing Council consists of national representatives is likely to give rise to regional influences on policy decisions.

Committee decisions can be made in several ways. There is a large literature on different types of collective decision-making procedure.2 In this paper, we will focus on four general types of decision-making procedure that are particularly relevant for interest rate decisions in a monetary union. These are: (i) "union rule", where the central bank only focuses on union-wide aggregates; (ii) 'Benthamite rule' (utilitarian rule), where the central bank minimizes the sum of national loss functions; (iii) "majority rule", where each board member votes for the interest rate that minimizes losses in their respective home country; and (iv) "consensus rule", where the interest rate is set as the average of the desired interest rates of each national board member.

The differences between (i) and (ii) have been analysed by De Grauwe (2000), De Grauwe and Piskorski (2001), Nolan (2002) and Gros and Hefeker (2002a, 2002b). Although the analytical results from De Grauwe (2000), Nolan (2002) and Gros and Hefeker (2002a, 2002b) point towards important differences between the two rules, the empirical results of De Grauwe and Piskorski (2001) suggest that the differences between the two rules are quantitatively unimportant. Aksoy et al. (2002) compare a union-wide

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1 Statement released at the press conference following the Governing Council meeting of 9 September 1999.
2 For a survey of the literature relevant to monetary policy, see Gerlach-Kristen (2002).
perspective on the interest rate with nationalistic voting, as well as a combination of the two procedures, which they interpret as the 'ECB rule'. Von Hagen and Sueppel (1994) and Brueckner (2000) also compare a union-wide perspective with decisions based on nationalistic voting.

The present paper contributes to the literature in four ways. First, we consider a broader set of decision rules than has previously been considered in this literature. Second, contrary to previous analytical work on the issue, we apply a 'New Keynesian' theoretical framework to discuss the implications of alternative decision rules. We then circumvent the less realistic assumption in the frequently used Barro-Gordon model, where it is assumed that the monetary policy instrument is the rate of inflation, assumed to be equal among the union member states. By contrast, in the standard open-economy New Keynesian model, it is assumed that the monetary policy instrument is the interest rate. This not only allows an analytical discussion of the implications of asymmetry in the interest rate elasticities among member states, but also adds new insights. Third, we show that the alternative decision rules have very different properties qualitatively, as each rule favours different types of country. Moreover, by calibrating the model with estimates of the divergences in the transmission mechanisms among EMU members, we demonstrate that the differences between the alternative decision rules in terms of welfare could be quantitatively important. This suggests that the results in De Grauwe and Piskorski (2001) may not be very robust. Finally, we analyse the consequences of applying the different decision rules when there are asymmetries in both the transmission mechanism and in shocks to the different economies. The earlier literature has only explored these asymmetries separately, whereas we show that it is the combination of these two types of asymmetry that matters.

The paper is organized as follows. Section 2 presents the basic theoretical framework, and Section 3 presents the alternative decision rules. In Section 4, we compare the welfare implications of the alternative decision rules within our basic framework. This is done both analytically, with the help of a 'stress indicator', and by means of a quantitative exercise. Section 5 extends our model to include asymmetries in both the transmission mechanisms and in the shocks that affect the economies. Section 6 summarizes our results.

2 The baseline model

The union consists of $n$ countries. Initially, we focus solely on differences in the transmission mechanism and assume that countries are identical except for their responsiveness to monetary policy. That is, to begin with, we assume that shocks are common to all countries. In Section 5 below, we extend

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3This assumption is made in De Grauwe (2000) and Nolan (2002).
the model to the situation where there are asymmetries in both transmission mechanisms and shocks. As we will demonstrate there, a necessary condition for the different decision rules to have non-common effects on welfare is that the transmission mechanism differs across countries. Hence, we find it natural to focus first on the effects of differences in the transmission mechanism alone.

Output gap and inflation for country $j$ are given by:

$$y_{j,t} = E_t y_{j,t+1} - \varphi_j (i_{j,t} - E_t \pi_{j,t+1}) + g_t, \quad j = 1, 2, ..., n$$  \hspace{1cm} (1)

$$\pi_{j,t} = \delta E_t \pi_{j,t+1} + \alpha y_{j,t} + u_t, \quad j = 1, 2, ..., n$$  \hspace{1cm} (2)

where $y_{j,t}$ is the output gap in country $j$, $i_{j,t}$ is the nominal interest rate, $\pi_{j,t}$ is the inflation rate, $E_t$ is the expectations operator based on period $t$ information, $\delta \leq 1$ is a common discount factor, and $g_t$ and $u_t$ are iid aggregate demand shocks and cost-push shocks respectively, which satisfy $E_t g_{t+1} = E_t u_{t+1} = 0$. We assume that $n$ is ‘large’, so that economic developments in a given country have a negligible effect on the rest of the countries. As shown by Galí and Monacelli (2002) and Clarida, Galí and Gertler (2001), the standard New Keynesian model for a small open economy is isomorphic to the closed economy model and can be represented as in equations (1) and (2), with appropriate definitions of the shocks.\textsuperscript{4} The real exchange rate does not enter the reduced form explicitly, since there is a linear relationship between the real exchange rate and the output gap (see Galí and Monacelli, 2002).

As we focus attention on national divergences in the effect of interest rate changes, we assume that the only parameter that is country specific is the interest rate elasticity of the output gap $\varphi_j$.\textsuperscript{5} The empirical evidence points towards substantial differences in the output response to the interest rate both among European countries (see e.g., Dornbusch et al., 1998 and Ehrmann et al., 2003)\textsuperscript{6} and among regions within the United States (see e.g., Carlino and DeFina, 1998).

Output gap and inflation in the union as a whole are given by:

\textsuperscript{4}In Galí and Monacelli (2002) and Clarida, Galí and Gertler (2000), the real interest rate deviates from its flexible price counterpart. The flexible price real interest rate depends, among other things, on productivity shocks. In our specification, $g_t$ includes the stochastic part of the flexible price real interest rate. In addition, for simplicity we have assumed that the steady state real interest rate is zero. We follow Clarida, Galí and Gertler (2000) by adding a cost-push shock to the Phillips curve.

\textsuperscript{5}One could also consider differences in the slope of the Phillips curve, $\alpha$, but this would not add anything substantial beyond differences in $\varphi$.

\textsuperscript{6}See also the papers published by the ECB’s “European Transmission Network” for evidence on differences across euro area member countries with respect to monetary transmission (www.ecb.int).
\[ y_t = \frac{1}{n} \sum_{j=1}^{n} y_{j,t} = E_t y_{t+1} - \varphi_t + \frac{1}{n} \sum_{j=1}^{n} \varphi_j E_t \pi_{t+1} + g_t \]

\[ \pi_t = \frac{1}{n} \sum_{j=1}^{n} \pi_{j,t} = \delta E_t \pi_{t+1} + \alpha y_t + u_t \]

where \( \varphi \equiv \frac{1}{n} \sum_{j=1}^{n} \varphi_j \).

