

**A bioeconomic sheep-vegetation trade-off model.  
An analysis of the Nordic sheep farming system**

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**Abstract**

The paper studies the economy and ecology of sheep farming at the farm level where two different categories of the animals, ewes (adult females) and lambs, are included. The model is analyzed within a Nordic economic and biological setting with a crucial distinction between the outdoors grazing season and the indoors season. During the outdoors grazing season the animals face limiting grazing resources so that the weight gain of the lambs is determined by the per animal vegetation consumption. On the other hand, the number of grazing animals, lambs as well as ewes, determine the grazing pressure. This problem is studied under the assumptions of a rational and well-informed farmer that aims to maximize profit in ecological equilibrium with zero animal and vegetation growth.

Keywords: sheep farming, grazing, stage model, stocking problem

## 1. Introduction

In this paper, a bioeconomic sheep-vegetation trade-off model is analyzed. The main content of this trade-off is that high sheep densities yield high farm output in number of animals slaughtered. On the other hand, high sheep densities relative to pasture productivity are expected to cause a reduction in the per animal meat production and thus also reduced income per animal. This problem has similarities with the standard predator-prey renewable natural resource problem (see, e.g., Clark, 1990) where the sheep is the predator while the vegetation is the prey. However, whereas the standard predator-prey problem is formulated within a biomass framework, the different *age categories* of the sheep are central in the following analysis. The study is carried out with a crucial distinction between the outdoors grazing season (spring, summer and fall) and the indoors winter feeding period and where the lambs are born in late winter to early spring, just before the grazing season starts. This is the typical situation found in many strongly seasonal environments at northern latitudes such as in the Nordic countries as well as in areas at high altitudes in continental Europe (e.g., mountain areas in France and Spain). Sheep is the main livestock in animal husbandry in Norway, Iceland, The Faroe Islands and Greenland, and in both Norway and Iceland most of the cultivated land is used for winter fodder production (58% and 95% respectively, see e.g., Austrheim et al. 2008a ). As winter grazing is practiced in The Faroe Islands, the present analysis is essentially related to the economic and biological setting found in Norway, Iceland and Greenland.

Within this farming system, the individual farmer faces several decision problems. The problem to be analyzed here is, for a given farm capacity (i.e., size of the farm), to find the capacity utilization that gives the optimal number of animals to be fed and kept indoors during the winter season. A corollary of this problem is to find what effect the summer grazing sheep density has on the vegetation productivity and hence on the per animal meat production. The problem includes two categories, or stages, of the sheep population, lambs and ewes (adult females) and is analyzed as an equilibrium harvest problem with zero animal and vegetation growth under the assumption that the farmer aims to do it 'as good as possible', represented by current profit maximizing. Analyzing the dynamic problem where present value profit is maximized is hence left out of the present exposition. However, it is well known that the steady state of this problem coincides with our static problem except for the discount rent; that is, for zero discount rent these solutions are similar.

There is extensive literature on the economics of livestock management (see, e.g., Kennedy 1986 and Jarvis 1974), but most of this literature has little relevance for a farming system with a distinct seasonal subdivision between the winter indoors season and the outdoors grazing. The problem of the typical cow-calf operator in the western United States, however, has some similarities with the Nordic sheep farming system, but one crucial question here is to determine the length of the grazing season, in addition to determine the stocking level (see, e.g., Huffaker and Wilen 1991). In contrast to this, the length of the grazing season is fixed due to climate conditions in our problem. The spring lambing scheme is also taken for granted due to climatic conditions as well. The animal growth model builds on Skonhøft (2008), but is extended to take into account that the outdoors grazing conditions represent a constraint on the animal weight growth. How to balance the number of animals and the animal weight is indeed seen as a crucial management problem in the Nordic countries (e.g., Thomson et al. 2005, Olafsdóttir and Juliusson 2000, Mysterud and Austrheim 2005). The contribution of this paper is, from a theoretical point of view, to see how the stocking decision of the individual farmer and the balance between the number of animals and animal weight are influenced by various economic as well as ecological factors.

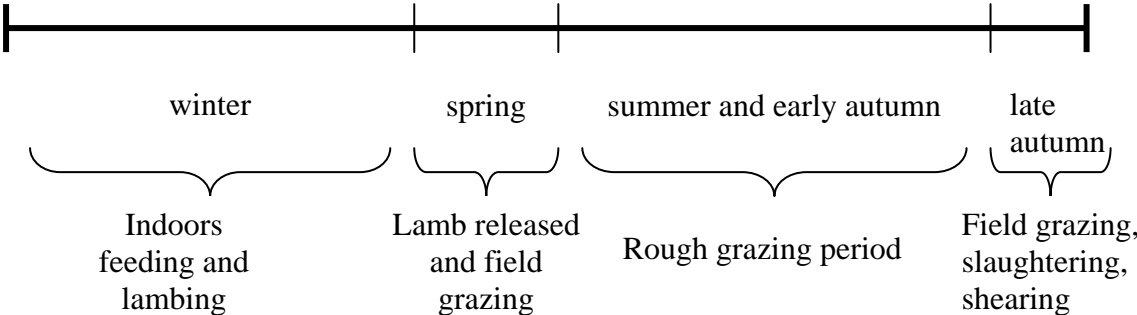
The paper is organized as follows. We first briefly present the Nordic sheep farming system in section two. Section three provides information about the sheep vegetation interaction and the simplified ecological model is presented. In section four, we take a closer look at this system in equilibrium with zero animal and vegetation growth. The revenue and cost functions are described in section five, and the stocking problem of the farmer is solved in section five under the assumption of current profit maximization. Section six provides a numerical illustration while section seven summarizes our findings.

## **2. The Nordic sheep farming system**

The following analysis is basically related to economic and ecological conditions found in Norway, but also in Iceland and Greenland. There are about 16,000 sheep farms in Norway. These are all family farms. As there are around 2.1 million animals during the outdoors grazing season, the average farm size therefore just counts somewhat about one hundred and thirty animals during the summer. Norwegian farms are either located close to mountain areas and other sparsely populated areas or along the coast with transport of the sheep to more distant alpine areas. The main product is meat, which accounts for about 80% of the average

farmer’s income .The rest comes from wool, as sheep milk production is virtually nonexistent today (Nersten et al. 2003). In Iceland there are about 450,000 winterfed animals today. The number has decreased significantly during the last few decades due to overgrazing. Meat is also the most important product from sheep farming here (Austrheim et al. 2008a). In Greenland the land for sheep grazing is much more restricted, and the population is estimated to count about some 25,000 ewes in 2007 (Austrheim et al. 2008a).

Housing and indoor feeding is required throughout the winter because of snow and harsh weather conditions (Figure 1). In Norway, winter-feeding typically consists of hay grown on pastures close to the farm (80%), with the addition of concentrate pellets provided by the industry (20%). The lambs are born during late winter to early spring, and in late spring and early summer the animals usually graze on fenced land close to the farm at low elevation, typically on the areas where winter food for the sheep is harvested during summer. When the weather conditions allow (linked to plant phenological development), the sheep are released into rough grazing areas in the valleys and mountains. In Norway, most sheep (about 75% of the total metabolic biomass) graze in the northern boreal and alpine region (Austrheim et al. 2008b). The outdoors grazing season in mountain areas ends around late August to the middle of September and do normally not exceed 130 days. The animals are then gathered, the wool is cut and slaughtering takes place or the animals spend some time grazing on the farmland before being sent to slaughtering (more details in Austrheim et al. 2008a). The seasonal subdivision is more or less the same in Iceland and Greenland. During the summer rough grazing period, the flocks may be vulnerable to accidents and to disease, and in some regions also to large predators. Aunsmo et al. (1998) and Nersten et al. (2003) provide more details.



**Figure 1: Seasonal subdivision Nordic sheep farming system**

### 3. Ecological model

There is a dynamic relationship between large herbivores and the plants that they forage (e.g., Hobbs 1996, Augustine and McNaughton 1998, Danell et al. 2006) since grazing affect the quantity and quality of vegetation which in turn affect the performance of the herbivores (Choquetot 1991, Simard et al. 2008). Removal of plant tissue affects individual fitness (e.g., plant growth) directly, and may cause biomass reduction of preferred plant species (i.e., fodder plant; Bråthen and Oksanen 2001, Eskelinen and Oksanen 2006). Indirect effects, operating by changing the competitive balance with other species, may be even more important for the vegetation community development. In particular, in low productive ecosystems as the one considered, heavy grazing may favor heavily defended, non-palatable plant species to the detriment of palatable species (Austrheim et al. 2007). Invasion of such species will affect strength of density dependent effects on the weight growth of the sheep in the long term. The farmer may thus increase the stock today at the cost of reduced growth in subsequent years. Indeed, with increasing density of sheep on pasture, a higher proportion of low quality plants (Kausrud et al. 2006) and vegetation types (Mobæk et al. 2008) may result. In ruminants, even slight changes in plant quality can lower body growth substantially since it causes fewer nutrients per bite as well as increases rumination time (White 1983). However, moderate grazing is expected to facilitate plant biomass production in productive habitats and thus the fodder availability at moderate grazing as compared to no grazing (McNaughton 1979).

In the simplified sheep-vegetation model to be formulated, we assume just a single plant species, or composite homogenous vegetation, expressed as vegetation quantity, and measured in tonne vegetation biomass. This composite vegetation biomass is consumed by the sheep during the outdoors grazing season while it regenerates through a natural growth process. The model is formulated in a discrete time with a seasonal subdivision between the outdoors grazing period (spring, summer and fall) and indoors winter feeding period (Figure 1). The sheep population is structured (e.g., Caswell 2001) as ewes (adult females), and young females and males (lambs). As already indicated, the lambs are born in late winter to early spring, just before the grazing season starts. Lambs not slaughtered enter the adult population after the slaughtering period (i.e., September–October). All male lambs are assumed slaughtered since only very few (or none when artificial insemination is practiced) are kept for

breeding. Therefore, only female adults are considered. Fertility is assumed fixed, which is a reasonable assumption as farmers give extra feed to buffer environmental effects (e.g., due to a poor year; high density relative to food resources in the pasture). Natural mortality differs between adults and lambs and is considered fixed and density independent as well. All natural mortality is assumed to occur during the grazing season. Demographic data on sheep are available in Mysterud et al. (2002).

The number of adult females in year  $(t + 1)$  after slaughtering is made up of the previous year's adults surviving natural mortality and not slaughtered, and the female lambs surviving natural mortality and not slaughtered. This is written as  $X_{t+1} = Y_t s_Y (1 - h_{Y,t}) + X_t s_X (1 - h_{X,t})$ , where  $Y_t$  is the number of female lambs,  $s_X$  and  $s_Y$  are the natural survival fractions of adult females and lambs, respectively, and  $h_{X,t}$  and  $h_{Y,t}$  are the fractions slaughtered. With the fecundity rate  $b$  (lambs per adult female) and  $\psi$  as the fraction of female lambs being recruited ( $\psi$  is usually close to 0.5),  $Y_t = \psi b X_t$  yields the number of female lambs. Therefore, when ignoring the possibility of adding new animals from outside, the ewe population growth is governed by:

$$(1) \quad X_{t+1} = \psi b X_t s_Y (1 - h_{Y,t}) + X_t s_X (1 - h_{X,t}).$$

Vegetation growth is made up of natural growth and consumption by the grazing sheep and follows the Noy-Meir (1975) model where the *per animal* vegetation consumption increases with the vegetation availability. It is assumed that the *number* of grazing animals influences the vegetation consumption while the consumption, in a next step, determines the weight gain of the animals during the grazing season. See also, e.g., Huffaker and Wilen (1991). While the food intake of the ewes may be above that of the lambs, it is for simplicity supposed that all animals influence the vegetation consumption in a similar manner. In addition to consumption, the vegetation regenerates through a natural growth process represented by a one-peaked value function. The vegetation growth may then be written as<sup>1</sup>:

$$(2) \quad V_{t+1} - V_t = f(V_t) - g(V_t)(1 + b)X_t$$

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<sup>1</sup> The vegetation consumption (or grazing pressure) is given by the number of animals in the *beginning* of the grazing season. An average over the season possible describes the actual grazing pressure better, but comes at the cost of considerable notational clutter without altering the qualitative aspect of the model. How to decide on this, and similar questions, is an inherent problem of time discrete models.

and where  $g(V_t)$  is the sheep per capita consumption function and  $f(V_t)$  yields the natural growth function. In the numerical analysis as well as in the theoretical reasoning, we are thinking of a consumption function specified as  $g(V_t) = kV_t/(V_t + c)$ , where  $k > 0$  is the maximum vegetation biomass intake per animal and  $c > 0$  determines the shape of the consumption pattern. Natural growth is described by the logistic function

$f(V_t) = rV_t(1 - V_t/Q)$ , with  $r > 0$  as the maximum specific vegetation growth rate (vegetation productivity) and  $Q > 0$  as the carrying capacity<sup>2</sup>.