Welfare in country \( j \) is represented by a standard (period) loss function:

\[ L_{j,t} = \frac{1}{2} \left( \pi_{j,t}^2 + \lambda y_{j,t}^2 \right), \quad j = 1, 2, \ldots, n \] (3)

where we assume that the preferences concerning inflation stability versus output stability, as captured by the parameter \( \lambda \geq 0 \), are identical across the countries. The policy objective is to minimize the discounted sum of all current and expected future period losses. Galí and Monacelli (2002) show that the true welfare loss function for a small open economy under certain assumptions can be approximated by (3).

When considering monetary policy we assume that the central bank follows a time-consistent policy by optimizing in each period (discretion). As argued by Svensson (1999), the way that central banks operate in practice is best described as decision making under discretion rather than commitment. In the model considered here, the reaction function under a discretionary policy implies that the interest rate is a (linear) function of the state variables, that is, \( g_t \) and \( u_t \), and private sector expectations. Since there is no intrinsic inertia in the model, either in terms of lagged responses or auto-correlated shocks, the minimum-state-variable solution, which is the natural solution to consider for the purpose of this paper, is then characterized by:

\[ E_t y_{j,t+1} = E_t \pi_{j,t+1} = 0 \quad \forall j = 1, 2, \ldots, n. \]

We can therefore neglect expected future losses and focus solely on the period loss function (3) when evaluating welfare.

### 3 Monetary Policy

#### 3.1 Independent monetary policy

Consider first the case where the countries conduct independent monetary policies. The monetary policy instrument is the nominal interest rate \( i_{j,t} \).

Minimizing the loss function (3) with respect to \( i_{j,t} \) gives the following first-order condition:

\[ \alpha \pi_{j,t} + \lambda y_{j,t} = 0. \] (4)

Inserting equations (1) and (2) into (4), and remembering that expected future output gap and inflation are equal to zero, gives the following solution
for the optimal independent policy:

$$i_{j,t}^* = \frac{1}{\varphi_j} \left[ g_t + \frac{\alpha}{\alpha^2 + \lambda} u_t \right] \quad j = 1, 2, \ldots, n. \quad (5)$$

This gives the following solutions for output gap and inflation under the optimal (time-consistent) monetary policy:

$$y_{j,t} = -\frac{\alpha}{\alpha^2 + \lambda} u_t \quad j = 1, 2, \ldots, n.$$

$$\pi_{j,t} = \frac{\lambda}{\alpha^2 + \lambda} u_t \quad j = 1, 2, \ldots, n.$$

Under the optimal independent policy, output and inflation fluctuations are not affected by the interest rate responsiveness of the country. $\varphi_j$ only enters the solution for the interest rate. The intuition is that whatever the interest rate responsiveness is, the interest rate is always adjusted to keep output insulated from aggregate demand shocks and to keep a constant relationship between inflation and output when cost-push shocks occur. A low interest rate elasticity simply leads the central bank to change the interest rate more aggressively.

### 3.2 Common monetary policy

Consider then the case where the $n$ countries form a monetary union. Then, the union central bank must decide whether regional divergences in the transmission mechanism should influence policy and, if so, how the central bank should take account of such regional divergences when setting the interest rate. There are several alternatives that the union central bank can choose concerning interest rate decisions. We consider four types of decision-making model that have been suggested in the literature. As described in the Introduction, these are: (i) the "union rule", (ii) the "Benthamite rule", (iii) the "majority rule", and (iv) the "consensus rule". We analyse each decision rule in turn and then compare them.

#### 3.2.1 Union rule

The "union rule" is defined as the interest rate that minimizes the loss function (3), but where national inflation and output gap are replaced by union inflation and output gap, i.e.:

$$i_t^U = \arg \left\{ \min_i \frac{1}{2} \left[ \pi_i^2 + \lambda y_i^2 \right] \right\} \quad (6)$$

This rule seems to correspond to the official policy rule of the ECB. The solution for the interest rate under the "union rule" is found by simply
removing the country subscript from equation (5). The solution for the interest rate and for output gap and inflation in each country are given by:

\[ i^U_t = \frac{1}{\varphi} \left[ g_t + \frac{\alpha}{\alpha^2 + \lambda} u_t \right] \]  

\[ y^U_{j,t} = \frac{\varphi - \varphi_j}{\varphi} g_t - \frac{\varphi_j^2}{\varphi(\alpha^2 + \lambda)} u_t \quad j = 1, 2, ..., n. \]

\[ \pi^U_{j,t} = \frac{\alpha(\varphi - \varphi_j)}{\varphi} g_t + \frac{\alpha^2 (\varphi - \varphi_j) + \varphi \lambda}{\varphi(\alpha^2 + \lambda)} u_t \quad j = 1, 2, ..., n. \]

where \( y^U_{j,t} \) and \( \pi^U_{j,t} \) denote output gap and inflation in country \( j \) under the "union rule". We see that aggregate demand shocks affect output gap and inflation to the extent that the country’s interest rate responsiveness deviates from the union average. Likewise, the cost-push shocks are not optimally distributed between output and inflation if the interest rate responsiveness deviates from the union average. If \( \varphi_j < \varphi \), too much of the cost-push shock shows up in inflation variability. If \( \varphi_j > \varphi \), output gap variability is too high compared to inflation variability.

3.2.2 Benthamite rule

An alternative to aggregating the arguments in the loss function is to aggregate the individual loss functions. This "Benthamite rule", based on a utilitarian approach to utility aggregation, can be specified as:

\[ i^B_t = \arg \left\{ \min \frac{1}{2} \left[ \frac{1}{n} \sum_{j=1}^{n} L_{j,t} \right] \right\} \]  

The first-order condition for minimizing (8) is:

\[ \sum_{j=1}^{n} (\varphi_j \alpha \pi_{j,t} + \varphi_j \lambda y_{j,t}) = 0. \]  

As shown in Appendix A.1, the solution for the interest rate under the "Benthamite rule" is given by:

\[ i^B_t = \frac{1}{\varphi + d} \left[ g_t + \frac{\alpha}{\alpha^2 + \lambda} u_t \right], \]  

where

\[ d \equiv \frac{1}{\varphi n^2} \sum_{j=1}^{n-1} \sum_{h>j}^{n} (\varphi_j - \varphi_h)^2 \geq 0. \]

\( d \) measures the degree of divergence in interest rate responsiveness among the union members. If there is no divergence, \( d = 0 \), while \( d > 0 \) if at least
one member differs from the other. We see from (10) that monetary policy is less activist under the "Benthamite rule" than under the "union rule", and that the responsiveness to shocks is lower the more heterogenous the union members are.