The weight gain of the lambs during the grazing season coincides with the weight at the end of the season: that is, the slaughter weight (kg per animal). It is assumed to be proportional to per animal vegetation consumption:

$$(3) \quad w_{Y,t} = qg(V_t)$$

so that the parameter  $0 < q < 1$  translates grazing biomass into meat biomass. For the adults there is generally no weight change during the grazing season (Mysterud and Austrheim 2005). For this category we thus simply neglect any possible connection between the amount of vegetation and weight, and the adult slaughter weight is fixed and determined outside the model:

$$(4) \quad w_{X,t} = w_X.$$

In line with reality, and irrespective of the grazing conditions,  $w_X > qg(V_t)$  is assumed to hold (see also numerical section). Equation (3) and (4) complete the ecological model.

#### 4. Ecological equilibrium

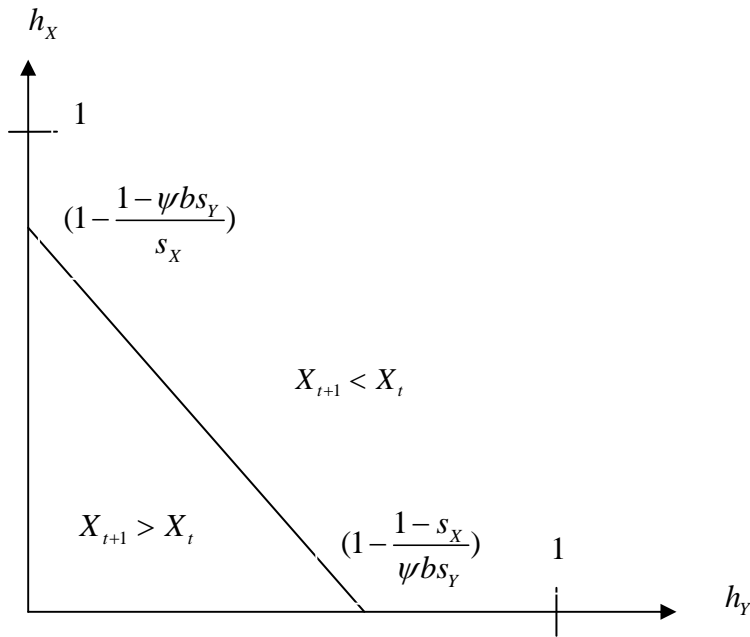
As mentioned, the stocking decision of the farmer is analyzed in ecological equilibrium, i.e., when vegetation and animal growth equalize zero. As the population growth equation (1) is linear for number of animals, there will be infinite combinations of harvesting fractions that sustain a stable population. Therefore, for a constant number of animals  $X_{t+1} = X_t = X$ , we have:

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<sup>2</sup> In high productive vegetation areas in Norway, it has been observed that the vegetation quantity may increase between years when the sheep grazing pressure is low (Mysterud and Austrheim 2005) This may be interpreted as if the vegetation growth function shifts up between years. This possible effect is not taken into account in the present analysis. Our model and reasoning hence fit best to low productive areas and/or areas where the grazing pressure is moderate or high.

$$(1') \quad X = \psi b X s_Y (1 - h_Y) + X s_X (1 - h_X),$$

or simply  $1 = \psi b s_Y (1 - h_Y) + s_X (1 - h_X)$  when  $X > 0$  (see Figure 2). It intersects with the  $h_X$ -axis at  $[1 - (1 - \psi b s_Y) / s_X]$ , which may be above or below one. Therefore, the highest adult slaughtering rate compatible with zero animal growth is  $\min\{1, [1 - (1 - \psi b s_Y) / s_X]\}$ . For all realistic parameter values, it is below one (see numerical section) and this is assumed to hold in the subsequent analysis. It intersects with the  $h_Y$ -axis at  $[1 - (1 - s_X) / \psi b s_Y] < 1$  and is hence the highest lamb slaughtering rate compatible with equilibrium.



**Figure 2: Equilibrium (constant animal population) harvesting relationship (Eq. 1').**  
 $h_Y$  female lamb slaughtering fraction,  $h_X$  ewe (adult female) slaughtering fraction.

The equilibrium vegetation growth condition  $V_{t+1} = V_t = V$  next yields:

$$(2') \quad f(V) = g(V)(1 + b)X.$$

Depending on the slopes of the natural growth function  $f(V) = rV(1 - V/Q)$  and the sheep consumption curve  $g(V)(1 + b)X = (kV/(V + c))(1 + b)X$  (see above), there may be one equilibrium or two equilibria (see also Noy-Meir 1975). A necessary and sufficient condition for a unique equilibrium is that the consumption curve intersects the natural growth function from below and where more animals, *ceteris paribus*, means less vegetation biomass. In the opposite case there are two interior equilibria. However, the lower vegetation level equilibrium, for a given number of animals, is for obvious reasons not stable, and is not

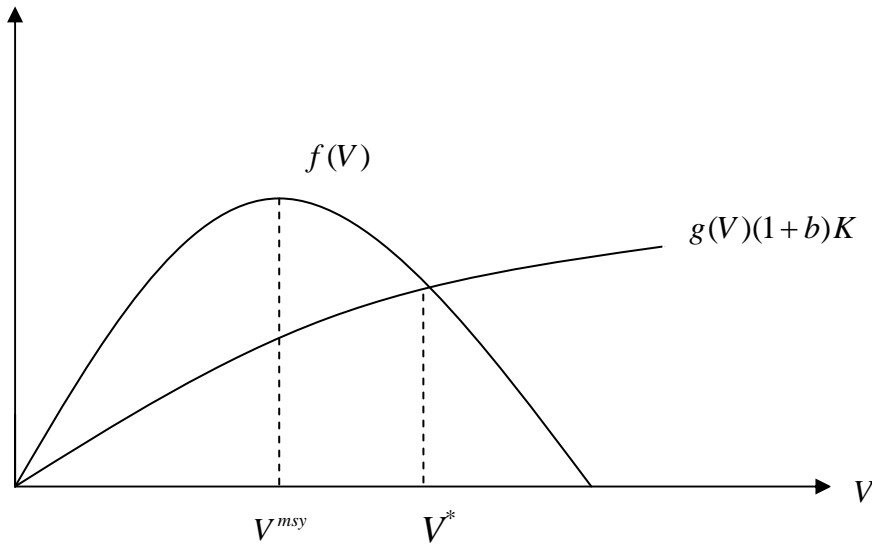
considered. Therefore, in what follows, these functions are scaled such that the consumption curve intersects the natural vegetation growth curve from below; that is,

$f'(V) < g'(V)(1+b)X$  holds at the unique (interior) equilibrium (cf. Figure 3). For the given

specific functional forms, the sheep-vegetation equilibrium reads

$r(1-V/Q) = [k/(V+c)](1+b)X$ , and is defined for  $0 < X < rc/k(1+b)$  and  $0 < V < Q$ .