To understand the intuition for this result, assume that \( n = 3 \) and that \( \varphi_1 < \varphi_2 = \varphi < \varphi_3 \) and \( g_t > 0 \). Note that under the "union rule", the central bank completely offsets the effect of demand shocks on union output, so that country 1 (the least interest rate sensitive) faces a positive output gap, country 2 faces a zero output gap, and country 3 faces a negative output gap that exactly offsets country 1’s positive gap. Consider then a marginal reduction of the rate from the level implied by the "union rule". Since country 2 is already at its optimum, the loss from a marginal reduction of the interest rate is of second order and can be ignored. However, country 3 (the most interest rate sensitive) would experience a (first-order) gain in terms of inflation and an output gap that are less negative, while country 1 would experience a (first-order) loss. However, country 3’s gain more than outweighs country 1’s loss because of the higher interest rate elasticity in country 3. Hence, the "Benthamite rule", which minimizes the sum of the national losses, implies a lower interest rate than the "union rule".

output gap and inflation under the "Benthamite rule" are found by inserting (10) into (1) and (2):

\[
y_{j,t}^B = \frac{\varphi + d - \varphi_j}{\varphi + d} g_t - \frac{\varphi_j \alpha}{(\varphi + d)(\alpha^2 + \lambda)} u_t \quad j = 1, 2, ..., n.
\]

\[
\pi_{j,t}^B = \frac{\alpha(\varphi + d - \varphi_j)}{\varphi + d} g_t + \frac{\alpha^2(\varphi + d - \varphi_j) + (\varphi + d)\lambda}{(\varphi + d)(\alpha^2 + \lambda)} u_t \quad j = 1, 2, ..., n.
\]

### 3.2.3 Majority rule

It was implicitly assumed above that the interest rate decision was taken by a single person. In most central banks, however, interest rate decisions are made by a board. Blinder and Morgan (2000) list three general types of collective decision procedure: (i) letting the median voter decide; (ii) reaching a consensus where each member has the same influence on the decision (averaging); and (iii) letting the most skilled member decide. Since we assume that all members are equally skilled, we will not consider type (iii). If all the board members have a union-wide perspective, either represented by the "union rule" or the "Benthamite rule", the decisions made by a single person and those made by a board will be identical. If board members instead have a national perspective, the type of decision-making procedure may matter.

To analyse this case, suppose that the interest rate is decided by a board of \( n \) members, each representing his or her home country. Suppose further
that the individual board members are only concerned about economic conditions in their home countries and that the interest rate decision is taken by majority voting, i.e., type (i) in Blinder and Morgan’s general listing. We label this rule the "majority rule". Since the median voter theorem applies, the "majority rule" is specified as:

\[ i^M_t = \text{med} \left[ i^*_1, i^*_2, \ldots, i^*_n \right], \]

where \( i^*_j \) is given by equation (5). From Section 3.1, the median voter’s preferred interest rate is given by:

\[ i^M_t = \frac{1}{\varphi_m} \left[ g_t + \frac{\alpha}{\alpha^2 + \lambda} u_t \right], \quad (11) \]

where \( \varphi_m \) is the median of \( \varphi_1, \varphi_2, \ldots, \varphi_n \).

Does nationalistic voting result in a different policy from the situation where board members only care about union-wide economic conditions? The answer depends on how asymmetric the distribution of \( \varphi \) is—that is, whether the median differs from the mean. If \( \varphi_m < \varphi \), monetary policy is more activist under the "majority rule" than under the "union rule", whereas the opposite is true if \( \varphi_m > \varphi \). As a baseline case, however, it is natural to assume that the distribution of \( \varphi \) within the union is symmetric, so that \( \varphi_m = \varphi \).\(^7\) Then, the interest rate under nationalistic voting is equal to the interest rate under the union-wide perspective represented by the "union rule". Thus, divergent transmission mechanisms among union members do result in conflicts of interest among members, but do not necessarily lead to a different policy outcome compared to the situation where all member countries are equally affected by interest rate changes. Thus, in the case of symmetrically distributed \( \varphi \), the policy outcome of both the "majority rule" and the "union rule" is not affected by the type of divergence considered here.\(^8\) However, the "Benthamite rule" exploits divergence among member countries in order to minimize the sum of national losses.

### 3.2.4 Consensus rule

Although interest rate decisions in most central banks are formally taken by majority voting, there is also a tradition for consensus decisions (type (ii) in Blinder and Morgan’s categorization). For example, as far as we know, the ECB council has never taken a formal vote on interest rate decisions, but agrees on all its moves by consensus. We will interpret a consensus decision

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\(^7\)Ehrmann et al. (2003) presents estimates of the interest rate elasticity in nine EMU countries, where the mean and the median are very close to each other. See Section 4.2 below, where we apply their estimates for a numerical illustration.

\(^8\)In general, however, the 'majority rule' and the 'union rule' will result in a different policy outcome if there is less than perfect correlation of shocks among the countries, see Section 5.
as a compromise between conflicting interests. Specifically, we assume that the ”consensus rule” is given by:

\[ i_t^C = \frac{1}{n} \sum_{j=1}^{n} i_{j,t}^* \]  

(12)

where \( i_{j,t}^* \) is given by (5). The ”consensus rule” may also be interpreted as a bargaining solution where all members have equal bargaining power.

Gerlach-Kristen (2002) compares ‘majority rules’ and ‘consensus rules’ (the latter denoted as ‘averaging’) when board members have equal preferences but differ in their ability to estimate the output gap. Aksoy et al. (2002) also consider the ”consensus rule”, but their motivation for the rule is that (12) is, in their view, a good proxy for the union-wide rule represented by (6), which they do not estimate owing to data limitations. However, we shall see that (12) is not a good proxy for (6) if the interest rate responsiveness differs among union members.

We show in Appendix A.2 that the interest rate under the ”consensus rule” is given by:

\[ i_t^C = \frac{1}{\varphi - d'} \left[ g_t + \frac{\alpha}{\alpha^2 + \lambda} u_t \right], \]

(13)

where:

\[ 0 \leq d' = \frac{\sum_{j=1}^{n-1} \sum_{h>j} (\varphi_j - \varphi_h)^2 \prod_{k \neq j,h} \varphi_k}{n \prod_{j=1}^{n} \varphi_j \sum_{j=1}^{n} \varphi_j^{-1}} < \varphi. \]

Note that \( d' \) is also a measure of the degree of divergence in the interest rate responsiveness among the union members, and is thus closely related to \( d \). Since \( d' > 0 \) if at least one country differs in interest rate responsiveness from the others, we see that monetary policy under the ”consensus rule” is more activist than monetary policy under the ”union rule” and the ”majority rule”. This is a result of Jensen’s inequality: since \( \frac{1}{\varphi} \) is convex in \( \varphi \), the mean of \( \frac{1}{\varphi} \) is larger than the inverse of the mean of \( \varphi \). More intuition can be provided by again considering the case where \( n = 3 \), \( \varphi_1 < \varphi_2 = \varphi < \varphi_3 \), and \( g_t > 0 \). Country 2 would then prefer the interest rate set under the ”union rule”, whereas country 1 prefers a higher interest rate, and country 3 prefers a lower interest rate. Since country 1 is the least interest rate sensitive, the difference between its desired rate and country 2’s is higher than the corresponding difference between countries 2 and 3. It follows that average preferred interest rate among the member countries (i.e., the rate set under the ”consensus rule”) is higher than the one set under the ”union rule”.