Within these intervals, the vegetation quantity is hence a decreasing function of the stocking rate.



**Figure 3: The natural vegetation growth-consumption relationship with an unique vegetation equilibrium. Equilibrium indicated when binding farm capacity,  $X^* = K$**

## 5. Revenue and Costs

We are neglecting any income from wool production and meat sale is the only revenue component of the farmer. Because slaughtering takes place after natural mortality, the number of ewes and female lambs removed are  $X_t s_x h_{x,t}$  and  $\psi b X_t s_y h_{y,t}$ , respectively. As mentioned, the entire male lamb subpopulation  $(1-\psi)b X_t s_y$  is slaughtered. The number of animals removed is then  $H_t = b X_t s_y (\psi h_{y,t} + 1 - \psi) + X_t s_x h_{x,t}$ . With  $p_x$  as the net (net of slaughtering costs) ewe slaughtering price (NOK per kg) and  $p_y$  the lamb slaughtering price, both assumed

to be fixed and independent of the number of animals supplied at the farm level, the current meat income of the farmer reads  $R_t = [p_Y w_{Y,t} b X_t s_Y (\psi h_{Y,t} + 1 - \psi) + p_X w_{X,t} X_t s_X h_{X,t}]$ .

The cost structure differs sharply between the outdoors grazing season and the indoors feeding season, and the indoors costs are substantial higher. Throughout this analysis, we assume a given farm capacity. Therefore, the costs of buildings, machinery and so forth, are fixed (see also below). The indoor season variable costs include labor cost (typically as an opportunity cost), electricity, and veterinarian costs, in addition to fodder and vary with the given length of the indoors season (section one). The cost is assumed to increase linearly with the size of the winter population,  $C_t = \alpha X_t$  with  $\alpha > 0$ .

As indicated, during the grazing period the sheep may graze on communally owned lands ('commons'), or private land. Within the Nordic sheep farming system, such land may be available cost free, or the farmer may pay a *fixed* yearly rental (Asheim 2007). There may be some transportation and maintenance costs, but such costs are neglected since they most often are rather low. The total yearly variable cost is hence simply assumed to be the indoors season cost. Therefore, when ignoring discounting within the year, the current profit of the farmer is:

$$(5) \quad \pi_t = R_t - C_t = p_Y q g(V_t) b X_t s_Y (\psi h_{Y,t} + 1 - \psi) + p_X w_X X_t s_X h_{X,t} - \alpha X_t$$

when inserting for equations (3) and (4).

As mentioned, the farm capacity is assumed given. However, while not allowing for investment in farm capacity, the capacity will be taken into account as a possible constraint<sup>3</sup>.

The capacity is related to the number of animals kept during the winter and reads:

$$(6) \quad X_t \leq K.$$

The capacity may be binding, or not. When binding, we obviously find that the current profit of the last animal to be kept during the winter is positive. However, when not binding, the marginal profit of the last animal will be positive as well. This is due to the shadow cost of vegetation consumption. The subsequent analysis will explain this more detailed.

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<sup>3</sup> The problem of also allowing for physical capital accumulation and changing farm capacity is progressively more difficult to analyse because one has to account for irreversibility (see the pioneering work of Clark et al. 1979 in a fishery context).

## 6. The Optimal Sheep-Vegetation Trade off

The farmer is assumed to be ‘rational’ and well-informed with the goal of maximizing current profit (5) subject to the animal equilibrium condition (1’) and the vegetation equilibrium condition (2’), together with the farm capacity constraint (6) (see also introductory section).

When omitting the time subscript, the Lagrangean of this problem reads

$$L = p_Y q g(V) b X s_Y (\psi h_Y + 1 - \psi) + p_X w_X X s_X h_X - \alpha X$$

$$- \lambda [X - \psi b X s_Y (1 - h_Y) - X s_X (1 - h_X)] - \mu [g(V)(1 + b)X - f(V)] - \eta (X - K) \text{ where } \lambda > 0 \text{ is}$$

the animal resource shadow price,  $\mu > 0$  is the vegetation resource shadow price and  $\eta \geq 0$  is the farm capacity constraint shadow price. The first order conditions of this problem with  $X > 0$  and  $V > 0$  and both harvest mortalities below one (see above) are:

$$(7) \quad \partial L / \partial h_Y = p_Y q g(V) - \lambda \leq 0; \quad 0 \leq h_Y < 1,$$

$$(8) \quad \partial L / \partial h_X = p_X w_X - \lambda \leq 0; \quad 0 \leq h_X < 1,$$

$$(9) \quad \partial L / \partial X = p_Y q g(V) b s_Y (\psi h_Y + 1 - \psi) + p_X w_X s_X h_X - \alpha - \mu g(V)(1 + b) - \eta = 0$$

and

$$(10) \quad \partial L / \partial V = p_Y q g'(V) b X s_Y (\psi h_Y + 1 - \psi) + \mu [f'(V) - g'(V)(1 + b)X] = 0.$$

The interpretation of control condition (7) is that lamb slaughtering should take place up to the point where the marginal meat income (NOK per animal) is equal to, or below, the animal resource shadow price. Following the Kuhn-Tucker theorem, it holds as an equation when the removal of this subpopulation is optimal. The adult control condition (8) has the same interpretation. The animal stock equation (9) states that the number of ewes (adult females) should be maintained so that the value of one more animal on the margin equalizes the marginal cost of doing so, plus the marginal grazing cost evaluated at the shadow price and the shadow price of the farm capacity constraint. Finally, the vegetation condition (10) states that the marginal benefit of more vegetation through higher lamb weight should equalize its marginal cost given by the difference between the marginal vegetation growth and marginal consumption, evaluated at its shadow price. As the vegetation consumption curve intersects with the natural growth function from below (see above), the condition for a stable vegetation equilibrium (for a given number of animals) implies a positive vegetation shadow price,  $\mu > 0$ . Therefore, as expected, the profit of adding one more animal to the stock (Eq. 9) is always strictly positive in an optimal program.