The solutions for output gap and inflation under the ”consensus rule” are given by:

\[ y_{j,t}^C = \frac{\varphi - d' - \varphi_j}{\varphi - d'} g_t - \frac{\varphi_j \alpha}{(\varphi - d')(\alpha^2 + \lambda)} u_t, \quad j = 1, 2, ..., n. \]
\[
\pi^U_{j,t} = \frac{\alpha(\varphi - d^l - \varphi_j)}{\varphi - d^l} g_t + \frac{\alpha^2(\varphi - d^l - \varphi_j) + (\varphi - d^l)\lambda}{(\varphi - d^l)(\alpha^2 + \lambda)} u_t \quad j = 1, 2, \ldots, n.
\]

4 Welfare implications of alternative decision rules

The ultimate judgement of the alternative decision rules should be how they affect welfare, which is measured by the loss function for the individual country in our model. In this section, we analyse welfare implications both qualitatively and quantitatively.

4.1 The stress of living with a common interest rate

From the above solutions for output gap and inflation under the alternative rules, we see that the alternative decision rules give the same outcome if the countries in the union are perfectly symmetric. However, if there are asymmetries in the transmission mechanisms, conflicts of interest may emerge in regard to which rule to apply. In order to perform welfare comparisons under the different rules, we derive a ‘stress indicator’ based on a Taylor approximation around the optimal (time-consistent) policy under monetary autonomy. Then, we have:

\[
L_{j,t}(i_{j,t}) \approx L_{j,t}(i_{j,t}^*) + L'_{j,t}(i_{j,t}^*)(i_{j,t} - i_{j,t}^*) + \frac{1}{2} L''_{j,t}(i_{j,t}^*)(i_{j,t} - i_{j,t}^*)^2
\]

(14)

where \(L'_{j,t}(i_{j,t}^*) = 0\) under the optimal policy and terms higher than order two are equal to zero owing to the linear-quadratic structure of the model. The ‘stress indicator’ is then given by:

\[
S_{j,t} = L^h_{j,t} - L^*_{j,t} = \frac{1}{2} L''_{j,t}(i_{j,t}^*) (i_{j,t}^h - i_{j,t}^*)^2 = \frac{1}{2} \varphi^2(\alpha^2 + \lambda)(i_{j,t}^h - i_{j,t}^*)^2,
\]

(15)

\(h = U, B, M, C\). Even if the \(S\) is derived from a Taylor approximation, it can easily be verified that (15) gives an exact measure of the welfare loss for static linear-quadratic models of the type considered here. We see that the cost of giving up monetary autonomy is not only related to how much the common monetary policy differs from the optimal independent policy, but also to how strongly the economy is affected by monetary policy. A given deviation from the optimal interest rate is less costly for the country if output and inflation are less affected by interest rate changes.

To simplify notation and draw attention to the important differences between the decision rules, we now aggregate the different shocks according to
\[ z_t \equiv g_t + \frac{\alpha}{\alpha^2 + \lambda} u_t. \]

Making use of equations (7), (10), (11) and (13) and taking the expectation through the expression for \( S_{j,t} \) yields the following expressions for the ‘expected stress’ for country \( j \) under the alternative decision rules for the union central bank:

\[
ES_{U,j,t} = \frac{1}{2} \varphi^{-2}(\alpha^2 + \lambda)\text{var}(z_t)[\varphi_j - \varphi]^2, \tag{16}
\]

\[
ES_{B,j,t} = \frac{1}{2}(\varphi + d)^{-2}(\alpha^2 + \lambda)\text{var}(z_t)[\varphi_j - (\varphi + d)]^2, \tag{17}
\]

\[
ES_{M,j,t} = \frac{1}{2} \varphi_m^{-2}(\alpha^2 + \lambda)\text{var}(z_t)[\varphi_j - \varphi_m]^2 \tag{18}
\]

\[
ES_{C,j,t} = \frac{1}{2}(\varphi - d')^{-2}(\alpha^2 + \lambda)\text{var}(z_t)[\varphi_j - (\varphi - d')]^2. \tag{19}
\]

By comparing the above expressions, we see that a country that has the same interest rate elasticity as the union average would prefer the "union rule" (or the "majority rule" if the mean is equal to the median). A country that is more interest rate elastic than the union average would prefer the "Benthamite rule", since the "optimal" interest rate elasticity under the "Benthamite rule" is \( \varphi_j = \varphi + d \). A country that is less interest rate elastic would prefer the "consensus rule", since the "optimal" elasticity under this rule is \( \varphi_j = \varphi - d' \). Thus, the alternative decision rules favour different types of country.

From a political economy point of view, the "union rule", which describes the Euro-wide perspective of the ECB, has the advantage that regional differences per se are not taken into account when the interest rate is set. This may limit the scope for regional lobbyism and make the monetary policy decision less subject to political pressure. As shown above, the "union rule" can also be regarded as a decision rule that does not favour countries that are very different from the union average concerning the transmission mechanism. Of the alternative decision rules considered, it may therefore be easier to gain acceptance for the "union rule", as well as the "majority rule". Nevertheless, the total welfare for the union can be higher under the "Benthamite rule", although some of the welfare gain comes from sacrificing welfare in less interest-rate elastic countries. If the regional welfare distribution is considered important, the "Benthamite rule" could be less problematic if appropriate compensation schemes were feasible.

### 4.2 A quantitative illustration

The preceding subsection demonstrated that, when the impact of monetary policy differs among member states, the effects on welfare of joining a monetary union depend on the decision rule followed by the union central bank.

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9See Lohmann (1998) for further discussion on channels through which elected politicians may influence monetary policy.
Table 1. Percentage increase in loss for various decision rules compared to optimal independent policy

<table>
<thead>
<tr>
<th>Country</th>
<th>$\varphi_j$</th>
<th>$\frac{EL^{1j}}{L^*_1}$</th>
<th>$\frac{EL^{2j}}{L^*_2}$</th>
<th>$\frac{EL^{3j}}{L^*_3}$</th>
<th>$\frac{EL^{4j}}{L^*_4}$</th>
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<td>Italy</td>
<td>0.12</td>
<td>31.3</td>
<td>27.2</td>
<td>39.1</td>
<td>20.4</td>
</tr>
<tr>
<td>Spain</td>
<td>0.14</td>
<td>23.6</td>
<td>19.5</td>
<td>31.7</td>
<td>13.0</td>
</tr>
<tr>
<td>France</td>
<td>0.20</td>
<td>6.9</td>
<td>4.4</td>
<td>14.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Germany</td>
<td>0.20</td>
<td>6.9</td>
<td>4.4</td>
<td>14.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Austria</td>
<td>0.25</td>
<td>0.6</td>
<td>0.0</td>
<td>4.7</td>
<td>2.1</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.32</td>
<td>3.3</td>
<td>7.9</td>
<td>0.0</td>
<td>21.8</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.32</td>
<td>3.3</td>
<td>7.9</td>
<td>0.0</td>
<td>21.8</td>
</tr>
<tr>
<td>Finland</td>
<td>0.44</td>
<td>39.1</td>
<td>58.2</td>
<td>14.6</td>
<td>103.8</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.45</td>
<td>43.9</td>
<td>64.5</td>
<td>17.1</td>
<td>113.4</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td>158.9</td>
<td>193.3</td>
<td>135.2</td>
<td>297.9</td>
</tr>
</tbody>
</table>

Notes: Calculations of $ES^h_j/L^*_j$ use equations (3) and (16)-(19). Estimates of $\varphi_j$ are from Ehrmann et al. (2003). Besides $\varphi_j$, we use the parameters $\lambda = 1$, $\alpha = 0.05$, $var(u_t) = var(g_t) = 0.02^2$.

In this subsection, we calibrate the model to investigate whether our results could be of quantitative importance. We calibrate the parameters of the model by drawing on existing literature on policy rules.

Following Rudebusch and Svensson (1999), we set the relative weight on output gap fluctuations $\lambda$ equal to 1. The sacrifice ratio $\alpha$ is set to 0.05 as in Jensen (2002). Galí and Monacelli (2002) calibrate their model so that it delivers a standard deviation of the natural rate of output gap of 0.02. We use this value in calibrating the standard deviation of our demand shock, $\sigma_g = 0.02$. As a (reasonable) baseline value, we also use 0.02 for the standard deviation of inflation shocks $\sigma_u$. In calibrating the elasticities of output with respect to the interest rate, we adapt the empirical estimates for nine EMU nations reported in Ehrmann et al. (2003). For concreteness, we assign country names to our nine members, but we assume (for simplicity) that they are of the same size. The elasticities are in the range of 0.12 (Italy) to 0.45 (Netherlands). The mean elasticity is 0.27, while Austria is the median country with $\varphi$ equal to 0.25. Table 1 summarizes the elasticities.

The loss measure reported in table 1 is $ES^h_j/L^*_j$, i.e. expected percentage increase in loss under the various decision rule compared to optimal independent monetary policy.

Several aspects of these numbers are noteworthy. First, there are large variations across countries for given decision rules. While the increase in loss is zero or negligible for the countries that are least affected, the loss of giving up monetary independence is quantitatively important for some countries, regardless of the decision rule. Generally, it is the countries at the tails of the distribution of elasticities that ‘suffer’ the most for all rules. Second, the increase in loss for a given country varies substantially across
decision rules. The Netherlands (the most interest rate-sensitive country), for instance, experiences an increase in loss that is almost seven times bigger under the "consensus rule" compared to the "Benthamite rule". Third, and confirming our results in the previous subsection, members with elasticities above the average prefer the union central bank to apply the "Benthamite rule", while countries with \( \phi \)s below the average prefer the "consensus rule". Fourth, the "union rule" provides the most evenly distributed increase in loss, while the "consensus rule" gives the most uneven distribution. As an illustration of this latter point, we note that the four countries with above-average elasticities bear close to 90 per cent of the burden (of the increases in loss) under the "consensus rule". Finally, the sum of the increase in loss across countries is larger under the "consensus rule" than under the "union rule" and the "majority rule". (It is obviously lowest in a 'Benthamite regime', as the central bank would then minimize the sum of losses.) The intuition is that the "consensus rule" pulls the interest rate further away from the one preferred by the most interest rate-sensitive countries than does the "union rule". Since these countries are interest rate sensitive, this creates 'big' losses for them. The "union rule" implies an interest rate that is further away from the preferred one for the least sensitive countries than does the "consensus rule". However, since these countries are not very interest rate sensitive, their losses are relatively small, giving a moderate increase in aggregate losses relative to the "Benthamite rule". In this particular example, the increase in aggregate loss under the "consensus rule" is almost twice as large as with that under "union rule".

5 Extension: Asymmetric shocks

In order to focus solely on the transmission mechanism, it was assumed above that shocks were common to all members of the union. Here, we depart from this assumption and consider asymmetric shocks. When analysing the alternative decision rules under asymmetric shocks, it is important to consider asymmetric shocks and asymmetric transmission mechanisms in combination. As will be clear from the analysis below, the results of the alternative decision rules will be identical if the transmission mechanisms are symmetric, even though shocks are asymmetric.

Suppose that the aggregate shocks \( z \) consist of an idiosyncratic part and a common part, given by:

\[
Z_{j,t} = z_{j,t}^i + \beta_{j} z_{t},
\]

where \( z_{j,t}^i \) is a pure idiosyncratic shock, assumed to be white noise, \( z_{t} \) is a common shock and \( \beta_{j} \) measures the systematic covariance of shocks between country \( j \) and the union. By varying \( \beta_{j} \) and the variance of \( z_{j,t}^i \), the decomposition enables us to consider a general set of potential asymmetries.
Accordingly, $\beta_j = \frac{\text{cov}(z_{j,t}, z_t)}{\text{var}(z_t)}$ and thus measures the "systematic risk" of the country as a member of the "union portfolio" of countries. That is, $\beta_j$ may be positive or negative, and it satisfies $\frac{1}{n} \sum_{j=1}^{n} \beta_j = 1$. We assume for simplicity that a given country’s $\beta$ is the same concerning demand shocks $g_t$ as for cost-push shocks $u_t$, so that we can multiply the combined shock $z_t$ by the common factor $\beta$. To facilitate comparison with the results under common shocks presented above, we also make the assumption that the cross-country distributions of idiosyncratic shocks $z_{i,j,t}$, the "systematic risks" $\beta_j$, and the interest rate elasticities $\varphi_j$ are independent of each other.

As $\frac{1}{n} \sum_{j=1}^{n} \beta_j = 1$, the solution of the interest rate under the "majority rule" is unaffected by introducing asymmetric shocks. Formally,

$$i_t^U = \frac{1}{\varphi} n \sum_{j=1}^{n} \beta_j z_{j,t} + \frac{1}{\varphi} z_t = \frac{1}{\varphi} z_t.$$(21)

Since we have assumed that $n$ is 'large', the average idiosyncratic shock, $\frac{1}{n} \sum_{j=1}^{n} z_{i,t}$, approaches zero.