One striking point with the solution of the model is that the control conditions (7) and (8) cannot generally be satisfied simultaneously as equations. Due to demand conditions, the meat price (NOK per kg) is higher for the lambs,  $p_Y > p_X$ . On the other hand, the per animal weight is higher for the ewes. However, the meat price difference dominates the weight difference, and the lamb slaughter price (NOK per animal) is above that of the ewe slaughter price (more details in the numerical section).  $p_Y qg(V) > p_X w_X$  is therefore assumed to hold for all possible vegetation quantities. As a consequence and because slaughtering mortalities are below one, condition (7) must hold as an equation while (8) holds as an inequality. Lamb only slaughtering is hence optimal, and the animal shadow price is given by  $\lambda^* = p_Y qg(V^*)$  (superscript  $*$  indicates optimal values). A corollary of this result is that lamb slaughtering should take place at the highest level compatible with the sheep population equilibrium, cf. equation (1') and Figure 2.

An important result of this equilibrium stocking problem hence boils down to a simple principle, and one-stage slaughtering only results because the harvest benefit is linear in both harvest controls. This result has similarities with the well-known finding of Reed (1980) who studied the maximum sustainable yield problem of a fishery. On the other hand, the reason for slaughtering at the highest level compatible with ecological equilibrium follows from the lack of any density dependent effects in the animal growth equation (1). The fact that there is an animal-vegetation interaction and that vegetation growth is density dependent does not affect this. At the same time, this means that the optimal slaughter rate, in contrast to the result in most bioeconomic models, depends on biological conditions (fertility and mortality) only.

Therefore, the optimal equilibrium slaughtering rates are  $h_X^* = 0$  and  $h_Y^* = 1 - (1 - s_X) / \psi b s_Y$ .

This is stated as:

*Result 1. Slaughtering is contingent upon the per animal meat value only. Lamb slaughtering only is optimal, and slaughtering should take place at the highest level compatible with population equilibrium determined by (sheep) biological factors alone.*

The stock conditions (9) and (10) are next considered, and where we make a distinction between two cases; a binding and non-binding farm capacity constraint. Suppose first that the capacity constraint (6) is binds; that is,  $X^* = K$  and  $\eta^* > 0$ . This may intuitively happen if the capacity is small, the farm profitability is high (e.g., a high lamb meat price), or both (see also

below). The vegetation utilization is then determined through the equilibrium condition (2') as  $f(V^*) = g(V^*)(1+b)K$ . Therefore, neither economic factors nor sheep biological factors, except fertility, influence the vegetation quantity  $V^*$ . Not surprisingly, a higher farm capacity, when binding, and more animals, means reduced  $V^*$ .

The optimal number of removed animals, consisting of lambs only (female and male), is found through  $H^* = bX^*s_Y(\psi h_Y^* + 1 - \psi) + X^*s_X h_X^* = K(bs_Y - (1 - s_X))$ , and is determined by farm capacity and sheep biological factors alone. Furthermore, in this case when the farm capacity binds, the capacity directly determines the lamb slaughter weight through the vegetation equilibrium condition,  $w_Y^* = qg(V^*)$ . The sheep-vegetation trade-off is then straightforward as higher farm capacity and more animals reduce the equilibrium vegetation quantity and hence the per animal slaughter weight. On the other hand, the capacity effect on farm output (in kg meat)  $w_Y^*H^* = qg(V^*)K(bs_Y - (1 - s_X))$  seems ambiguous. However, by taking the total differential,  $d(w_Y^*H^*) = q[bs_Y - (1 - s_X)](g'KdV^* + gdK)$ , and next combine with the shifting vegetation equilibrium condition  $f'dV^* = (1+b)(g'KdV^* + gdK)$ , we find  $d(w_Y^*H^*) = q[bs_Y - (1 - s_X)][f'/(1+b)]dV^*$ . Therefore, higher capacity and hence lower vegetation quantity mean higher output if the vegetation consumption curve intersects the vegetation natural growth function to the right hand side of its peak value,  $f' < 0$  and  $V^* > V^{msy}$ . In the opposite case, output, and hence farm revenue

$R = p_Y qg(V^*)K(bs_Y - (1 - s_X))$  reduce with a higher capacity  $K$ . As the cost  $\alpha K$  at the same time increases, the farm profit clearly decreases as well. However, because higher farm capacity, if binding, implies higher current profit following the logic of the optimization, it is evident that farm capacity cannot bind in this last case. The optimal solution is therefore characterized by  $V^* > V^{msy}$ . This is stated as:

*Result 2. Higher farm capacity, when binding, reduces per animal slaughter weight, but increases farm output.*

We next consider the situation with high farm capacity, low profitability, or both, so that the capacity no longer binds; that is,  $X^* < K$  and  $\eta^* = 0$ . The first order condition (9), as already mentioned, indicates higher profit on the margin. The marginal animal hence also contributes to increased revenue and farm output. Therefore, as explained above, the stocking rate can

never exceed a size coexisting with  $V^* < V^{msy}$ . If a vegetation quantity lower than  $V^{msy}$  is defined as ‘overgrazing’ (but see Myrsetrud 2006), we may state:

*Result 3. Irrespective of ecological and economic conditions, it is never beneficial for the farmer to keep an animal stock overgrazing the pasture.*

This result can also be demonstrated as follows. If we first insert the optimal slaughter rates  $h_X^* = 0$  and  $h_Y^* = 1 - (1 - s_X) / \psi b s_Y$  into the profit function (5), it reads

$\pi = p_Y q [b s_Y - (1 - s_X)] g(V) X - \alpha X$ . When next replacing  $X$  by using the vegetation equilibrium condition (2') and differentiation, we find after some small rearrangements  $d\pi / dV = [1 / (1 + b)] \{ p_Y q [b s_Y - (1 - s_X)] f' - (\alpha / g^2) (f' g - f g') \}$ .

Therefore,  $d\pi / dV = 0$  is characterized by

$\{ p_Y q [b s_Y - (1 - s_X)] g(V^*) - \alpha \} f'(V^*) g(V^*) = -\alpha f(V^*) g'(V^*)$ . Because  $g'(V) > 0$ , this condition holds only when  $f'(V^*) < 0$ , or  $V^* > V^{msy}$ . Therefore, as stated above, a stocking rate overgrazing the pasture is not economically beneficial for the farmer.