To find the interest rate under the "Benthamite rule", we solve for the interest rate from the first-order condition (9) and find that:

$$i_t^B = \sum_{j=1}^{n} \left( \frac{\varphi_j \beta_j}{\varphi_j^2} z_{j,t} + \frac{\varphi_j \beta_j}{\varphi_j^2} z_t \right) \approx \sum_{j=1}^{n} \frac{\varphi_j \beta_j}{\varphi_j^2} z_t,$$(22)

where the second (approximate) equality follows from the law of large numbers.10 The assumption of independent $\beta$'s and $\varphi$'s implies that:

$$\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \varphi_i \beta_i = \frac{1}{n} \sum_{i=1}^{n} \varphi_i = \varphi.$$ (23)

We will assume that $n$ is sufficiently large to apply this limit. Then, the coefficient on the shock is identical to the one under symmetric shocks, i.e.:

$$i_t^B = \frac{1}{\varphi + d} z_t,$$(24)

where $d$ is defined as in Section 4.

---

10 We observe that the interest rate response to shocks is not necessarily the same in the case where both shocks and the transmission mechanisms are asymmetric as in the case where only the transmission mechanisms are asymmetric. Depending on the combinations of $\varphi$'s and $\beta$'s, the coefficient on $z_t$ may be higher or lower than in the case of symmetric shocks. For example, if countries with high interest rate sensitivity also tend to have $\beta$'s above unity, the "Benthamite" response would be larger under asymmetric shocks than under symmetric shocks. Moreover, it is possible to construct combinations of $\varphi$'s and $\beta$'s that imply that the interest rate response to shocks is higher under the "Benthamite rule" than in the "union rule". Although this ambiguity is an interesting result, we focus on the case where the distribution of $\beta$ is independent of the distribution of $\varphi$, as this facilitates comparison with our earlier results.
To find the solution for the interest rate under the "majority rule", note that the desired interest rate for each country is given by:

\[ i_{j,t}^* = \frac{1}{\varphi_j}z_{j,t} + \frac{\beta_j}{\varphi_j}z_t. \]  

(25)

Then, the "majority rule" gives:

\[ i_t^M = \text{med} \left[ \frac{1}{\varphi_1}z_{1,t} + \frac{\beta_1}{\varphi_1}z_t, \frac{1}{\varphi_2}z_{2,t} + \frac{\beta_2}{\varphi_2}z_t, \ldots, \frac{1}{\varphi_n}z_{n,t} + \frac{\beta_n}{\varphi_n}z_t \right]. \]

Unfortunately, an explicit solution for the "majority rule" cannot be found in the general case. However, if we, in addition to the assumption that \( z_{i,j,t}, \beta_j, \) and \( \varphi_j \) are independent, assume that they are symmetrically distributed around the mean among the member countries, the solution for the interest rate under the "majority rule" becomes\(^{11}\)

\[ i_t^M = \frac{1}{\varphi}z_t. \]  

(26)

Thus, under these assumptions the "majority rule" coincides with the "union rule".

The interest rate under the "consensus rule" is given by:

\[ i_t^C = \frac{1}{n} \sum_{j=1}^{n} \left( \frac{\beta_j}{\varphi_j}z_{j,t} + \frac{\beta_j}{\varphi_j}z_t \right) = \frac{1}{n} \sum_{j=1}^{n} \frac{\beta_j}{\varphi_j}z_t, \]  

(27)

Exploiting the limit in (23) again, our independence assumption implies that this expression can be reduced to:

\[ i_t^C = \frac{1}{n} \sum_{j=1}^{n} \frac{1}{\varphi_j}z_t = \frac{1}{\varphi - \bar{d}'}z_t, \]  

(28)

where \( \bar{d}' \) is defined in Section 4.

We are now able to solve for the cost of union participation for a given country under the assumptions of \( n \) being 'large' and \( \beta \) and \( \varphi \) being uncorrelated. Inserting equations (21), (24), (25), (26) and (28) into (15) and taking the expectations gives:

\[ \begin{align*}
ES_{j,t}^U &= \frac{1}{2}(\alpha^2 + \lambda)[\text{var}(z_{j,t}^i) + \frac{(\varphi_j - \beta_j\varphi)^2}{\varphi^2}\text{var}(z_t)], \\
ES_{j,t}^M &= \frac{1}{2}(\alpha^2 + \lambda)[\text{var}(z_{j,t}^i) + \frac{(\varphi_j - \beta_j(\varphi + d))^2}{(\varphi + d)^2}\text{var}(z_t)], \\
ES_{j,t}^B &= \frac{1}{2}(\alpha^2 + \lambda)[\text{var}(z_{j,t}^i) + \frac{(\varphi_j - \beta_j(\varphi - d'))^2}{(\varphi - d')^2}\text{var}(z_t)].
\end{align*} \]

\(^{11}\)See Appendix A.3 for a proof.
The variance of pure idiosyncratic shocks enters similarly under the alternative decision rules because the union central bank does not respond to pure idiosyncratic shocks, as these tend to average out. From (29)-(31), we also see that with symmetric elasticities across union members (i.e., \( \bar{d} = \bar{d}' = 0 \)), welfare is independent of the decision rules followed by the common central bank. Thus, asymmetric shocks alone do not create differences between the rules in terms of welfare.\(^{12}\)

By considering asymmetric transmission mechanisms and asymmetric shocks in combination, the results from Section 4 can be modified in an intuitive way. With common shocks, we saw that it is the elasticity of country \( j \) compared to the average of the union that determines which rule \( j \) prefers. Under asymmetric shocks, (29)-(31) demonstrate that it is country \( j \)'s ratio of elasticity to "systematic risk" (i.e., \( \varphi_j/\beta_j \)) compared to the average union elasticity that governs the preferred rule. A country where this ratio is equal to \( \varphi \) prefers the "union rule" or "majority rule". For instance, with idiosyncratic shocks, a country with \( \varphi_j > \varphi \) may still prefer the "union rule" or the "majority rule" if it is a "high-risk" country (\( \beta_j > 1 \)). By the same token, the "Benthamite rule" now favours countries with average interest rate responsiveness, given that they are less affected by common shocks than the union average. The reason is that the "Benthamite rule" is less activist than the "union rule" and the "majority rule", so the interest rate response will tend to be too weak for a country with average interest rate sensitivity, unless the country is less affected by common shocks than the union average, i.e., \( \beta_j < 1 \). The result for "consensus rule" is the opposite. Since this rule is more activist than the "union rule" and the "majority rule", the interest rate response tends to be too strong for a country with average interest rate sensitivity, unless the country is more affected by common shocks than the average, i.e., \( \beta_j > 1 \).

Let us finally illustrate the possible importance of interaction between asymmetries in shocks and transmission mechanisms by a simple extension of our numerical example from Section 4.2.\(^{13}\) We continue using \( \alpha = 0.05 \) and \( \lambda = 1 \). In addition, we assume that the standard deviation of all shocks \((g_{j,t}, g_t, u_{j,t}, u_t)\) is 0.02. To calibrate the \( \beta_j \)s, we use estimates of volatility and correlation of GDP growth rates among EU nations for the period 1979-96, taken from Table 3 in Fatás (1998). (Recall that our derivations above assumed equal \( \beta_j \)s for demand- and cost-push shocks.) For the nine nations included in our union this results in the \( \beta_j \)s reported in the third column of Table 2. We observe that Ireland has the lowest "systematic risk", whereas Finland is at the other end of the spectrum. Ireland had a particularly low correlation with other EU countries in the underlying data (see Fatás, 1998).\(^{17}\)

\(^{12}\)This is also noted by De Grauwe (2000).

\(^{13}\)Since the assumptions needed to have an explicit solution for the "majority rule" is not satisfied in this numerical example, the "majority rule" is disregarded here.
Table 2. Asymmetries on both transmission mechanisms and shocks: Percentage increase in loss for various decision rules compared to optimal independent policy

<table>
<thead>
<tr>
<th>Country</th>
<th>$\varphi_j$</th>
<th>$\beta_j$</th>
<th>$EL_j^U/L_j^*$</th>
<th>$EL_j^H/L_j^*$</th>
<th>$EL_j^C/L_j^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>0.12</td>
<td>1.10</td>
<td>65.3</td>
<td>69.4</td>
<td>59.4</td>
</tr>
<tr>
<td>Spain</td>
<td>0.14</td>
<td>1.12</td>
<td>61.0</td>
<td>65.4</td>
<td>54.9</td>
</tr>
<tr>
<td>France</td>
<td>0.20</td>
<td>0.93</td>
<td>56.1</td>
<td>59.0</td>
<td>54.1</td>
</tr>
<tr>
<td>Germany</td>
<td>0.20</td>
<td>0.73</td>
<td>65.6</td>
<td>66.3</td>
<td>67.8</td>
</tr>
<tr>
<td>Austria</td>
<td>0.25</td>
<td>0.96</td>
<td>52.4</td>
<td>54.0</td>
<td>54.1</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.32</td>
<td>0.98</td>
<td>53.2</td>
<td>51.2</td>
<td>63.1</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.32</td>
<td>0.61</td>
<td>97.8</td>
<td>85.3</td>
<td>127.9</td>
</tr>
<tr>
<td>Finland</td>
<td>0.44</td>
<td>1.58</td>
<td>29.0</td>
<td>30.0</td>
<td>34.5</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.45</td>
<td>0.99</td>
<td>73.2</td>
<td>59.6</td>
<td>108.4</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td>553.6</td>
<td>540.2</td>
<td>624.4</td>
</tr>
</tbody>
</table>

Notes: Calculations of $ES_j^h/L_j^*$ use equations (3) and (29)-(31). Estimates of $\varphi_j$ are from Ehrmann et al. (2003), while $\beta_j$ is calculated with data from Fatás (1998). Other parameters used are $\lambda = 1$, $\alpha = 0.05$, $\var(u_j^i, t) = \var(u_t) = \var(g_j^i, t) = \var(g_t) = 0.02^2$.  

1998), while Finland had a very volatile GDP path for the sample period in question.

Table 2 also report the expected loss under the "union-", "Benthamite-", and "compromise rule" measured relative to the loss under independent monetary policy. By comparing with Table 1 in section 4.2, we note the following: First, losses are generally significantly higher when shocks have idiosyncratic components; imperfect comovement is costly in a currency union. Second, for any given country the difference between the rules in table 2 is smaller than in table 1 (except for Ireland). The type of imperfect comovement introduced in this example thus tend to diminish the importance of differences in interest rate responsiveness. Third, of the three rules in table 2, all countries prefer the same rule as in table 1. Thus, again in this specific example, asymmetric shocks tend not to affect ranking of rules for different nations, although some countries (e.g. Germany) would now change order between their second- and third-best rule. Fourth, it is no longer the case that countries at the tails of the $\varphi_j$ distribution generally experience larger loss for any given rule. Finland is a particularly compelling case. It has the second highest interest rate elasticity, but with the lowest increase in loss relative to independence for all rules. A high $\beta_j$ means that union monetary policy will tend to be stabilizing (positive correlation of shocks), but also that the economy has big shocks. As the union central bank take only common shocks into account when it sets the interest rate (see discussion.

\footnote{We do not report for the "majority rule" as we do not have an explicit solution.}
(above) regardless of the employed rule, it is optimal for an economy with a high $\beta_j$ to be relatively responsive to monetary policy (compared to the rest of the union). Ireland is an example of a country that have very high losses for all rules, due to a unwarranted combination of high $\varphi_j$ and low $\beta_j$.

6 Summary and final remarks

We have examined four monetary policy decision rules in a monetary union where members have different interest rate elasticities. Our main findings are as follows. First, the "Benthamite rule" gives the least activist monetary policy, while the "consensus rule" implies the highest variability of the interest rate. The 'union' and 'majority' rules give variability somewhere in between.\footnote{This ranking of the 'majority rule' requires that the distribution of elasticities among the member states not be 'too skewed'.} Second, there are important welfare effects of these differences in interest rate decisions. Unambiguous insights can be reached when shocks are (largely) common in nature. Then, countries with an interest rate elasticity close to the union average (median) prefer the "union rule" ("majority rule"). Countries with elasticities (significantly) above the average are better off with the "Benthamite rule", whereas those with elasticities (significantly) below the average prefer the "consensus rule". In effect, the "Benthamite rule" gives more weight to the preferences of the most interest rate-sensitive member states, while the opposite is true for the "consensus rule". Our calibration exercise indicates that these welfare effects could be quantitatively important. Third, if, in addition, there are (important) asymmetric shocks, an individual country’s ratio of interest rate elasticity to "systematic risks" should be compared to the average union elasticity. Given this modification, the ranking of the different rules for a given country is the same as for purely common shocks.