When  $\alpha$  becomes small and negligible, it is also seen that profit maximizing implies  $f'(V^*) = 0$  and  $V^* = V^{msy}$ . Given the specific vegetation natural growth function  $f(V) = rV(1 - V/Q)$  (see above),  $V^* = Q/2$  hence yields the minimum optimal vegetation quantity. When inserted into the vegetation equilibrium condition (2')  $rV(1 - V/Q) = [kV / (V + c)](1 + b)X$ , we find  $X^* = r(Q + 2c) / 4k(1 + b)$ . Therefore, for these specific functional forms, this number yields the highest possible stocking rate under the present assumption of a well-informed, profit-maximizing farmer. Note that no economic parameters are included here, and the fertility parameter  $b$  is the only sheep biological parameter included.

From the above analysis we may also suspect to find  $\partial V^* / \partial \alpha > 0$ , or equivalently,  $\partial V^* / \partial p_Y < 0$ . Not surprisingly, it can be shown that these results hold due to the second order condition for maximum<sup>4</sup>. As  $\partial V^* / \partial p_Y < 0$  implies  $\partial X^* / \partial p_Y > 0$  through the vegetation equilibrium condition (2'), we may also state:

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<sup>4</sup> Therefore, we find  $\partial V^* / \partial p_Y < 0$  suggested that  $\partial^2 \pi / \partial V^2 < 0$  holds when  $d\pi / dV = 0$ .

*Result 4. A higher slaughter price yields a larger flock size and lower vegetation quantity.*

This result contrasts standard bioeconomic harvesting theory (Clark 1990). The working of the price effect here is, however, different from the standard model as there is no stock dependent harvesting, or slaughter, cost included. Therefore, a higher price  $p_Y$  simply means that it becomes relatively less expensive to keep animals during the indoors feeding season, which motivates the farmer to increase the number of animals. Because the fraction of animals slaughtered  $h_Y^*$  is determined by biological factors alone, we also find that it is beneficial for the farmer to increase the number of slaughtered animals, and increase the farm output (in kg) for a higher lamb meat price. As more animals added and a higher pasture utilization means a lower per animal slaughter weight (see above), increased meat supply is hence met through a higher removal of animals dominating the reduced per animal (lamb) slaughter weight.

It may also be of interest to assess how vegetation productivity affects the stocking decision and pasture utilization. Under the present specification of the vegetation natural growth function,  $f(V) = rV(1 - V/Q)$ , the intrinsic growth rate parameter  $r$  steers the productivity. For this specific functional form, however, we find that it does not influence the optimal vegetation utilization. This is observed by studying the above expression

$d\pi/dV = [1/(1+b)]\{p_Y q[bs_Y - (1-s_X)]f' - (\alpha/g^2)(f'g - fg')\}$  where  $r$  is left out when characterizing  $d\pi/dV = 0$  because  $f'$ , as well as  $f$ , include this productivity parameter as a multiplicative term. On the other hand, through the vegetation equilibrium

condition  $f(V^*) = g(V^*)(1+b)X^*$ , or  $rV^*(1 - V^*/Q) = [(k/(V^* + c))(1+b)X^*$ , we find that higher productivity yields more animals. Therefore, again following the logic of our rational and well-informed farmer, different pasture productivity *ceteris paribus* translates into unchanged pasture utilization, but different farm sizes in number of animals. This is stated as:

*Result 5. With higher vegetation productivity, it is beneficial for the farmer to keep more animals when farm capacity constraint is unbinding. When the vegetation natural growth is described by a logistic growth function, higher vegetation productivity does not influence the optimal pasture utilization.*

The above result, as well as other results, can be confirmed more directly by inserting the specific functional forms of the vegetation consumption curve and the natural growth function

into the above expression  $d\pi/dV = 0$ . When solving for the vegetation quantity, we find

$$V^* = (Q/2) \frac{p_Y q k [b s_Y - (1 - s_X)] - \alpha (1 - c/Q)}{p_Y q k [b s_Y - (1 - s_X)] - \alpha}$$

. When next inserting into the vegetation equilibrium condition (2'), it is also possible to find an explicit expression for the sheep stocking rate as well.

## 7. Numerical Illustration

### *Data*

To shed some further light on the above analysis, the model is illustrated numerically where the above specified vegetation natural growth function and animal consumption curve are applied. Only simulations where the farm capacity is unbinding are reported. We consider a rather large sized farm (cf. introductory section), located in an area with about average vegetation productivity. The baseline parameter values, where sheep ecological data and economic values are related to Norwegian conditions, are shown in the Table A1 (Appendix). The size of the farm is scaled by the vegetation carrying capacity  $Q$ . With  $Q = 500$  (tonne vegetation biomass) we find  $V^{msy} = Q/2 = 250$ . Accordingly, for the baseline parameter values (Table A1), the highest possible stocking rate (and winter population size) is  $X^* = r(Q + 2c)/4k(1 + b) = 108$  (ewes). We contrast this farm with a farm located in an area with significant lower vegetation productivity, captured by different vegetation intrinsic growth rate. We also study the effects of changes in prices and costs as well as sheep biological factors.

### *Results*

In the results reported (Table 1), the lamb harvesting rate (fraction) is all the time  $h_Y^* = 0.93$ , determined by (sheep) biological parameters alone (Result 1). In the baseline scenario (row one), the optimal ewe flock size is 102 animals (ewes). The lamb slaughter weight is somewhat above 23 (kg) while the number of animals slaughtered (lambs) is 135. The vegetation quantity is 278 (tonne), slightly above that of the maximum sustainable yield value  $V^{msy} = 250$ . The lamb value (NOK per lamb) becomes  $p_Y w_Y^* = p_Y q g(V^*) = 1,153$ , compared to the ewe fixed value of  $p_X w_X = 35 \cdot 30 = 1,050$  (Table A1, Appendix).

**Table 1: Stocking rate  $X^*$  (number of ewes),  $V^*$  vegetation (tonne),  $H^*$  slaughtering (number of lambs),  $w_Y$  slaughter weight (kg/animal), and  $\pi^*$  profit (in NOK). Farm capacity not binding.**

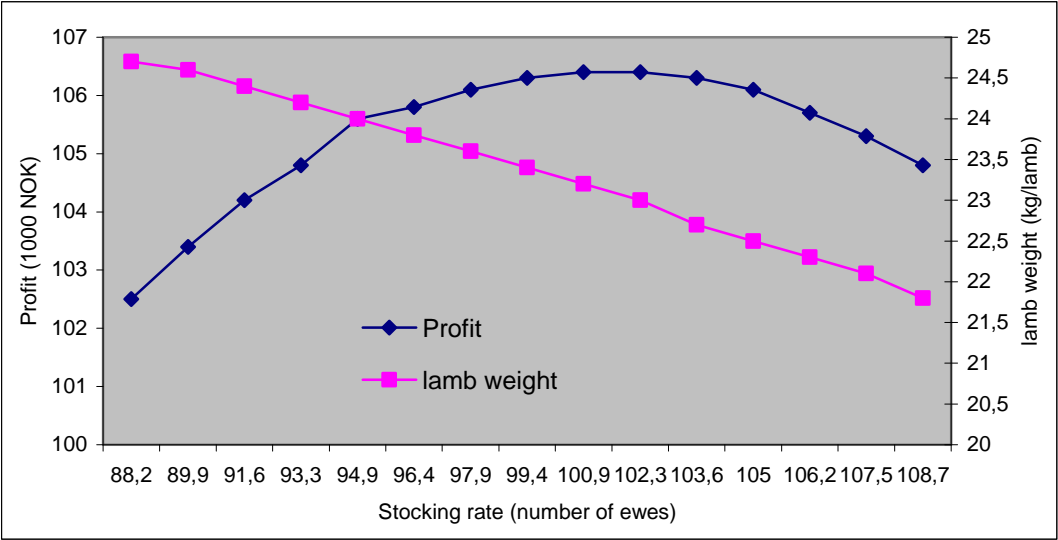
	$X^*$	$V^*$	$H^*$	$w_Y$	$\pi^*$
Baseline	102	278	135	23.1	106,400
50 % price increase ( $p_Y = 75$ )	104	267	140	22.5	185,200
50 % cost increase ( $\alpha = 750$ )	96	296	129	23.8	81,700
33 % reduction vegetation productivity ( $r = 0.33$ )	67	278	90	22.9	70,200

When the slaughter price shifts up while keeping all other parameters fixed (row two), the stocking rate increases while the vegetation quantity falls (Result 4). Increased indoors feeding cost (line three) works in the opposite manner. The numerical simulations also confirm (not shown in the table) that the hypothetical case of small and negligible feeding cost yields a stocking rate consistent with the highest possible vegetation natural growth (Result 3). The effects of changing sheep biological factors are studied as well, and higher fertility and reduced mortality increase the lamb harvesting rate and number of animals slaughtered while the size of the optimal flock decreases (not reported).

Table 1 (last row) also illustrates the effects of a shift in the vegetation productivity (intrinsic growth rate) parameter. The stocking rate  $X^*$  becomes lower when the vegetation productivity shifts down while the vegetation quantity stays unchanged compared to the baseline scenario (Result 5). For the given specific functional forms, we also find that the stocking rate increases linearly with an increasing vegetation productivity. This is recognized when writing the vegetation equilibrium condition as  $X^* = [(V^* + c)(1 - V^*/Q)/k(1 + b)]r$  (see section five) which for the baseline parameter values (except  $r$ ) yields

$X^* = [(278 + 300)(1 - 278/500)/0.50(1 + 1.53)]r = 202.9r$  (animals). Accordingly, we find  $X^* = 67$  when  $r = 0.33$  while the baseline value  $r = 0.50$  yields  $X^* = 102$  (first line Table 1).

A striking point with these calculations, are the modest changes in stocking rate, number of animals slaughtered and vegetation utilization due to shifting economic conditions. This picture is confirmed when also applying other economic parameter values. Therefore, price and cost variations more or less spill over to profitability changes only (last column Table 1). This indicates that the profit function  $\pi^* = p_Y q [b s_Y - (1 - s_X)] g(V^*) X^* - \alpha X^*$  (section six above) is only weakly non-linear (at least in the actual range of parameter values) in  $p_Y$  and  $\alpha$ . Figure 4 demonstrates this aspect of our model in another way where profit is depicted for different stocking values under the baseline parameter scenario. The optimal ewe stocking rate is  $X^* = 102$  (Table 1), but the figure demonstrates a rather flat profitability curve in the neighborhood of this optimum. For example, 90 animals instead of the optimal one of 102, reduces the profit from (NOK) 106,400 (Table 1) to just 103,200 (left hand scale). At the same time, the number of animals slaughtered changes from 135 to 121. This animal output reduction is hence counterbalanced by a higher, albeit quite modest, per animal (lamb) slaughter weight (right hand scale Figure 4).



**Figure 4: Variations stocking rate. Baseline parameter values. Profitability (left hand scale) and per lamb productivity (right hand scale).**

Figure 4 illustrates at the same time the basic sheep-vegetation trade-off taking place in our ecological-economic system without a binding farm capacity and when the slaughtering policy is fixed by the difference in the per animal value. For an initial low stocking rate, expansion and more animals added is beneficial for the farmer as increased production in

number of animals slaughtered (lambs) more than outweighs reduced productivity in weight per slaughtered animal, together with additional winter fodder costs. However, when expanding the farm size further above the optimal stocking rate, reduced vegetation quantity translating into a still lower weight per slaughtered animal together with still higher winter fodder costs, dominate the additional income gain from more animals slaughtered.

## **8. Concluding Remarks**

This paper has analyzed the economics of sheep farming in a two-stage model of lambs and adult females (ewes). The analysis is at the farm level in a Nordic context with a crucial distinction between the outdoors grazing season and the winter indoors feeding season, and where a Noy-Meir (1975) type model describes the animal vegetation interaction. The farmer is assumed to be 'rational' and well-informed and aims to find the animal slaughtering that maximizes profit, the accompanying number of summer grazing animals and the number of animals to be kept indoors during the winter. This problem is analyzed as an equilibrium harvesting problem with zero vegetation and animal growth.