The four types of decision rules considered here are somewhat stylized. In practice, actual interest decisions may not follow the formal description exactly, and they may have elements from more than one type of rule. Increased transparency among central banks may help identify how interest rate decisions are made in practice. This paper shows that the manner in which interest rate decisions in a monetary union are made has some important implications for the choice of interest rate and also sheds some light on which types of country would benefit from the various types of decision rule. In our "consensus rule", we assumed that each union member would not act strategically by reporting a false desired interest rate in order to affect the collective decision. A topic for future work is to consider how and when strategic behaviour among union members can affect the collective decision. In an extension of this, future research should also analyse institutional arrangements that can prevent such strategic behaviour.
A. Appendix

A.1 Derivation of equation (10)

Inserting equations (1) and (2) into the first-order condition (9) and solving for $i_t$ yields:

$$i_t^B = \frac{\sum_{j=1}^{n} \varphi_j}{\sum_{j=1}^{n} \varphi_j^2} [g_t + (\alpha^2 + \lambda)^{-1} u_t]$$

(32)

To be consistent with equation (10), we must have:

$$\frac{\sum_{j=1}^{n} \varphi_j}{\sum_{j=1}^{n} \varphi_j^2} = \frac{1}{\frac{1}{n} \sum_{j=1}^{n} \varphi_j + d}$$

(33)

Solving for $d$ gives:

$$d = \frac{n \sum_{j=1}^{n} \varphi_j^2 - (\sum_{j=1}^{n} \varphi_j)(\sum_{j=1}^{n} \varphi_j)}{n \sum_{j=1}^{n} \varphi_j} = \frac{\sum_{j=1}^{n-1} \sum_{h>j} \varphi_j \varphi_h}{\varphi_n^2}$$

(34)

A.2 Derivation of equation (13)

Inserting equation (5) into (12) gives:

$$i_t^C = \frac{1}{n} \sum_{j=1}^{n} \frac{1}{\varphi_j} [g_t + (\alpha^2 + \lambda)^{-1} u_t]$$

$$= \frac{\varphi_2 \varphi_3 \varphi_4 \ldots \varphi_n + \varphi_1 \varphi_3 \varphi_4 \ldots \varphi_n + \varphi_1 \varphi_2 \varphi_4 \ldots \varphi_n + \ldots + \varphi_1 \varphi_2 \varphi_3 \varphi_{n-1}}{n \prod_{j=1}^{n} \varphi_j} [g_t + (\alpha^2 + \lambda)^{-1} u_t]$$

(35)

To be consistent with (13), we must have:

$$\frac{\varphi_2 \varphi_3 \varphi_4 \ldots \varphi_n + \varphi_1 \varphi_3 \varphi_4 \ldots \varphi_n + \varphi_1 \varphi_2 \varphi_4 \ldots \varphi_n + \ldots + \varphi_1 \varphi_2 \varphi_3 \varphi_{n-1}}{n \prod_{j=1}^{n} \varphi_j} = \frac{1}{\frac{1}{n} \sum_{j=1}^{n} \varphi_j - d'}$$

(36)
Solving for \( d' \) yields:

\[
d' = \left( \frac{1}{n} \sum_{j=1}^{n} \varphi_j \right) \left( \varphi_2 \varphi_3 \varphi_4 \ldots \varphi_n + \varphi_1 \varphi_3 \varphi_4 \ldots \varphi_n + \varphi_1 \varphi_2 \varphi_4 \ldots \varphi_n + \ldots + \varphi_1 \varphi_2 \varphi_3 \varphi_{n-1} \right) - n \prod_{j=1}^{n} \varphi_j
\]

\[
= \frac{1}{n} (\varphi_1 - \varphi_2)^2 \varphi_3 \varphi_4 \ldots \varphi_n + (\varphi_1 - \varphi_3)^2 \varphi_2 \varphi_4 \ldots \varphi_n + (\varphi_1 - \varphi_4)^2 \varphi_2 \varphi_3 \varphi_4 \ldots \varphi_n + \ldots - n \prod_{j=1}^{n} \varphi_j
\]

\[
= \frac{1}{n} (\varphi_2 - \varphi_3)^2 \varphi_1 \varphi_4 \ldots \varphi_n + (\varphi_2 - \varphi_4)^2 \varphi_1 \varphi_3 \varphi_4 \ldots \varphi_n + (\varphi_2 - \varphi_5)^2 \varphi_1 \varphi_3 \varphi_4 \ldots \varphi_n + \ldots - n \prod_{j=1}^{n} \varphi_j
\]

\[
= \frac{1}{n} (\varphi_3 - \varphi_4)^2 \varphi_1 \varphi_2 \varphi_5 \ldots \varphi_n + (\varphi_3 - \varphi_5)^2 \varphi_1 \varphi_2 \varphi_4 \ldots \varphi_n + (\varphi_3 - \varphi_6)^2 \varphi_1 \varphi_2 \varphi_3 \varphi_4 \ldots \varphi_n + \ldots - n \prod_{j=1}^{n} \varphi_j
\]

\[
= \frac{1}{n} (\varphi_4 - \varphi_5)^2 \varphi_1 \varphi_2 \varphi_3 \varphi_6 \ldots \varphi_n + \ldots - n \prod_{j=1}^{n} \varphi_j
\]

\[
= \frac{1}{n} \left( \sum_{j=1}^{n-1} \sum_{h>j} ((\varphi_j - \varphi_h)^2 \prod_{k \neq j, h} \varphi_k) \right) - n \prod_{j=1}^{n} \varphi_j \sum_{j=1}^{n} \varphi_j^{-1}
\]

A.3 Proof of equation (26)

Each member’s desired interest rate is given by \( i_{j,t}^* = \frac{z_{j,t} + \beta_j z_t}{\varphi_j} \). \( z_{j,t}, \beta_j, \varphi_j \) are assumed to be independent and symmetrically distributed. Since the sum of two independent symmetric distribution is symmetric, we have that \( z_{j,t} + \beta_j z_t \) is symmetrically distributed around \( E_t(z_{j,t} + \beta_j z_t) = z_t \) (since \( E_t(z_{j,t}) = 0 \) and \( E_t(\beta_j) = 1 \)). Due to the symmetry, we know that the number of members characterized by \( z_{j,t} + \beta_j z_t = z_t + k_j \) is equal to the number of members characterized by \( z_{j,t} + \beta_j z_t = z_t - k_j \) for any \( k_j \). Likewise, the number of members characterized by \( \varphi_j = \varphi - l_j \) is equal to the number of members characterized by \( \varphi_j = \varphi + l_j \), where \( 0 < l_j < \varphi \). For each pair \((k_j, l_j)\) there are nine possible outcomes for \( \frac{z_{j,t} + \beta_j z_t}{\varphi_j} \) given by the following matrix:

\[
\begin{pmatrix}
\frac{z_t - k_j}{\varphi} & \frac{z_t + k_j}{\varphi}, & \frac{z_t + k_j}{\varphi} & \frac{z_t - k_j}{\varphi}, & \frac{z_t - k_j}{\varphi} & \frac{z_t + k_j}{\varphi}, & \frac{z_t + k_j}{\varphi} & \frac{z_t - k_j}{\varphi} & \frac{z_t - k_j}{\varphi}
\end{pmatrix}
\]

Among the elements outside the diagonal, there are as many cases above \( \frac{z_t}{\varphi} \) as there are cases below. In the diagonal, we have that if \( \frac{z_t - k_j}{\varphi - l_j} > \frac{z_t}{\varphi} \), then \( \frac{z_t + k_j}{\varphi + l_j} < \frac{z_t}{\varphi} \) and the other way around. Since this is true for any \((k_j, l_j)\), we have that \( \frac{z_t}{\varphi} \) must be the median of \( \frac{z_{j,t} + \beta_j z_t}{\varphi_j} \).
References