In this two-stage model of lambs and ewes, the harvesting decision is determined by economic factors alone. For the given price and market conditions where lamb value is higher than the ewe value, lamb-only slaughtering at the highest possible level is the optimal strategy. On the other hand, the optimal lamb slaughter fraction is steered by sheep biological factors only. The reason for this sharp distinction between the effects of economic and biological forces is the lack of any density-dependent factors regulating sheep population growth. In contrast to this, and in line with standard bioeconomic harvesting theory, biological and economic factors jointly determine the optimal flock size and the vegetation utilization. Our stocking problem is analyzed when the farm capacity constraint binds, as well as not being binding. In the last case, as explained, the basic trade-off mechanism taking place is that for an initial low stocking rate, expansion and more animals added is beneficial as increased production in number of animals slaughtered more than outweighs reduced productivity in weight per slaughtered animal (lambs). At the optimum, the marginal meat income should be equal the marginal cost of keeping the stock plus the user cost of the vegetation pasture, evaluated by its shadow price.

The numerical illustrations indicate that shifting economic conditions for the farmer has small effects on the stocking rate and vegetation utilization. Such shifts, at least within the actual range of parameter values, basically spill over to changing farm profitability. On the other hand, we find vegetation productivity to have crucial allocation effects. For example, when comparing two equalized sized farms located in areas with different productivity, the farmer that benefits from high productivity will find it rewarding to keep a significant higher stocking rate than the other one. The high productivity farmer will benefit substantially economically as well.

Meat production only is included in our study as this account for most of the income of the Nordic sheep farmer. The rest comes from wool. When also adding the wool value, however, we will find that the slaughtering decision will be the same as without wool (see Skonhoft 2008). On the other hand, including the wool value will generally influence the optimal stocking decision and the optimal number of animals to be kept during the winter. For this reason, the pasture utilization will be affected as well. Adding more stages of the sheep population, and where natural mortality as well as fertility generally differ among the stages (Mysterud et al. 2002), also represents a possible extension that makes the analysis more realistic. However, such extension would neither change the principal working of our model as differences in the per animal economic value, and not natural mortality and fertility, steer the optimal slaughtering decision of the farmer.

Equilibrium harvesting only is analyzed in this paper. If not being at the optimal equilibrium stock size due to shifting economic and ecological conditions, or otherwise, the question arises how the farmer should adjust the slaughtering to reach equilibrium. If initially being below the optimal stocking rate, slaughtering below the equilibrium lamb slaughter rate should take place for a period of time. Due to the high animal growth potential, however, this adjustment period will typically be short, possibly only one year. For example, following the animal growth equation (1), we find the ewe number to increase with more than 60 percent within one year without any slaughtering (cf. parameter values Table A1). On the other hand, if the stock initially is too high stock relatively to the vegetation resources, some ewe harvesting, in addition to slaughtering all the lambs, should possibly be included if the farmer aims to adjust to the optimal equilibrium as rapid as possible. While it is quite simple to find harvesting policies leading to an equilibrium when initially being outside this equilibrium, it

is a more difficult to find optimal transitional paths leading toward equilibrium, or steady state. To find such paths a complete dynamic analysis of our farm problem is required.

## Appendix

### *Data and parameter values*

The sheep biological parameter values are based on Mysterud et al. (2002) and Aunsmo et al. (1998). Aunsmo and Nersten et al. (2003) and Asheim (2007) provide economic data. Prices and cost are in 2003-values. The vegetation parameter values builds on detailed animal food intake data, but also calibration based on weight and price data. First, the ewe slaughter weight is assumed to dominate the lamb weight for all vegetation quantities,

$w_X \geq w_Y = qg(V)$ . With  $Q$  as the vegetation carrying capacity, we simply assume

$w_X = qg(V = Q)$ . For the specific functional form  $g(V) = kV/(V + c)$ , this yields:

$$(a1) \quad w_X = qkQ/(Q + c).$$

Next, the lamb slaughter price is higher than the ewe slaughter price,  $p_Y qg(V) > p_X w_X$ .

Following the equilibrium optimization model (main text section five), the lowest possible vegetation quantity is  $V^{msy} = Q/2$ . Inserted into the vegetation consumption function  $g(V)$ ,  $p_Y qkQ/(Q + 2c)$  yields the lowest possible lamb price. Therefore,

$p_Y qkQ/(Q + 2c) > p_X w_X$  should hold. Inserted for (a1), we then find

$c < Q(p_Y / p_X - 1)/(2 - p_Y / p_X)$  after a small rearrangement. A necessarily and sufficient condition for an unique vegetation equilibrium is  $r > (k/c)(1 + b)X$  (main text section four).

After a small rearrangement and also using the above slaughter price constraint, we have:

$$(a2) \quad (k/r)(1 + b)X < c < Q(p_Y / p_X - 1)/(2 - p_Y / p_X).$$

Based on food intake data, the vegetation saturation parameter is fixed as  $k = 0.50$  (tonne vegetation biomass/animal) while the intrinsic vegetation productivity parameter value (baseline) is assumed to be  $r = 0.5$ . When further scaling the farm size through the vegetation carrying capacity given as  $Q = 500$  (tonne), and inserting for the slaughter price ratio and fertility (Table 2), condition (a2) reads  $2.53X < c < 375$ . As indicated above (section seven), the highest possible stocking rate is  $X = 108$  (ewes). Based on this inequality, the animal consumption shape baseline parameter value is simply scaled to  $c = 300$  (tonne vegetation

biomass). Finally, using (a1) and inserting for  $Q$ ,  $k$ ,  $c$  and  $w_x = 30$  (kg/animal) (Table A1), we find the value of the biomass translation parameter to be  $q = 96$  (kg meat/tonne vegetation biomass).

**Table A1: Baseline ecological and economic parameter values**

Parameter	Parameter description	Value
$s_Y$	-Natural survival fraction lambs	0.91
$s_X$	-Natural survival fraction ewes	0.95
$b$	-Fertility rate	1.53 (lamb/ewe)
$\psi$	-Sex fraction female lambs	0.50
$w_x$	-Adult (ewes) slaughter weight	30 (kg/animal)
$q$	-biomass translation parameter	96 (kg meat/ Tonne vegetation biomass)
$r$	-Intrinsic vegetation growth rate (pasture productivity)	0.50
$Q$	-Vegetation carrying capacity	500 (tonne vegetation biomass)
$k$	-Vegetation saturation parameter	0.50 (tonne vegetation biomass/ animal)
$c$	-Shape animal consumption parameter	300 (tonne vegetation biomass)
$p_x$	-Adult (ewe) slaughter price	35 (NOK/kg)
$p_Y$	-Lamb slaughter price	50 (NOK/kg)
$\alpha$	-Marginal cost winter	500 (NOK/animal)

Table note: Exchange rate: 1 Euro  $\approx$  8.70 NOK (Sept. 2009).

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